→ Reasoning as before the number of molecules simultaneously having velocities in the range v_x and $v_x + dv_x$, v_y and $v_y + dv_y$, v_z and $v_z + dv_z$ is given by

$$dN^3(v_x, v_y v_z) = N f(v_x) f(v_y) f(v)_z \, dV_x dV_y dV_z \quad \text{—— (4)}$$

where $dN^3(v_x, v_y, v_z)$ is a differential of 3rd ord

eq (3) the number of endpoints in the volume elements $(dv_x\, dv_y\, dv_z)$ and (density of pointin (points is unit volume). n)

velocity space is given by

$$\rho = \frac{dN^3(v_x, v_y v_z)}{dV_x \cdot dV_y\, dV_z}$$

$$= N f(v_x) f(v_y) f(v_z) \quad \text{—— (5)}$$

Since velocity is isotropic (same structure), density should be constant at all points on the sphere

$$v_x^2 + v_y^2 + v_z^2 = v^2$$ of radius v

is the velocity space.

Hence $N f(v_x) f(v_y) f(v_z) = \text{constant}$ —— (6)

when $v_x^2 + v_y^2 + v_z^2 = \text{constant}$ —— (7)

The eq (7) is called the eq of constraint as it reduces independent variables from three to two.

Important: Partial differentiation of the eq (6) & (7)

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gives $f(v_y)f(v_z)\frac{\partial f(v_x)}{\partial v_x}dv_x + f(v_x)f(v_z)\frac{\partial f(v_y)}{\partial f(v_y)}$
$dv_y + f(v_x)f(v_y)\frac{\partial f(v_z)}{\partial f(v_z)}dv_z = 0$

Representing the derivatives by prime notation & dividing through out by $f(v_x)f(v_y)f(v_z)$

$$\frac{f'(v_x)}{f(v_x)}dv_x + \frac{f'(v_y)}{f(v_y)}dv_y + \frac{f'(v_z)}{f(v_z)}dv_z = 0 \quad \text{—— (8)}$$

Now from eq (7)

$$2v_x dv_x + 2v_y dv_y + 2v_z dv_z = 0$$
$$(v_x dv_x + v_y dv_y + v_z dv_z) = 0 \quad \text{—— (9)}$$

let us multiply the eq (9) with $m\beta$ called Laggrange undetermined multiplier. (where m is mass of molecule and β some arbitrary unknown function) and add to equation (8)

$$\left[\frac{f'(v_x)}{f(v_x)} + m\beta v_x\right]dv_x + \left[\frac{f'(v_y)}{f(v_y)} + m\beta v_y\right]dv_y$$
$$+\left[\frac{f'(v_z)}{f(v_z)} + m\beta v_z\right]dv_z = 0 \quad \text{—— (10)}$$

Since eq (10) is an identity and for this identity to hold all the terms on the L.H.S of above eq should seperately vanish

i.e

$$\left[\frac{f'(v_x)}{f(v_x)} + m\beta v_x\right]dv_x = 0 \quad \text{—— (11)}$$

$$\left[\frac{f'(v_y)}{f(v_y)} + m\beta v_y\right]dv_z = 0 \quad \text{—— (12)}$$

Important:

$$\left[\frac{f'(v_z)}{f(v_z)} + m\beta v_z\right] dv_z = 0 \quad \text{--- (13)}$$

lit us consider eq (11)

$$\left[\frac{f'(v_x)}{f(v_x)} + m\beta v_x\right] dv_x = 0$$

$$\frac{f'(v_x)}{f(v_x)} dv_x + m\beta v_x \, dv_x = 0$$

$$\frac{f'(v_x)}{f(v_x)} dv_x = -m\beta v_x \, dv_x \quad \text{--- (14)}$$

Integrating it we get

$$\int \frac{f'(v_x)}{f(v_x)} dv_x = -\int m\beta v_x \, dv_x$$

$$\ln|f(v_x)| = -\frac{m\beta v_x^2}{2} + \ln A$$

$$\ln|f(v_x)| - \ln A = -\frac{m\beta}{2} v_x^2$$

$$\ln\left|\frac{f(v_x)}{A}\right| = -\frac{m\beta v_x^2}{2}$$

$$\frac{f(v_x)}{A} = e^{-\frac{1}{2} m\beta v_x^2}$$

$$\left.\begin{aligned} f(v_x) &= A e^{-\frac{1}{2} m\beta v_x^2} \\ \text{Similarly} \quad & \\ f(v_y) &= A e^{-\frac{1}{2} m\beta v_y^2} \\ f(v_z) &= A e^{-\frac{1}{2} m\beta v_z^2} \end{aligned}\right\} \quad \text{--- (15)}$$

setting these values in eq (5)

$$\rho(v) = NA\, e^{-\frac{1}{2}m\beta v_x^2} \cdot A e^{-\frac{1}{2}m\beta v_y^2} \cdot A e^{-\frac{1}{2}m\beta v_z^2}$$

$$= NA^3\, e^{-\frac{1}{2}m\beta(v_x^2+v_y^2+v_z^2)}$$

$$= NA^3\, e^{-\frac{1}{2}m\beta v^2}$$

$$= NA^3\, e^{-\beta(\frac{1}{2}mv^2)} \quad \text{—— (6)}$$

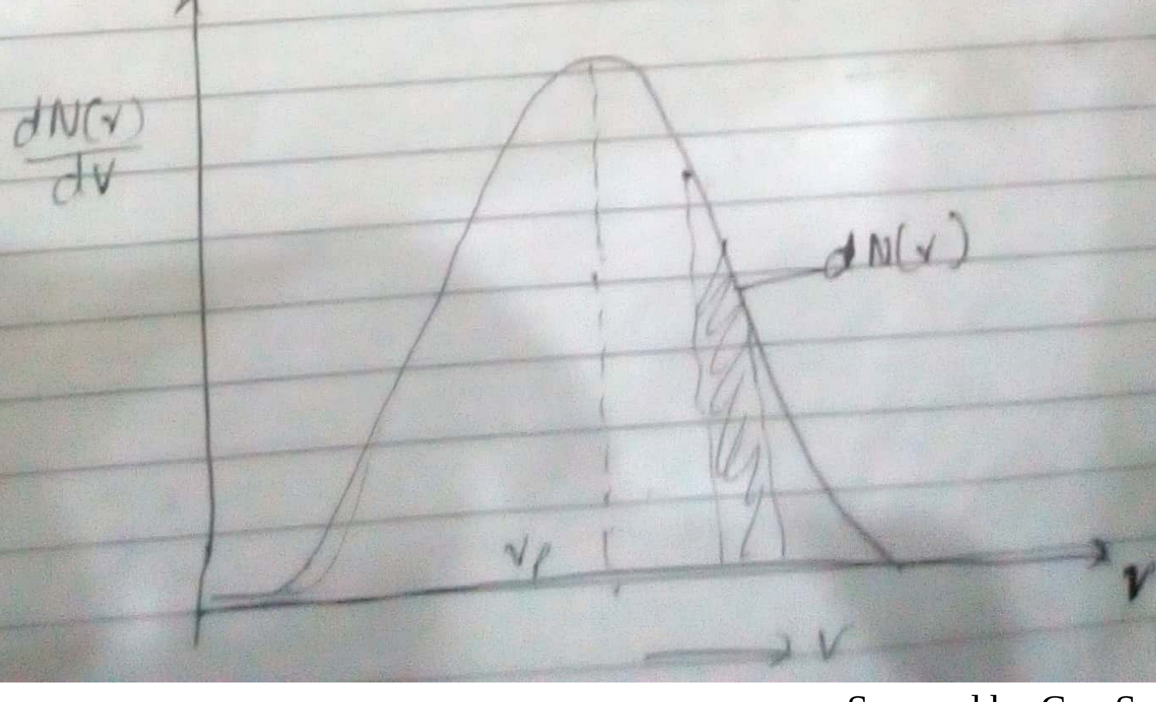
$= \cancel{NA^3}$

But ρ is constant at all points within the infatisenal spherical shell enclosed by the surface of radii v and $v+dv$ in the velocity space.
the volume of shell is $(4\pi v^2)dv$ which contains velocity point of $dN(v)$ molecules hence eq (4) can be put in the form

$$\rho(v) = \frac{dN(v)}{4\pi v^2 dv} = NA^3 e^{-\beta(\frac{1}{2}mv^2)}$$

$$\boxed{\frac{dN(v)}{dv} = 4\pi N A^3 v^2 e^{-\beta(\frac{1}{2}mv^2)}} \quad \text{—— (7)}$$

which is the Maxwell law of velocities distribution plotted as shown in fig



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The shaded area gives the number of molecules in the range v & $v+dv$.

N_v

→ The number of molecules having x components of velocity in the range v_x & v_x+dv_x & it can be obtained by combining eq (1) & (15) thus

$$dN(v_x) = NAe^{-\beta(\frac{1}{2}mv_x^2)}dv_x \quad \text{—(18)}$$

Evaluation of β and A:-

we use that fact

$$\int_0^\infty dN(v) = N \quad \text{(total number of molecules)}$$

Substituting for $dN(v)$ for eq (17)

$$\int_0^\infty 4\pi NA^3 v^2 e^{-\beta(\frac{1}{2}mv^2)} dv = N$$

$$\int_0^\infty 4\pi A^3 v^2 e^{-\beta(\frac{1}{2}mv^2)} = 1 \quad \text{—(19)}$$

The integral on L.H.S of the form

$$\int_0^\infty x^n e^{-ax^2} dx$$

when $n=2$ $\quad a = \frac{1}{2}m\beta$ & its value is

$$\frac{1}{4}\sqrt{\frac{\pi}{a^3}} = \frac{1}{4}\sqrt{\frac{\pi}{(\frac{1}{2}m\beta)^3}} =$$

$$= \frac{1}{4}\frac{\sqrt{\pi}}{(\frac{1}{2}m\beta)^{3/2}}$$

using this value in above relation (19) we get