5 Nested Designs and Nested Factorial Designs

5.1 Two-Stage Nested Designs

- The following example is from *Fundamental Concepts in the Design of Experiments* (C. Hicks). In a training course, the members of the class were engineers and were assigned a final problem. Each engineer went into the manufacturing plant and designed an experiment. One engineer studied the strain (stress) of glass cathode supports on the production machines:
  - There were 5 production machines (fixed effect).
  - Each machine has 4 components called 'heads' which produces the glass. The heads represent a random sample from a population of heads (random effect).
  - She took 4 samples from each. Data collection of the $5 \times 4 \times 4 = 80$ measurements was completely randomized. The data is presented in the table below:

	Machine																			
Head	A				B				C				D				E			
1	6	13	1	7	10	2	4	0	0	10	8	7	11	5	1	0	1	6	3	3
2	2	3	10	4	9	1	1	3	0	11	5	2	0	10	8	8	4	7	0	7
3	0	9	0	7	7	1	7	4	5	6	0	5	6	8	9	6	7	0	2	4
4	8	8	6	9	12	10	9	1	5	7	7	4	4	3	4	5	9	3	2	0

She analyzed the data as a two-factor factorial design. **Is this correct?**

  - To be a two-factor factorial design, the same 4 heads must be used in each of the 5 machines. This was not the case. The 4 heads in Machine A are different from the 4 heads in Machine B, and so on. 20 different heads were used in this experiment (not 4).
  - Therefore, we **do not have a factorial experiment**. When the levels of a factor are unique to the levels of one or more other factors, we have a **nested factor**. In this experiment, we say the "heads are nested within machines".

- A proper format for presenting the data is in the following table:

	Machine																			
	A				B				C				D				E			
Head	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	6	13	1	7	10	2	4	0	0	10	8	7	11	5	1	0	1	6	3	3
	2	3	10	4	9	1	1	3	0	11	5	2	0	10	8	8	4	7	0	7
	0	9	0	7	7	1	7	4	5	6	0	5	6	8	9	6	7	0	2	4
	8	8	6	9	12	10	9	1	5	7	7	4	4	3	4	5	9	3	2	0
Head $\sum$	16	33	17	27	38	14	21	8	10	34	20	18	21	26	22	19	21	16	7	14
Machine $\sum$	93				81				82				88				58			

- The design for the previous experiment is an example of a **two-stage nested design**. The factor in the first stage is Machine. The nested factor in the second stage is head within machine (denoted Head(Machine)).

- Notation for a balanced two-stage nested design with factors A and $B(A)$.

  $a =$ number of levels of factor A

  $b =$ number of levels of factor B within the i^{th} level of factor A

  $n =$ number of replicates for the j^{th} level of B within the i^{th} level of A

- A two-stage nested design can also be unbalanced with
  - Unequal b_i $(i = 1, 2, \ldots, a)$ where b_i is the number of number of levels of factor B within the i^{th} level of factor A, or
  - Unequal n_{ij} where n_{ij} is the number of replicates within the j^{th} level of factor B and the i^{th} level of factor A
- Statistical software (such as *SAS*) can easily handle the unbalanced case. We will initially focus on the balanced case.

5.1.1 The Two-Stage Nested Effects Model

- The **two-stage nested effects model** is:

$$y_{ijk} \quad = \qquad\qquad (36)$$

where μ is the overall mean, α_i is the i^{th} factor A effect,

$\beta_{j(i)}$ is the j^{th} effect of factor B nested within the i^{th} level of factor A,

ϵ_{ijk} is the random error of the k^{th} observation from the j^{th} level of B within the i^{th} level of A.

We assume $\epsilon_{ijk} \sim IID\, N(0, \sigma^2)$.

- If we impose the constraints

$$\sum_{i=1}^{a} \alpha_i = 0 \qquad \sum_{j=1}^{b} \beta_{j(i)} = 0 \quad \text{for } i = 1, 2, \ldots, a \qquad (37)$$

then the least squares estimates of the model parameters are

$$\widehat{\mu} \;= \qquad\qquad \widehat{\alpha}_i \;= \qquad\qquad \widehat{\beta}_{j(i)} \;=$$

- If we substitute these estimates into (36) we get

$$y_{ijk} \;=\; \widehat{\mu} \;+\; \widehat{\alpha}_i \;+\; \widehat{\beta}_{j(i)} \;+\; e_{ijk}$$

$$=\; \overline{y}_{\cdots} \;+\; (\overline{y}_{i\cdot\cdot} - \overline{y}_{\cdots}) \;+\; (\overline{y}_{ij\cdot} - \overline{y}_{i\cdot\cdot}) \;+\; e_{ijk}$$

where e_{ijk} is the k^{th} residual from the $(i,j)^{th}$ nested treatment. Thus $e_{ijk} \;=$.

Notation for an ANOVA

- $SS_A = nb \sum_{i=1}^{a} (\overline{y}_{i\cdot\cdot} - \overline{y}_{\cdots})^2$ = the sum of squares for A $(df = a - 1)$
- $MS_A = SS_A/(a-1)$ = the mean square for A
- $SS_{B(A)} = n \sum_{i=1}^{a} \sum_{j=1}^{b} (\overline{y}_{ij\cdot} - \overline{y}_{i\cdot\cdot})^2$ = the sum of squares for B nested within A $(df = a(b-1))$
- $MS_{B(A)} = SS_B/[a(b-1)]$ = the mean square for B nested within A

- $SS_E = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(y_{ijk} - \overline{y}_{ij.}\right)^2$ = the error sum of squares ($df = ab(n-1)$)
- $MS_E = SS_E/ab(n-1)$= the mean square error
- $SS_T = \displaystyle\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \overline{y}_{...})^2$ = the total sum of squares ($df = abn - 1$)
- Like previous designs, the total sum of squares for the two factor CRD is partitioned into components corresponding to the terms in the model:

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \overline{y}_{...})^2 = nb\sum_{i=1}^{a}(\overline{y}_{i..} - \overline{y}_{...})^2 + n\sum_{i=1}^{a}\sum_{j=1}^{b}(\overline{y}_{ij.} - \overline{y}_{i..})^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \overline{y}_{ij.})^2$$

OR

$$SS_T = SS_A + SS_{B(A)} + SS_E$$

- The alternate SS formulas for the balanced two stage nested design are:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn} \qquad SS_A = \sum_{i=1}^{a}\frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn} \qquad SS_{B(A)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\left(\frac{y_{ij.}^2}{n} - \frac{y_{i..}^2}{bn}\right)$$

$$SS_E = SS_T - SS_A - SS_{B(A)}$$

ANOVA Table for Two-Stage Nested Design

Source of Variation	Sum of Squares	d.f.	Mean Square	F Ratio
A	SS_A	$a-1$	$MS_A = SS_A/(a-1)$	$F_A =$ (see ‡ below)
$B(A)$	$SS_{B(A)}$	$a(b-1)$	$MS_B = SS_{B(A)}/[a(b-1)]$	$F_B = MS_{B(A)}/MS_E$
Error	SS_E	$ab(n-1)$	$MS_E = SS_E/[ab(n-1)]$	——
Total	SS_{total}	$abn-1$	——	——

‡ If $B(A)$ is a fixed factor then $F_A = MS_A/MS_E$
If $B(A)$ is a random factor then $F_A = MS_A/MS_{B(A)}$

- To estimate variance components, we use the same approach that was used for the one- and two-factor random effects models:

  If A and $B(A)$ are random, replace $E(MS_A)$, $E(MS_{B((A)})$, and $E(MS_E)$ in the expected means square equations with the calculated values of MS_A, $MS_{B(A)}$, and MS_E.

- Solving the system of equations produces estimates of the variance components:

$$\widehat{\sigma}^2 = MS_E \qquad \widehat{\sigma}_\beta^2 = \frac{MS_{B(A)} - MS_E}{n} \qquad \widehat{\sigma}_\alpha^2 = \frac{MS_A - MS_{B(A)}}{bn}$$

- Consider the example with factor A = Machines and nested factor $B(A)$ = Heads(Machines). The following table summarizes totals for for the levels of A and $B(A)$:

	Machine A				Machine B				Machine C				Machine D				Machine E			
Head $y_{ij.}$	16	33	17	27	38	14	21	8	10	34	20	18	21	26	22	19	21	16	7	14
Machine $y_{i..}$	93				81				82				88				58			

- Then the sums of squares are:

$$SS_T = (6^2 + 2^2 + \cdots + 4^2 + 0^2) - \frac{402^2}{80} =$$

$$SS_A = \frac{93^2 + 81^2 + 82^2 + 88^2 + 58^2}{16} - \frac{402^2}{80} =$$

$$\begin{aligned} SS_{B(A)} = & \left(\frac{16^2 + 33^2 + 17^2 + 27^2}{4} - \frac{93^2}{16}\right) + \left(\frac{38^2 + 14^2 + 21^2 + 8^2}{4} - \frac{81^2}{16}\right) \\ & + \left(\frac{10^2 + 34^2 + 20^2 + 18^2}{4} - \frac{82^2}{16}\right) + \left(\frac{21^2 + 26^2 + 22^2 + 19^2}{16} - \frac{88^2}{16}\right) \\ & + \left(\frac{21^2 + 16^2 + 7^2 + 14^2}{4} - \frac{58^2}{16}\right) \\ = & \; 50.1875 + 126.1875 + 74.75 + 6.50 + 25.25 = \end{aligned}$$

$$SS_E = 969.95 - 45.075 - 282.875 =$$

ANOVA Table for Two-Stage Nested Design Example

Source of Variation	Sum of Squares	d.f.	Mean Square	F Ratio	p-value
Machines	45.075	4	11.269	$F_A = 0.60$	.6700
Heads(Machine)	282.875	15	18.858	$F_B = 1.76$	.0625
Error	642	60	10.70		
Total	969.95 79				

- Both F-tests are not significant at the $\alpha = .05$ significance level. The F-test for the Head(Machine) is significant, however, at the $\alpha = .10$ level.
- From the residual diagnostic plots, we see there are no serious problems with the homogeneity of variance (HOV) and the normality assumptions.
- To perform Levene's HOV Test, use the same approach presented with a two-factor factorial design: Create a single factor with one level for each combination of factors.
  - For this example, there are 20 Heads within Machine combinations. Levene's Test would compare the 20 sample variances.
  - The *SAS* code contains an example of using Levene's Test.

TWO-STAGE NESTED DESIGN (HICKS P.173-178)

The GLM Procedure

Class Level Information		
Class	**Levels**	**Values**
machine	5	A B C D E
head	20	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Variable: strain

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	19	327.9500000	17.2605263	1.61	0.0823
Error	60	642.0000000	10.7000000		
Corrected Total	79	969.9500000			

R-Square	Coeff Var	Root MSE	strain Mean
0.338110	65.09623	3.271085	5.025000

Source	DF	Type III SS	Mean Square	F Value	Pr > F
machine	4	45.0750000	11.2687500	1.05	0.3876
head(machine)	15	282.8750000	18.8583333	1.76	0.0625

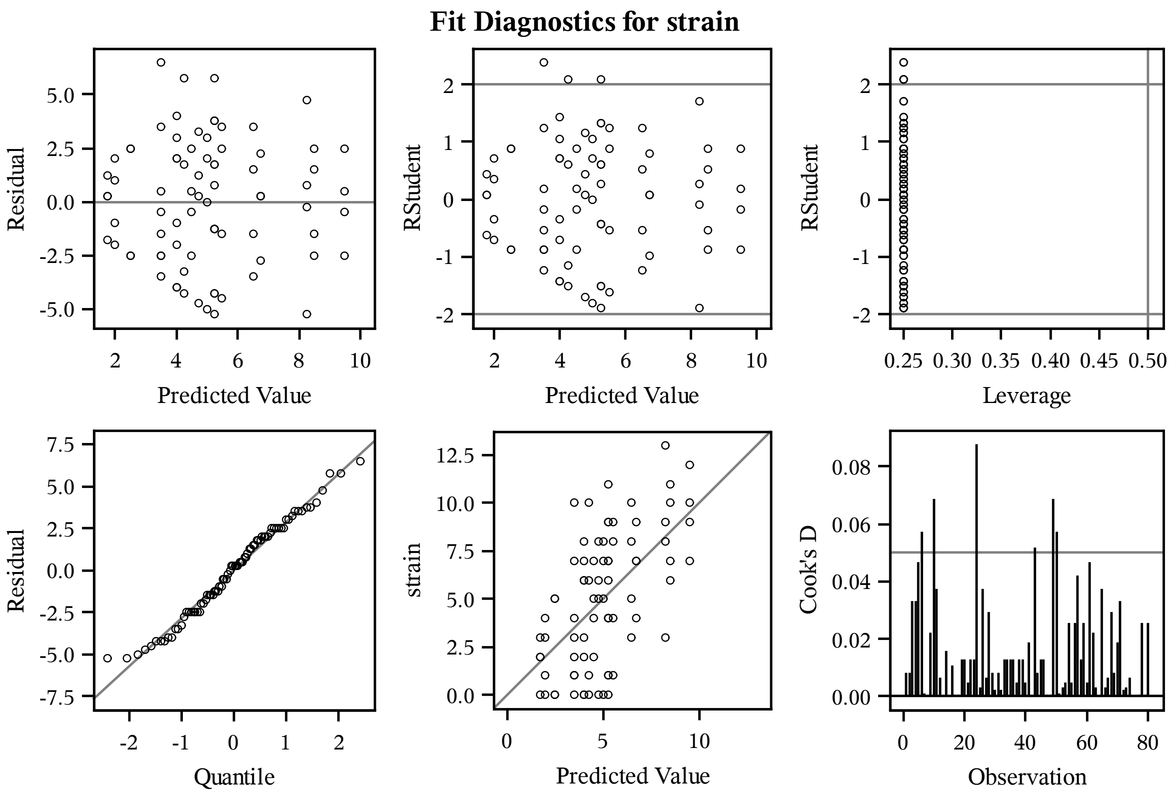
Source	Type III Expected Mean Square
machine	Var(Error) + 4 Var(head(machine)) + Q(machine)
head(machine)	Var(Error) + 4 Var(head(machine))

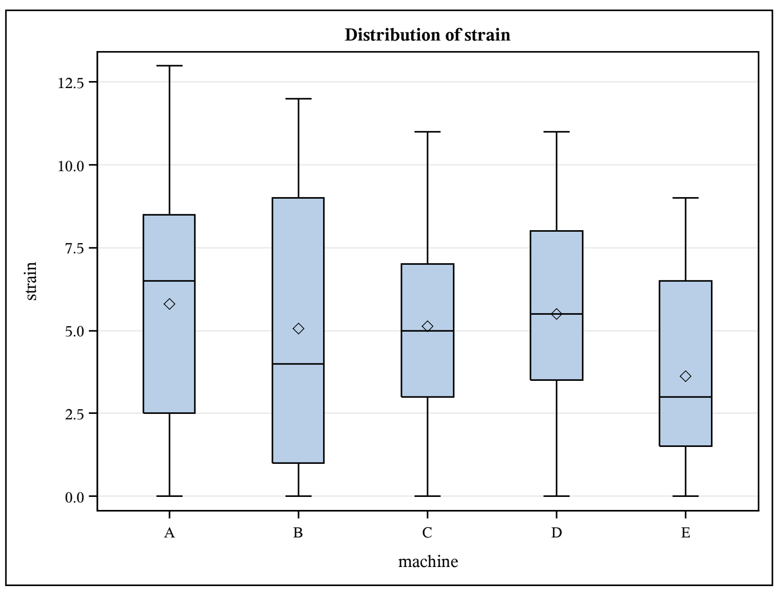
The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance

Variable: strain

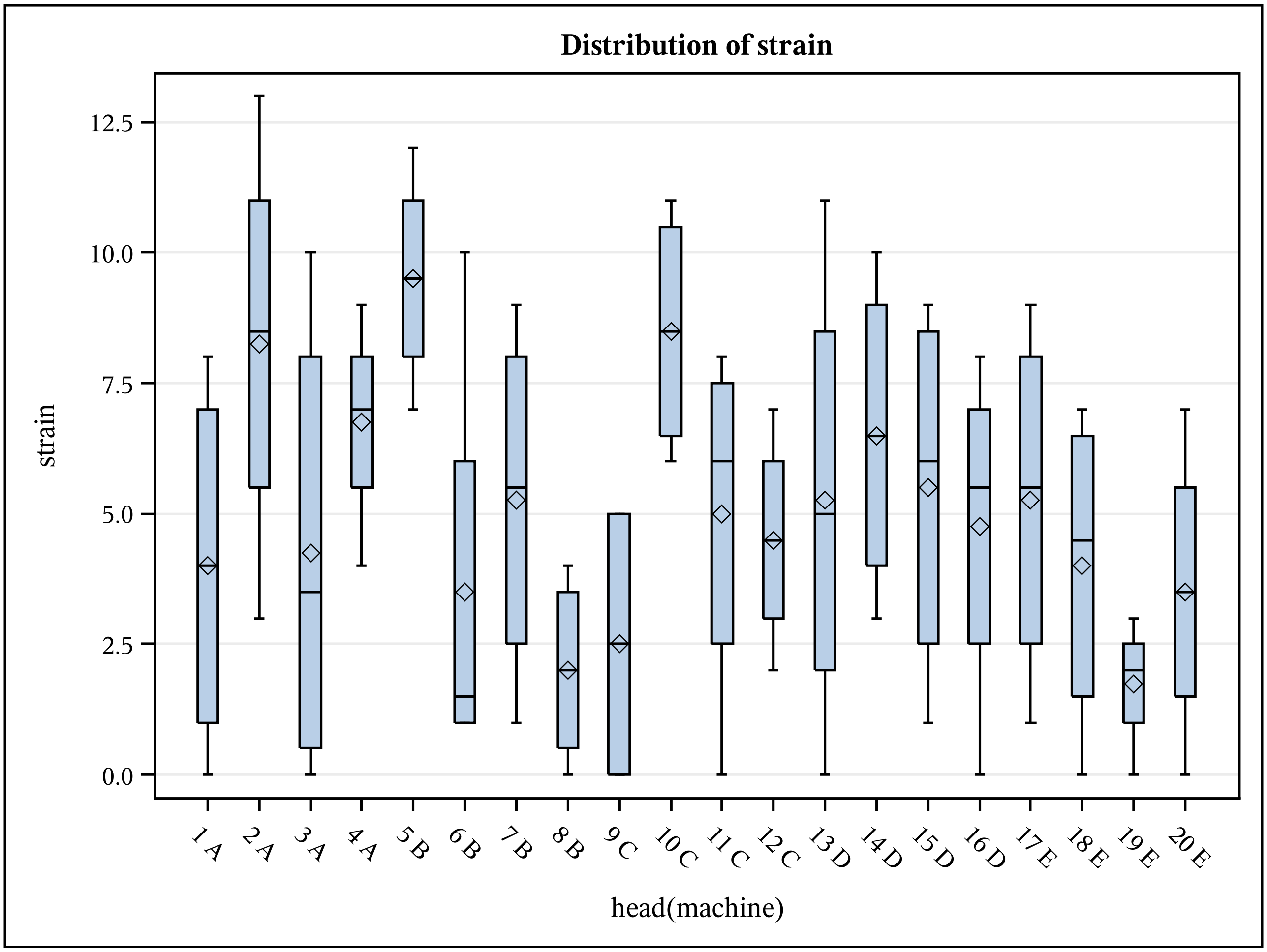
Source	DF	Type III SS	Mean Square	F Value	Pr > F
machine	4	45.075000	11.268750	0.60	0.6700
Error	15	282.875000	18.858333		
Error: MS(head(machine))					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
head(machine)	15	282.875000	18.858333	1.76	0.0625
Error: MS(Error)	60	642.000000	10.700000		





Level of machine	N	strain Mean	strain Std Dev
A	16	5.81250000	3.81608438
B	16	5.06250000	4.02440472
C	16	5.12500000	3.34414912
D	16	5.50000000	3.40587727
E	16	3.62500000	2.84897642



Level of head	Level of machine	N	strain Mean	strain Std Dev
1	**A**	4	4.00000000	3.65148372
2	**A**	4	8.25000000	4.11298756
3	**A**	4	4.25000000	4.64578662
4	**A**	4	6.75000000	2.06155281
5	**B**	4	9.50000000	2.08166600
6	**B**	4	3.50000000	4.35889894
7	**B**	4	5.25000000	3.50000000
8	**B**	4	2.00000000	1.82574186
9	**C**	4	2.50000000	2.88675135
10	**C**	4	8.50000000	2.38047614
11	**C**	4	5.00000000	3.55902608
12	**C**	4	4.50000000	2.08166600
13	**D**	4	5.25000000	4.57347424
14	**D**	4	6.50000000	3.10912635
15	**D**	4	5.50000000	3.69684550
16	**D**	4	4.75000000	3.40342964
17	**E**	4	5.25000000	3.50000000
18	**E**	4	4.00000000	3.16227766
19	**E**	4	1.75000000	1.25830574
20	**E**	4	3.50000000	2.88675135

LEVENE TEST (COMPARING VARIANCES WITHIN MACHINE HEAD)

The GLM Procedure

Class Level Information		
Class	Levels	Values
head	20	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Tests for Normality				
Test	Statistic		p Value	
Shapiro-Wilk	W	0.979233	Pr < W	0.2187
Kolmogorov-Smirnov	D	0.072249	Pr > D	>0.1500
Cramer-von Mises	W-Sq	0.069051	Pr > W-Sq	>0.2500
Anderson-Darling	A-Sq	0.443911	Pr > A-Sq	>0.2500

Variable: strain

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	19	327.9500000	17.2605263	1.61	0.0823
Error	60	642.0000000	10.7000000		
Corrected Total	79	969.9500000			

R-Square	Coeff Var	Root MSE	strain Mean
0.338110	65.09623	3.271085	5.025000

Source	DF	Type III SS	Mean Square	F Value	Pr > F
head	19	327.9500000	17.2605263	1.61	0.0823

Levene's Test for Homogeneity of strain Variance ANOVA of Absolute Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
head	19	42.0594	2.2137	0.91	0.5758
Error	60	146.3	2.4385		

SAS Code for Two-Stage Nested Design

```
DM 'LOG; CLEAR; OUT; CLEAR;';
ODS GRAPHICS ON;
ODS PRINTER PDF file='C:\COURSES\ST541\NESTED2.PDF';
OPTIONS NODATE NONUMBER;

*******************************;
*** A TWO-STAGE NESTED DESIGN ***;
*******************************;
DATA IN;
  RETAIN head 0;
  DO machine='A', 'B', 'C', 'D', 'E';
     DO mhead=1 TO 4;
           head=head+1;
           DO rep=1 TO 4;
              INPUT strain @@;  OUTPUT;
  END; END; END;
  CARDS;
 6  2  0  8 13  3  9  8  1 10  0  6  7  4  7  9
10  9  7 12  2  1  1 10  4  1  7  9  0  3  4  1
 0  0  5  5 10 11  6  7  8  5  0  7  7  2  5  4
11  0  6  4  5 10  8  3  1  8  9  4  0  8  6  5
 1  4  7  9  6  7  0  3  3  0  2  2  3  7  4  0

PROC GLM DATA=in PLOTS=(ALL);
     CLASS machine head;
     MODEL strain = machine head(machine) /  SS3;
     RANDOM head(machine) / TEST;
     MEANS machine head(machine);
     ID mhead;
      OUTPUT OUT=diag R=resid;
TITLE 'TWO-STAGE NESTED DESIGN (HICKS P.173-178)';

PROC UNIVARIATE DATA=diag NORMAL;
     VAR resid;

PROC GLM DATA=in;
     CLASS head;
     MODEL strain = head /  SS3;
     MEANS head / HOVTEST=LEVENE(TYPE=ABS);
TITLE 'LEVENE TEST (COMPARING VARIANCES WITHIN MACHINE HEAD)';
RUN;
```

5.2 Expected Means Squares (EMS) for Two-Stage Nested Designs (Supplemental)

- We will use the same EMS rules presented in Chapter 5. Recall that a subscript is **dead** if it is present and is in parentheses. In each column we put 1 for all dead subcripts in that row.
- With nested effects $\beta_{j(i)}$, we will have a "dead" subscript i. Also, recall that the error ϵ_{ijk} is written $\epsilon_{k(ij)}$ to include dead subscripts i and j.

Case I:: A two-stage nested design with Factor A is **fixed** with a levels and factor B is **random** with b levels. n replicates were taken for each of the ab combinations of the levels of A and B.

Step 1: Set up the EMS table

		F	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$				
$\beta_{j(i)}$	σ_β^2				
$\epsilon_{k(ij)}$	σ^2				

STEP 2: Filling in the rows of the EMS Table:

1. Write 1 in each column containing dead subscripts.

		F	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$				
$\beta_{j(i)}$	σ_β^2	1			
$\epsilon_{k(ij)}$	σ^2	1	1		

2. If any row subscript corresponds to a random factor (R), then write 1 in all columns with a matching subscript. Otherwise, write 0 in all columns with a matching subscript.

		F	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$	0			
$\beta_{j(i)}$	σ_β^2	1	1		
$\epsilon_{k(ij)}$	σ^2	1	1	1	

3. For the remaining missing values, enter the number of factor levels for that column.

		F	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$	0	b	n	
$\beta_{j(i)}$	σ_β^2	1	1	n	
$\epsilon_{k(ij)}$	σ^2	1	1	1	

STEP 3: Obtaining the EMS

		F	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$	0	b	n	$\sigma^2 + n\sigma_\beta^2 + \frac{bn\sum \alpha_i^2}{a-1}$
$\beta_{j(i)}$	σ_β^2	1	1	n	$\sigma^2 + n\sigma_\beta^2$
$\epsilon_{k(ij)}$	σ^2	1	1	1	σ^2

The correct F-statistics are $F_A = MS_A/MS_{B(A)}$ $\quad F_{B(A)} = MS_{B(A)}/MS_E$

Case II:: A two-stage nested design with factor A is **fixed** with a levels and factor B is **fixed** with b levels. n replicates were taken for each of the ab combinations of the levels of A and B.

Step 1: Set up the EMS table

		F	F	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$				
$\beta_{j(i)}$	$\sum\sum \beta_{j(i)}^2/a(b-1)$				
$\epsilon_{k(ij)}$	σ^2				

STEP 2: Filling in the rows of the EMS Table:

1. Write 1 in each column containing dead subscripts.

		F	F	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$				
$\beta_{j(i)}$	$\sum\sum \beta_{j(i)}^2/a(b-1)$	1			
$\epsilon_{k(ij)}$	σ^2	1	1		

2. If any row subscript corresponds to a random factor (R), then write 1 in all columns with a matching subscript. Otherwise, write 0 in all columns with a matching subscript.

		F	F	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$	0			
$\beta_{j(i)}$	$\sum\sum \beta_{j(i)}^2/a(b-1)$	1	0		
$\epsilon_{k(ij)}$	σ^2	1	1	1	

3. For the remaining missing values, enter the number of factor levels for that column.

		F	F	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$	0	b	n	
$\beta_{j(i)}$	$\sum\sum \beta_{j(i)}^2/a(b-1)$	1	0	n	
$\epsilon_{k(ij)}$	σ^2	1	1	1	

STEP 3: Obtaining the EMS

		F	F	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$	0	b	n	$\sigma^2 + \frac{bn\sum \alpha_i^2}{a-1}$
$\beta_{j(i)}$	$\sum\sum \beta_{j(i)}^2/(b-1)$	1	0	n	$\sigma^2 + n\sum\sum \beta_{j(i)}^2/a(b-1)$
$\epsilon_{k(ij)}$	σ^2	1	1	1	σ^2

The correct F-statistics are $F_A = MS_A/MS_E$ $\quad F_{B(A)} = MS_{B(A)}/MS_E$

Case III:: A two-stage nested design with Factor A is **random** with a levels and factor B is **random** with b levels. n replicates were taken for each of the ab combinations of the levels of A and B.

Step 1: Set up the EMS table

		R	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	σ^2_α				
$\beta_{j(i)}$	σ^2_β				
$\epsilon_{k(ij)}$	σ^2				

STEP 2: Filling in the rows of the EMS Table:

1. Write 1 in each column containing dead subscripts.

		r	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	σ^2_α				
$\beta_{j(i)}$	σ^2_β	1			
$\epsilon_{k(ij)}$	σ^2	1	1		

2. If any row subscript corresponds to a random factor (R), then write 1 in all columns with a matching subscript. Otherwise, write 0 in all columns with a matching subscript.

		R	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	σ^2_α	1			
$\beta_{j(i)}$	σ^2_β	1	1		
$\epsilon_{k(ij)}$	σ^2	1	1	1	

3. For the remaining missing values, enter the number of factor levels for that column.

		R	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	σ^2_α	1	b	n	
$\beta_{j(i)}$	σ^2_β	1	1	n	
$\epsilon_{k(ij)}$	σ^2	1	1	1	

STEP 3: Obtaining the EMS

		R	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	σ^2_α	1	b	n	$\sigma^2 + n\sigma^2_\beta + bn\sigma^2_A$
$\beta_{j(i)}$	σ^2_β	1	1	n	$\sigma^2 + n\sigma^2_\beta$
$\epsilon_{k(ij)}$	σ^2	1	1	1	σ^2

The correct F-statistics are $F_A = MS_A/MS_{B(A)}$ $\quad F_{B(A)} = MS_{B(A)}/MS_E$

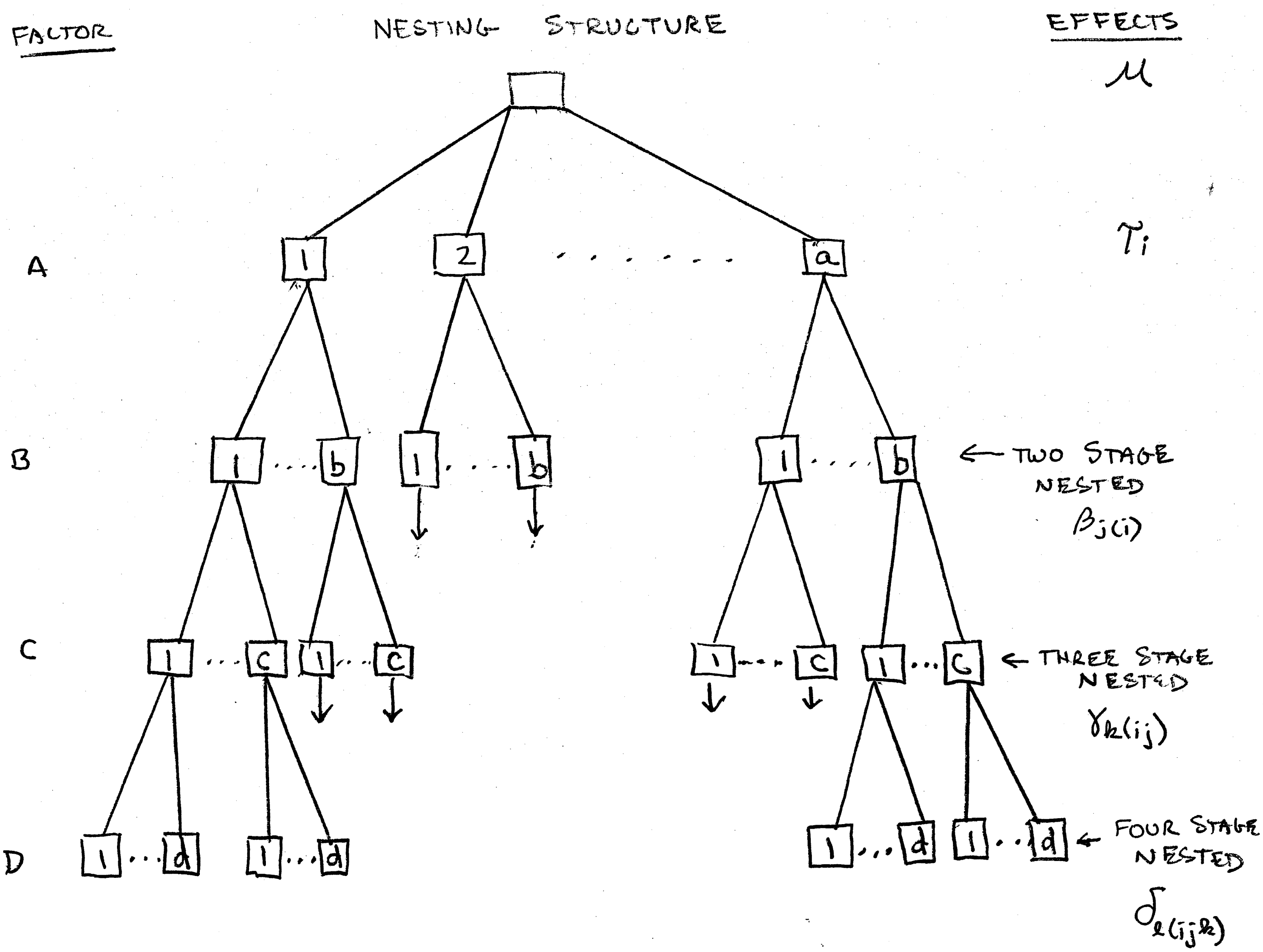
The General Balanced m-Stage Nested Design

Three-Stage Nested Design Model Equation

$$y_{ijkl} \; = \; \mu \; + \; \tau_i \; + \; \beta_{j(i)} \; + \; \gamma_{k(ij)} \; + \; \epsilon_{l(ijk)}$$

Four-Stage Nested Design Model Equation

$$y_{ijklm} \; = \; \mu \; + \; \tau_i \; + \; \beta_{j(i)} \; + \; \gamma_{k(ij)} \; + \; \delta_{l(ijk)} \; + \; \epsilon_{m(ijkl)}$$



Typically, all nested factors are random if the factor its levels are nested in are random. For example,

- **If A is random, then typically $B(A)$ is random.**
- **If $B(A)$ is random, then typically $C(AB)$ is random.**