Blocking and Confounding System for Two level factorial:-

① What is confounding?

It is design Technique, for arranging a complete factorial experiment in blocks.

where block size is smaller then the treatment combination in on replicate

OR

If the number of factors or levels increase in a factorial experiment, then the number of treatment combinations increase rapidly.

when the number of treatment combinations is large, then it may be difficult to get the blocks of sufficiently large size to accomodate all the treatment combination.

Example of Factorial Exp,
(i) Face wash use for skin < dry / oily

p^n =
n = factors
p = levels

Example:-

what is an ~~or~~ example of a factorial design.

⇒ "Gender" might be a factor with two levels,
"Male"
"Female" and

⇒ "Diet" might be a factor with three levels
low, medium, High Protein.

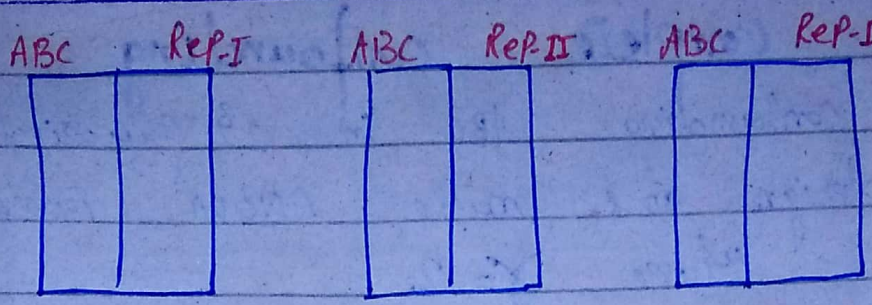
A factor design is one involving two or more factors in a single experiment.

Types of confounding

1. complete confounding
2. partial confounding

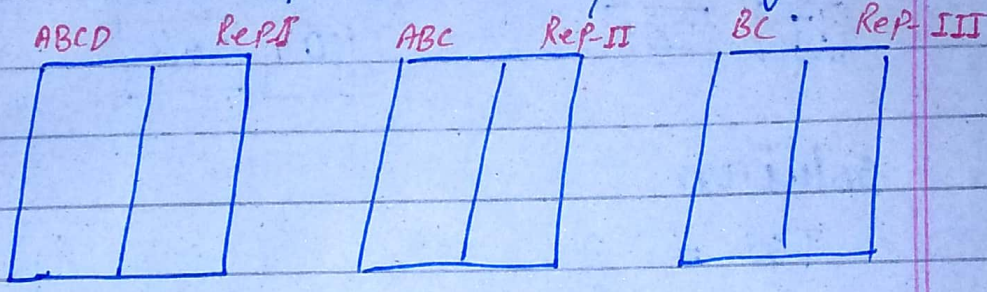
-1 complete confounding:-

when same. Treatment combination is confounding in different replicates, then it is called complete confounding.



-2 Partial Confounding:-

when different treatment combinations are confound in different replicate then it is called partial confounding



⇒ with two factors and two blocks

AB	Block I
+	−
1	a
ab	b

AB	Block (2)
+	1
1	a
ab	b

$2^2 = 4$

⇒ with three factors and two blocks

$2^3 = 8$

ABC	Rep-I
+	−
a	1
b	ab
c	ac
abc	bc

ABC	Rep-II
+	−
a	1
b	ab
c	ac
abc	bc

Question #01

Complete confounding

Confounding ABC in 2^3 factorial design and make ANOVA table where $\gamma = 2$

R-I

25	1	a	71
45	ab	b	48
40	ac	c	52
60	bc	abc	37
	170	208	

R-II

45	1	a	48
35	ab	b	52
55	ac	c	57
59	bc	abc	49
	194	206	

Solution

$TSS = ?$

$$TSS = Y'Y - \frac{G'G}{p^n \gamma}$$

$$Y'Y = [25^2 + 45^2 + 40^2 + 60^2 + 71^2 + 48^2 + 52^2 + 37^2 + 45^2 + 35^2 + 55^2 + 59^2 + 48^2 + 52^2 + 57^2 + 49^2]$$

$$\boxed{Y'Y = 39682}$$

$$\frac{G'G}{p^n r} = [25+45+40+60+71+48+52+37+45 +35+55+59+48+52+57+49]^2$$

$$= (778)^2$$

$$= 605284$$

$$TSS = Y'Y - \frac{GG}{p^n \cdot r} = 39682 - \frac{605284}{2^3(2)}$$

$$\boxed{TSS = 1851.75}$$

$$BSS = \frac{B'B}{n} - \frac{G'G}{p^n r}$$

$$= \frac{b_1^2 + b_2^2 + b_3^2 + b_4^2}{n} - \frac{605284}{16}$$

n ↙ no. of Treatment

$$= \frac{(170)^2 + (208)^2 + (194)^2 + (206)^2}{4} - 37830.25$$

$$= \frac{152236}{4} - 37830.25$$

$$\boxed{BSS = 228.75}$$

Effect of A

$[-1+a-b+ab-c+ac-bc+abc]$

$= [70+119-100+80-109+95-119+86] = -18$

Effect of B

$= [-1-a+b+ab-c-ac+bc+abc]$

$= [-70-119+100+80-109-95+119+86]$

$= -8$

Effect of AB

$= [+1-a-b+ab+c-ac-bc+abc]$

$= [+70-119-100+80+109-95-119+86]$

$= -88$

Effect of C

$= [-1-a-b-ab+c+ac+bc+abc]$

$= [-70-119-100-80+109+95+119+86]$

$= 40$

Effect of AC

$= [+1-a-b-ab-c+ac-bc+abc]$

$= [+70-119+100-80-109+95-119+86]$

$= -76$

Effect of BC

$= [1+a-b-ab-c-ac+bc+abc]$

$= [70+119-100-80-109-95+119+86]$

$= 10$

$$SSA = \frac{(\text{Effect of } A)^2}{p^n \cdot r} = \frac{(-18)^2}{(2^3)(2)} = \frac{324}{16}$$

$$SSA = 20.25$$

$$SSB = \frac{(\text{Effect of } B)^2}{p^n \cdot r} = \frac{(-8)^2}{(2^3)(2)} = \frac{64}{16} = 4$$

$$SSAB = \frac{(-88)^2}{16} = \frac{7744}{16}$$

$$= 484$$

$$SSC = \frac{(40)^2}{16} = \frac{1600}{16} = 100$$

$$SSAC = \frac{(-76)^2}{16} = 361$$

$$SSBC = \frac{(16)^2}{16} = \frac{100}{16} = 6.25$$

ANOVA Table

SOV	d.f	SS	MS	F
Block	4-1=3	228.75	76.25	0.70656
A	2-1=1	20.25	20.25	0.18764
B	1	4	4	0.03706
AB	1	484	484	4.4849
C	1	100	100	0.9266
AC	1	361	361	3.345
BC	1	6.25	6.25	0.0579
ERROR	15-9=6	647.5	107.917	
Total	16-1=15	1851.75		

SOV	d.f	SS	MS	F
A	level-1	$\frac{(\text{effect of A})^2}{p^n r}$	SSA/d.f	MSA/MSE
B	"	"	"	"
AB	"	"	"	"
ERROR	by sub	by sub	SSE/d.f	"
Total	$p^n r - 1$	$yy' - \frac{G'G}{p^n r}$	SST/d.f	MST/MSE

Example # 7.1

Block (1)	
1	28
a	36
b	18
ab	31

$B_1 = 113$

Block (2)	
1	25
a	32
b	19
ab	30

$B_2 = 106$

Block (3)	
1	27
a	32
b	23
ab	29

$B_3 = 111$

$$TSS = y'y - \frac{G'G}{p^n r} = 9398 - 9075 = 323$$

$$y'y = [28^2 - \cdots - 29^2] = 9398$$

$$\frac{G'G}{p^n r} = [28 - \cdots - 29]^2 = (330)^2 = \frac{108900}{(2^2)(3)}$$

$$= \frac{108900}{12} = 9075$$

$$TSS = 323$$

$$BSS = \frac{B'B}{n} - \frac{G'G}{p^n r}$$

$$= \frac{B_1^2 + B_2^2 + B_3^2}{n} - 9075$$

$$= 9081 - 9075$$

$$\boxed{BSS = 6.5}$$

Effect of (A)

$= [-8 + 100 - 60 + 90] = 50$

$SSA = \frac{(\text{effect of } A)^2}{p^n \gamma} = \frac{(50)^2}{12} = 208.33$

Effect of (B)

$= [-80 - 100 + 60 + 90] = -30$

$SSB = \frac{(-30)^2}{12} = 75$

Effect of (AB)

$= [80 - 100 - 60 + 90] = 10$

$SSAB = \frac{(10)^2}{12} = 8.33$

ANOVA Table

SOV	d.f	SS	MS	F
Blocks	3-1=2	6.50	3.25	
A	1	208.33	208.33	50.32
B	1	75	75	18.12
AB	1	8.33	8.33	20.1
ERROR	11-5=6	24.84	4.14	
Total	12-1=11	323		

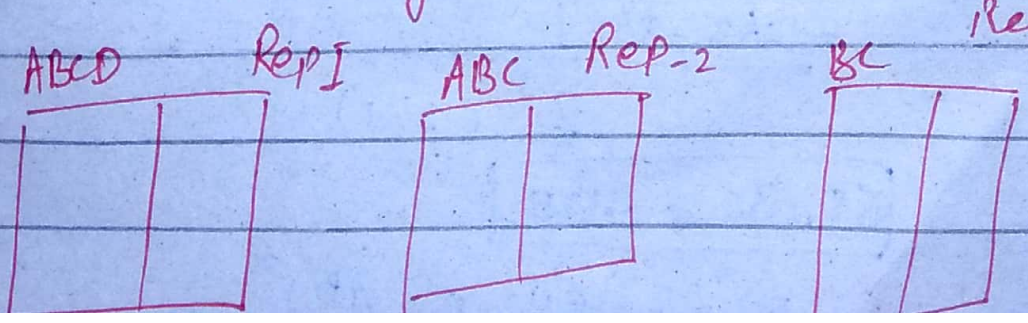
partial confounding

means that you confound different effects in every replication.

why we use partial conf.?

with partial confounding you can get non-zero efficiency for more effects.
while complete confounding gives you zero efficiency for the completely confounded effects.

Different treatments combination are confound in different replicates then it is called partial confounding.



Example 7.3

partial confounding 2^3

Rep=1	ABC		
550	1	a	669
642	ab	b	633
749	ac	c	1037
1075	bc	abc	729
↓ B_1 3016			↓ B_2 3668

$R_1 = 6084$

Rep-2	AB		
604	1	a	650
1052	c	b	601
635	ab	ac	868
860	abc	bc	1063
↓ B_3 3151			↓ B_4 3182

R_2 6333

↓

GG

$p^n \gamma$

$$SSABC = \frac{(\text{Effect of } ABC)^2}{2^k (\gamma - 1)}$$

$$SSABC = [-1 + a + b - ab + c - ac - bc + abc]$$

$$SSABC = [-604 + 650 + 601 - 635 + 1052 - 868 - 1063 + 860]$$

$$\frac{}{2^3 (2-1)}$$

$$\boxed{SSABC = 6.125}$$

$$SSAB = \frac{(\text{Effect of } AB)^2}{2^k (\gamma - 1)}$$

$$= [+1 - a - b + ab + c - ac - bc + abc]$$

$$= [550 - 669 - 633 + 642 + 1037 - 749 - 1075 + 729]$$

$$SSAB = \frac{(-168)^2}{2^3(1)} = 3528.0$$

$$\text{Repss} = \frac{R_1^2 + R_2^2}{2^K} - \frac{G'G}{p^n \gamma}$$

$$TSS = Y'Y - \frac{G'G}{p^n \gamma}$$

$$Y'Y = [550^2 + \ldots\ldots 1063^2]$$

$$Y'Y = 10167789$$

$$\frac{G'G}{p^n \gamma} = \left[\frac{550 + 642 + \ldots\ldots 1063}{2^3(2)}\right]^2$$

$$= \frac{154181889}{16}$$

$$TSS = Y'Y - \frac{G'G}{p^n \gamma}$$

$$\boxed{TSS = 531420.9375}$$

$$R_1 = 3016 + 3068 = 6084$$

$$R_2 = 3151 + 3182 = 6333$$

$$\text{Repss} = \frac{R_1^2 + R_2^2}{2^k} - \frac{G'G}{p^n r}$$

$$= \frac{(6084)^2 + (6333)^2}{2^3} - 9636368.063$$

$$= 9640243.125 - 9636388.063$$

$$\boxed{\text{Repss} = 3875.062}$$

Effect of (A)

$$= [-1 + a - b + ab - c + ac - bc + ab]$$

$$= [-1154 + 1319 - 1234 + 1277 - 2089 + 1617 - 2138 + 1589]$$

$$SSA = 41310.5625$$

Effect of (B)

$$= [-1 - a + b + ab - c - ac + bc + abc] = 59$$

$$ssB = \frac{(\text{effect of } B)^2}{2^3 (2)} = \frac{(59)^2}{16} = 217.5625$$

Effect of (C)

$$= [-1 - a - b - ab + c + ac + bc + abc]$$

$$SSC = 374850.0625$$

Effect of (AC)

$$= [+1 - a + b - ab - c + ac - bc + abc]$$

$$SS\,AC = 94402.5625$$

Effect of (BC)

$$[1 + a - b - ab - c - ac + bc + abc]$$

$$SSBC = \frac{(-17)^2}{16} = 18.0625$$

$$\text{Block SS} = \frac{B'B}{n} - \frac{G'G}{p^n r}$$

$$\text{Rep I} = \frac{b_1^2 + b_2^2}{n} - \frac{G'G}{p^n r}$$

$$\text{Rep II} = \frac{b_2^2 + b_3^2}{n} - \frac{G'G}{p^n r}$$

$$\text{Rep-I} = \frac{(3016)^2 + (3068)^2}{4} - \frac{(6084)^2}{8}$$

$$= 4627220$$

Rep-I = 338

Rep-II = 120.125

$$\text{Block SS} = \text{Rep}_{(1)} + \text{Rep}_{(2)}$$

$$= 338 + 120.125$$

Block SS = 458.125

ANOVA Table

SOV	d.f	SS	MS	F
Replicate	1			
Block	2	3875.0625	229.0625	—
A	1	458.1256		—
B	1			
C	1			16.26
AB	1			
AC	1			
BC	1			
ABC	1			
ERROR	15-10=5		2250.4625	0.002
Total	16-1=15	531420.4375		

Complete confounding with

Even odd method $2^3 = 8$, $r = 3$

ABC $r=1$

E	O
1	a
ab	b
ac	c
bc	abc

ABC $r=2$

E	O
1	a
ab	b
ac	c
bc	abc

ABC $r=3$

E	O
1	a
ab	b
ac	c
bc	abc

Effect of ABC

1	a	b	ab	c	ac	bc	abc
0	1	1	2	1	2	2	3

E	O
1	a
ab	b
ac	c
bc	abc

odd - even values

$a + b + c + abc - 1 - b - ac - bc$

Partial confounding:-

AC

E	O
1	a
b	ab
ac	c
abc	bc

BC

E	O
1	b
a	ab
bc	c
abc	ac

AB

E	O
1	a
ab	b
c	ac
abc	bc

(5)

Effect of AC

1	a	b	ab	c	ac	bc	abc
0	1	0	1	1	2	1	2

E	O
1	a
b	ab
ac	c
abc	bc

even - odd

1+b+ac+abc-a-ab-c-bc

Effect of BC

1	a	b	ab	c	ac	bc	abc
0	0	1	1	1	1	2	2

E	O
1	b
a	ab
bc	c
abc	ac

even - odd

1+a+bc+abc - b-ab-c-ac

Effect of AB

1	a	b	ab	c	ac	bc	abc
0	1	1	2	0	1	1	2

E	O
1	a
ab	b
c	ac
abc	bc

even - odd

1+ab+c+abc -a-b-ac-bc

For

$2^4 = 16$ Design

Complete confounding

$\gamma = 1$ $\gamma = 2$ $\gamma = 3$

ABCD

E	O
1	a
ab	b
ac	c
bc	abc
ad	d
bd	abd
cd	acd
abcd	bcd

E	O
1	a
ab	b
ac	c
bc	abc
ad	d
bd	abd
cd	acd
abcd	bcd

E	O
1	a
ab	b
ac	c
bc	abc
ad	d
bd	abd
cd	acd
abcd	bcd

Effect of ABCD

1	a	b	ab	c	ac	bc	abc
0	1	1	2	1	2	2	3

d	ad	bd	abd	cd	acd	bcd	abcd
1	2	2	3	2	3	3	4

Even - odd

$1 + ab + ac + bc + ad + bd + cd + abcd - a - b - c - abc - d - abd - acd - bcd$

Even	odd
1	a
ab	b
ac	c
bc	abc
ad	d
bd	abd
cd	acd
abcd	bcd

Partial confounding :-

ABC $\gamma = 1$

E	O
1	a
ab	b
ac	c
bc	abc
d	ad
abd	bd
acd	cd
bcd	abcd

BC $\gamma = 2$

E	O
1	b
a	ab
bc	c
abc	ac
ad	bd
bcd	abd
abcd	cd
	acd

AD $\gamma = 3$

E	O
1	a
b	ab
c	ac
abc	bc
ad	d
abd	bd
acd	cd
abcd	bcd

Effect of ABC

E	O
1	a
ab	b
ac	c
bc	abc
d	ad
abd	bd
acd	cd
bcd	abcd

1 (0)	a (1)	b (1)	ab (2)	c (1)	ac (2)	bc (2)
abc (3)	d (0)	acd (1)	bcd (1)	abcd (2)		
cd (1)	acd (2)	bcd (2)	abcd (3)			

odd - Even

a + b + c + abc + ad + bd + cd + abcd

-1 - ab - ac - bc - d - abd - acd - bcd

Effect of BC

1(0) a(0) b(1) ab(1) c(1) ac(1) bc(2) abc(2) d(0)

ad(0) bd(1) abd(1) cd(1) acd(1) bcd(2) abcd(2)

E	O
1	b
a	ab
bc	c
abc	ac
d	bd
ad	abd
bcd	cd
abcd	acd

even - odd

$1 + a + bc + abc + d + ad + bcd + abcd - b - ab - c - ac - bd - abd - cd - acd.$

Effect of AD

1(0) a(1) b(0) ab(1) c(0) ac(1) bc(0) abc(1) d(1) ad(2) bd(1) abd(2) cd(1) acd(2) bcd(1) abcd(2)

E.	O
1	a
b	ab
c	ac
bc	abc
ad	d
abd	bd
acd	cd
abcd	bcd

even - odd

$1 + b + c + bc + ad + abd + acd + abcd - a - ab - ac - abc - d - bd - cd - bcd$