LECTURE 10

Robust Design and Taguchi Methods

1. Robust Design ala Taguchi

G. Taguchi, a Japanese engineer, had a big effect on quality control and experimental design in the 1980s and 1990s.

Let's begin with the sources of product variation. Some example of the random influences or "noise," that affect a product's characteristics are

1. Manufacturing
    - Operator
    - raw materials
    - machine settings
    - environmental
2. Environmental (the customer's environment)
    - temperature
    - humidity
    - dust
    - load
3. Product Deterioration (Aging)

Taguchi suggested that "quality" should be thought of, not as a product being inside or outside of specifications, but as the variation from the target. Variation from the target can be broken into two components, production variation, and bias.
Picture

To quantify quality loss, write T for the target value and Y for the measured value. We want $E(Y) = T$. Write $L(Y)$ for the loss (in dollars, reputation, customer satisfaction, ...) for deviation of Y from T. A popular choice for the loss function is

$$L(Y) = k(Y - T)^2,$$

where k is some constant. If $E(Y)$ really is T, then $E(L(Y)) = k\sigma^2$, where $\sigma^2 = \mathrm{Var}(Y)$.

If the product is off target, so that $E(Y) = T + d$, then $E(L(Y)) = k(\sigma^2 + d^2)$.

Now consider the product development stages at which countermeasures against various sources of variation can be built into the product.

Development Stages	Sources of Variation: Environmental Variables	Product Deterioration	Manufacturing Variations
Product Design	O	O	O
Process Design	X	X	O
Manufactoring	X	X	O

O—Countermeasures possible
X—Countermeasures impossible

We can think of quality control during manufacturing as on-line quality control, while quality control efforts during product design and process design are off-line quality control. There are potentially bigger payoffs from off-line quality control.

Consider the example from Bell Labs. In designing a power circuit which is to have a target output voltage of 115V.

Draw picture.

Voltage depends on transistor gain nonlinearly. The engineers set the transistor gain at 350, where the voltage response curve was fairly flat. Then they adjusted the resistor to return the voltage to the target of 115.

Taguchi recommended a two-step design process, robust design followed by tolerance design. Robust Design is a technique that reduces variation in a product by reducing the sensitivity of the design of the product to sources of variation rather than by controlling their sources. Tolerance Design is concerned with how much variation of the design and noise factors is permissible. It is a method for determining tolerances that minimizes the sum of product manufacturing and lifetime costs. The basic idea is to set tolerances around nominal settingss identified by parameter design, not by convention.

One must first identify the control parameters. These are sometimes also called design parameters. They are the product design characteristics whose nominal settings can be specified by the product designers.

Next, one must state the problem and objectives. It works best to have a session that includes all the interested parties, and not to work in isolation.

Here are the operational steps for robust design.

1. State your problem and objective.
2. List responses, control parameters, and sources of noise.
3. Plan the experiment.
4. Run experiment and predict improved parameter settings.
5. Run confirmation experiment.

If the objective is not met, then it's back to step (2). Otherwise, you can adopt the improved design.

For the response variable, one begins by identifying important measures that are being targeted. One also wants a numerical representation of variability attributed to "noise." It is sometimes called a performance statistic. Two commonly used measures are

$$-\log(s^2),$$

where s^2 is the sample variance, and

$$\log(\text{mean}^2/s^2).$$

In general, the higher the performance measure the better.

Because of the need to estimate s^2, we are led to designs with two aspects. First one creates a design with design parameters, called by Taguchi the Inner Array. Then one creates the array of noise factors, called by Taguchi the Outer Array. Thus for each setting in the design matrix, one runs one run for each setting in the noise matrix. The measurements of the performance characteristic are then grouped to give the performance statistic for that run of the design matrix.

The Inner Array is generally a nearly-saturated array, usually a fractional factorial or Plackett-Burman design. The same kind of designs are used for for the Outer Array.

In the analysis, we'll look for factors that affect the targeted response only, those that affect variability only, those that affect both, and those that affect neither.

Let's consider an example from Bell Labs. The first step in silicon wafer fabrication is the growth of a smooth epitaxial layer onto a polished silicon wafer. The epitaxial layer is deposited on wafers while they are mounted on a rotating spindle called a susceptor. The problem: high drop-out rate caused by deviation in thickness, both between and within wafers, from the target value of 14.5 microns. Objective: reduce nonuniformity of epitaxial layer, and keep average thickness close to 14.5 microns. List responses: Epitaxial thickness, with a target value of 14.5 microns, a current average of 14.5, and a current std dev of 0.4. Control parameters: susceoptor rotation direction, arsenic flow rate, deposition time, nozzle position, and deposition temperature. List noise: uneven temperature in Bell Jar, nonuniform vapor concentration, nonuniform vapor composition, deviation in control parameter settings.

After settling on a 2^{5-2} factorial for the control variables, the experimenters placed four wafers with five sampling points each in the susceptor. In this wasy, they got a total of 20 measurements of epitaxial thickness, using the same plan for every run.

For each run, they calculated the mean, the variance, and the "robustness statistics," namely $-\log(\text{variance}/\text{mean}^2)$. The results were

Run	Rot Dir	As flow rate	dep temp	dep time	nozzle	mean	robustness stat
1	clock	55	1210	low	2	13.860	2.780
2	clock	55	1210	high	6	14.888	3.545
3	clock	59	1220	low	6	14.037	3.725
4	clock	59	1220	high	2	14.757	2.665
5	osci	55	1220	low	6	13.914	3.149
6	osci	55	1220	high	2	14.415	2.631
7	osci	59	1210	low	2	13.972	2.637
8	osci	59	1210	high	6	14.878	2.961

A larger robustness statistic corresponds to improved performance.

Draw pictures.

There are a number of issues that go into the choice of experimental design. These include

1. Number of levels of factors
2. Number of factors
3. Factor Interactions
4. Modeling versus Pick the Winner

The experimental designs commonly used include

1. Orthogonal Arrays
2. Plackett-Burman Designs
3. Fractional factorials
4. Response surface designs

Incidentally, the phrase Taguchi Methods is a trademark held by the American Supplier Institute. It encompasses all work due to Taguchi including quality engineering methods such as

1. Parameter Design
2. Tolerance Design
3. On-line quality control
4. The loss function
5. Signal-to-noise ratio

According to the ASI, if an experiment adheres to the following guidelines then it is a Taguchi experiment:

1. Best selection of quality characteristics (go/no go is not good).
2. Maximum possible number of control factors.
3. Comparison of existing conditions with predicted optimum
4. Use of signal-to-noise ratios.
5. Use of loss function.
6. Minimum interactions among control factors.
7. Control factors and noise factors separated.
8. Use of orthogonal arrays.

Strong Points of Taguchi Methods

1. Think about quality regarding closeness to target, not specification limits.
    - target
    - loss function
2. Transmission of error.
3. Analyze variation in addition to location.

Issues of Concern about Taguchi Methods and Taguchi Experiments

1. Blind use of orthogonal arrays.
2. No regard for interactions.
3. “Analysis” leads toward “pick-the-winner” rather than modeling.
4. Signal-to-noise ratio.

Summary

1. Examine both target and variability.
2. Find settings of factors to reduce sensitivity of product to fluctuations in noise parameters.

3. Use experimental design to determine factors that influence variability at target settings.
4. Attention needs to be paid to interactions.
5. Additional attention needs to be given to the interactive nature of experimentation.
6. Additional analysis should be placed on modeling system.

2. Improving Robust Design from a statistical point of view

2.1. Experimental Designs. Although the designs for the inner array and the outer array are economical individually, when they are crossed together they are not as economic.

For example, consider the crossed array when control factors A, B, C, in a 2^{3-1} crossed with another 2^{3-1} using D, E, and F in the outer array. The sixteen observations end up with one degree of freedom each for: A, B, C, D, E, F, AD, AE, AF, BD, BE, BF, CD, CE, CF. In other words, all the degrees of freedom go to main effects and noise $\times$ control interactions. These interactions are crucial, but interactions among the control variables may be just as crucial.

We would like a design to allow estimability of a reasonable model in both control and noise variables. To do that, we must first specify a reasonable model.

If we write x for the control variables and their settings, and z for the noise variables, then we might consider the response surface model

$$y(x,z) = \beta_0 + x'\beta + x'Bx + z'\gamma + x'\Delta z + \epsilon.$$

Notice that noise$\times$noise interactions are left out here.

We could use a crossed design for this model, but we could also use a "combined array," chosen specifically for this sort of model. The designs generally offer the concept of mixed resolution. For example, suppose there are three control factors A, B, C and three noise factors D, E, F. The usual design would be a crossed array

$$2^{3-1}_{III} \times 2^{3-1}_{I} II$$

with a total of 16 runs. We could view this as a single array with defining relations $I = ABC = DEF = ABCDEF$. A better alternative with the same number of runs would be a 2^{6-2} factorial with defining relations $I = ABCD = DEF = ABCDE$. This is resolution III for noise$\times$noise interactions and resolution IV for other interactions.

For a second order example, suppose we have 3 control variables and 2 noise variables. The crossed array might be a 1/3 fraction of a 3^3 and the noise array a 2^2, giving a total of 36 runs. A CCD with a 2^{5-1}, axial points in the control variables, and n_c center runs will total $22 + n_c$ runs.

2.2. Analysis. Taguchi's signal-to-noise approach, although easy to understand, is really sub-optimal. Modeling both the mean response and variance directly seems a much better idea. One can do this by:

1. Modeling the response as a function of control and noise variables, and then calculating the variance function from that, or

2. Take advantage of the crossed design, to calculate the variance at each control variable design point, and then simultaneously model the mean response and the variance. A classic paper by Bartlett and Kendall in 1946 suggests that using a log-linear model of the form

$$\log s_i^2 = x_i'\gamma + \epsilon,$$

will have approximately normal errors with constant variance.

Returning to the first point, from the given model the expected response, with the expectation taken over the distribution of the noise parameters, is

$$E(y(x,z)) = \beta_0 + x'\beta + x'Bx.$$

The variance is

$$\mathrm{Var}(y(x,z)) = \mathrm{Var}(z'\gamma + x'\Delta z) + \sigma^2 = \sigma_z^2(\gamma' + x'\Delta)(\gamma + \Delta' x) + \sigma^2).$$

Note that $(\gamma' + x'\Delta) = \partial y/\partial z$.

The text covers this point and many others quite thoroughly in Chapter 10.