

$$\bar{v} = 4\pi A^3 \cdot \frac{1}{2 \left(\frac{1}{2} m \beta\right)^{1/2}} = 4\pi A^3 \cdot \frac{1}{\sqrt{\frac{1}{2} m \beta}}$$

$$= 4\pi \left(\frac{1}{2} m \beta\right)^{3/2} \cdot \frac{1}{2 \left(\frac{1}{2} m \beta\right)^{1/2}}$$

$$= \frac{2\sqrt{2}}{(Am\beta)^{1/2}} \cdot \frac{4^2 \cdot 1}{2 \cdot 1 \cdot 2^{3/2}} = \frac{2 \cdot 1 \cdot 4}{2^{1/2}} = \frac{8}{\sqrt{8}} = \sqrt{8}$$

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{2.55kT}{m}} \quad \text{--- (2)}$$

PROBABLE VELOCITY:

So more probable velocity is computed by

$$\frac{d}{dv} \left[ \frac{dN(v)}{dv} \right] = 0$$

$$\frac{d}{dv} \left[ 4\pi N A^3 v^2 e^{-\beta \left(\frac{1}{2} m v^2\right)} \right] = 0$$

$$4\pi N A^3 \frac{d}{dv} \left[ v^2 e^{-\beta \left(\frac{1}{2} m v^2\right)} \right]$$

$$4\pi N A^3 \left[ 2v e^{-\beta \left(\frac{1}{2} m v^2\right)} - m\beta \frac{2v}{2} e^{-\beta \left(\frac{1}{2} m v^2\right)} v^2 \right]$$

$$\left[ 4\pi N A^3 \left( 2v e^{-\beta \left(\frac{1}{2} m v^2\right)} - m\beta v^3 e^{-\beta \left(\frac{1}{2} m v^2\right)} \right) \right] = 0$$

$$v^2 = \frac{2}{m\beta} = \frac{2kT}{m} \quad \left[ 4\pi N A^3 e^{-\beta \left(\frac{1}{2} m v^2\right)} \right]$$

$$v_{\text{prob}} = \sqrt{\frac{2kT}{m}}$$

$$\left[ 2v - m\beta v^3 \right] = 0$$

$$4\pi N A^3 e^{-\beta \left(\frac{1}{2} m v^2\right)} = 0 \quad \&$$

$$2v - m\beta v^3 = 0$$

$$m\beta v^3 = 2v$$

$$v = \frac{2}{m\beta}$$

$$v_{rms} > v_{avr} > v_p \text{ at given temperature}$$

## MAXWELL BOLTZMAN ENERGY DISTRIBUTION :-

The molecules of a gas are moving randomly in all directions. It makes no difference whether we bound molecules speed or by their k.E. The number of molecules  $n(E)dE$  with k.E between  $E$  and  $E+dE$  is the same as number of molecules  $n(v)dv$  with speeds  $v$  and  $v+dv$ .

$$\begin{aligned} n(E)dE &= n(v)dv \\ n(E) &= n(v) \frac{dv}{dE} \quad \text{--- (1)} \end{aligned}$$

Since the molecules of ideal gas have only k.E

$$E = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E}{m}}$$

$$v = \frac{\sqrt{2}}{\sqrt{m}} E^{1/2}$$

$$dv = \frac{\sqrt{2}}{\sqrt{m}} \cdot \frac{1}{2} E^{-1/2} dE \quad \text{--- (2)}$$

putting value from (2) in (1)

$$n(E) = n(v) \sqrt{\frac{2}{m}} \cdot \frac{1}{2} E^{-\frac{1}{2}} \quad (3)$$

We know that

$$\frac{dN(v)}{dv} = 4\pi N A^3 v^2 e^{-\beta \left(\frac{1}{2} m v^2\right)}$$

$$= 4\pi N \frac{(m\beta)^3}{2^3 \pi^3} v^2 e^{-\beta \left(\frac{1}{2} m v^2\right)}$$

$$= 4\pi N \left(\frac{m\beta}{2\pi}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} \quad (4)$$

putting eq — (4) in — (3)

$$n(E) = 4\pi N \left(\frac{m}{\pi 2kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} \cdot \sqrt{\frac{2}{m}} \cdot \frac{1}{2} E^{-\frac{1}{2}}$$

$$= 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} \cdot \left(\frac{2E}{m}\right) e^{-\frac{E}{kT}} \sqrt{\frac{2}{m}} \cdot \frac{1}{2} E^{-\frac{1}{2}}$$

$$n(E) = \frac{2N}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} E^{1/2} \cdot e^{-\frac{E}{kT}}$$

This is called Maxwell Boltzmann energy distribution.

It is same for any gas at a given temperature and doesn't depend upon mass of molecules.

This distribution applies only to monoatomic gas because we assumed that gas molecules have translation at k.E.

Now we can calculate the fraction of gas molecule  $\frac{n(E)dE}{N}$  having energy b/w  $E$  and  $E+dE$  because

$n(E)$  is known. The total number of molecule is given as  $N = \int_0^\infty n(E)dE$

⇒ Internal Energy :-

Consider there are  $n(E)dE$  molecules with energy b/w  $E$  and  $E+dE$ . Their contribution to integral energy of gas will  $E = n(E)dE$

$$E = n(E)dE$$

The total of all such contribution gives the internal energy of the gas

$$E_{int} = \int_0^\infty E n(E) dE$$

$$= \int_0^\infty \frac{2N}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \cdot E^{3/2} e^{-\frac{E}{kT}} dE \quad \text{putting } n(E)$$

$$\text{putting } \frac{E}{kT} = U$$

$$\Rightarrow E = kT U$$

$$dE = kT dU$$

so above eq - becomes

$$= \frac{2N}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \int_0^\infty (kT)^{3/2} U^{3/2} e^{-U} (kT dU)$$