Date: 21-5-17

$$\frac{4\pi A^3 \times 1 \sqrt{\pi}}{4\left(\frac{1}{2}m\beta\right)^{3/2}} = 1$$

$$A^3\left(\frac{\pi}{\frac{1}{2}m\beta}\right)^{3/2} = 1$$

$$A^3 = \left(\frac{\frac{1}{2}m\beta}{\pi}\right)^{3/2}$$

$$A = \left(\frac{\frac{1}{2}m\beta}{\pi}\right)^{1/2} \quad \text{—— (20)}$$

To find β we exploit the definition of mean square velocity

$$\text{(B) —— } \bar{V_i}^2 = \frac{1}{N}\int_0^\infty v_i^2\, dN(v) \quad \text{statistical dd}$$

from eq (16) (17)

$$dN(v) = 4\pi N A^3 v^2 e^{-\beta(\frac{1}{2}mv^2)}\, dv$$

put in above eq

$$\bar{v}^2 = 4\pi A^3 \int_0^\infty v^4 e^{-\beta(\frac{1}{2}mv^2)}\, dv$$

Substituting for A^3

$$\bar{v}^2 = 4\pi\left(\frac{\frac{1}{2}m\beta}{\pi}\right)^{3/2}\int_0^\infty v^4 e^{-\beta(\frac{1}{2}mv^2)}\, dv$$

$$= \frac{4}{\sqrt{\pi}}\left(\frac{1}{2}m\beta\right)^{3/2}\int_0^\infty v^4 e^{-\beta(\frac{1}{2}mv^2)}\, dv$$

The value of the integral is

$$\frac{3}{8}\cdot\sqrt{\frac{\pi}{a^5}} = \frac{3}{8}\sqrt{\frac{\pi}{(\frac{1}{2}m\beta)^5}}$$

$$\text{so } \bar{v}^2 = \frac{4}{\sqrt{\pi}}\left(\frac{1}{2}m\beta\right)^{3/2}\frac{3}{8}\sqrt{\frac{\pi}{(\frac{1}{2}m\beta)^5}}$$

Important:

Date:

$$\bar{v}^2 = \frac{3}{2\,\frac{1}{2}m\beta} = \frac{3}{m\beta} \quad \text{— (27)}$$

Putting the value in the relation

$$\frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT$$

$$\frac{1}{2}m\left(\frac{3}{m\beta}\right) = \frac{3}{2}kT$$

$$\frac{1}{\beta} = kT$$

$$\beta = \frac{1}{KT} \quad \text{— (28)}$$

combining eq (27) & (28) we get

$$\bar{v}^2 = \frac{3KT}{m}$$

$$v_{r.m.s} = \sqrt{\frac{3KT}{m}} \quad \text{— (29)}$$

using value of A & β in eq (17) (18)

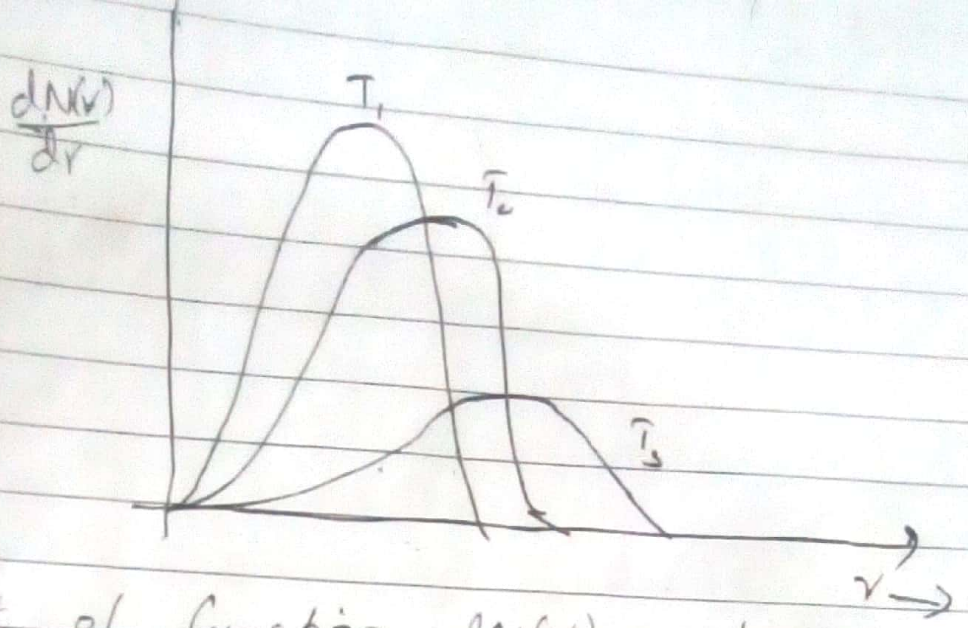
$$dN(v) = 4\pi N\left(\frac{\frac{1}{2}m\frac{1}{KT}}{\pi}\right)^{3/2} v^2 e^{-\frac{1}{KT}\left(\frac{1}{2}m\bar{v}^2\right)} dv$$

$$= \frac{4N}{\sqrt{\pi}}\left(\frac{m}{2KT}\right)^{3/2} \frac{3KT}{m} e^{-\frac{1}{KT}\left(\frac{1}{2}m\bar{v}^2\right)}$$

and $\boxed{dN(v_x) = \frac{N}{\sqrt{\pi}}\left(\frac{m}{(2kT)^{1/2}} e^{-\frac{mv_x^2}{2KT}}\right) dv_x}$ — (30)

$$= \frac{4N}{\sqrt{\pi}} \frac{m^{3/2-1}\, 3}{2^{3/2}(KT)^{3/2-1}} e^{-\frac{1}{KT}\left(\frac{1}{2}m\bar{v}^2\right)}$$

$$= \frac{6}{\sqrt{2}} \frac{m^{1/2}}{(KT)^{1/2}} e^{-m}$$



A plot of function $\frac{dN(v)}{dv}$ at three different temperatures $T_1 < T_2 < T_3$ is shown in figure.

The areas under the three curves are the same since the area represents the total number of molecules. It follows from the graph that velocity ~~speed~~ spread is wider at larger temperature.

⇒ EVALUATION OF AVERAGE & MOST PROBABLE VELOCITIES:

The average velocity of the molecules is defined by the relation

$$\bar{v} = \frac{1}{N}\int_0^\infty v \, dN(v)$$

$$= 4\pi A^3 \int_0^\infty v^3 e^{-\beta(\frac{1}{2}mv^2)} \, dv$$

where the definite integral is of the form $\int_0^\infty x^n e^{-ax^2} dx$ with $n = 3$ and $a = \frac{1}{2}m\beta$ & its value is

$$\frac{1}{2a^2} = \frac{1}{2(\frac{1}{2}m\beta)^2}$$

Important: Therefore