Factorial experiments

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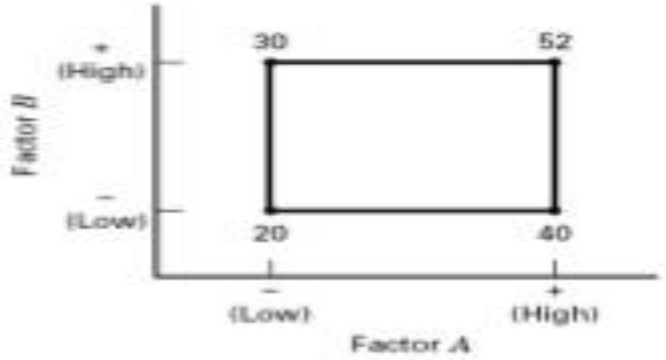
Contents

- Factorial experiments
- Informal way to find effects in Factorial Design
- Formal ways to find effects in Factorial Design
  1. Sign method
  2. Yates method
  3. Algebraic method

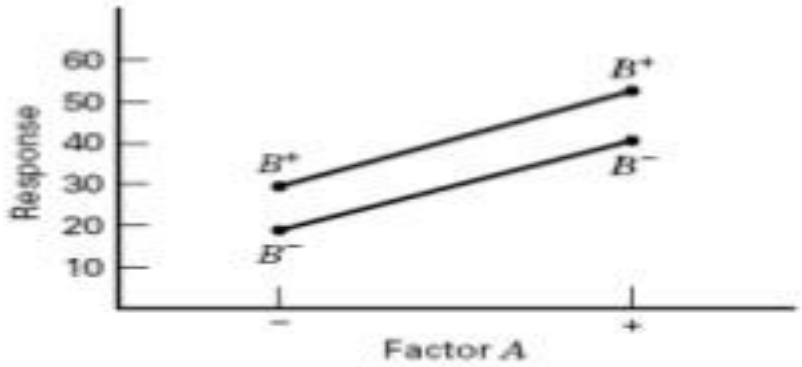
Basic Definition

- Many experiments involve the study of the effects of two or more factors.
- Factorial designs most efficient for this type of experiment.
- By a factorial design, we mean that in each complete trial or replicate of the experiment all possible combinations of the levels of the factors are investigated
- If there are a levels of factor A and b levels of factor B, each replicate contains all ab treatment combinations.

Informal way to find effects in Factorial Design



■ FIGURE 5.1 A two-factor factorial experiment, with the response (y) shown at the corners



■ FIGURE 5.3 A factorial experiment without interaction

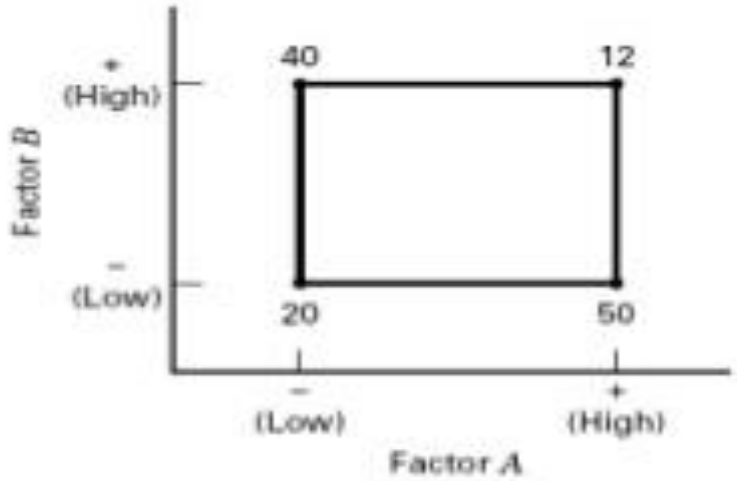
Definition of a factor effect: The change in the mean response when the factor is changed from low to high

$$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{40+52}{2} - \frac{20+30}{2} = 21$$

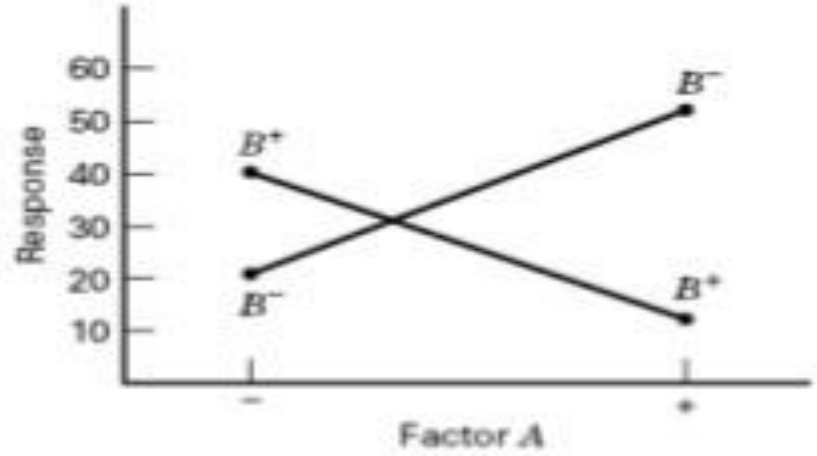
$$B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{30+52}{2} - \frac{20+40}{2} = 11$$

$$AB = \frac{52+20}{2} - \frac{30+40}{2} = -1$$

The Case of Interaction:



■ **FIGURE 5.2** A two-factor factorial experiment with interaction



■ **FIGURE 5.4** A factorial experiment with interaction

$$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{50+12}{2} - \frac{20+40}{2} = 1$$

$$B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{40+12}{2} - \frac{20+50}{2} = -9$$

$$AB = \frac{12+20}{2} - \frac{40+50}{2} = -29$$

Formal ways to find effects in Factorial Design

- **Sign Method:**

$$Effect\ of\ factor\ A = -1 + a - \mathrm{b} + \mathrm{ab}$$
$$Effect\ of\ factor\ B = -1 - a + \mathrm{b} + \mathrm{ab}$$
$$Interaction\ effect = +1 - a - \mathrm{b} + \mathrm{ab}$$

Sign Table				
Effect	**1**	**a**	**b**	**ab**
A	-	+	-	+
B	-	-	+	+
AB	+	-	-	+

Sign Table for 2^3 Factorial Experiment								
Effect	**1**	**a**	**b**	**ab**	**c**	**ac**	**bc**	**abc**
A	-	+	-	+	-	+	-	+
B	-	-	+	+	-	-	+	+
AB	+	-	-	+	+	-	-	+
C	-	-	-	-	+	+	+	+
AC	+	-	+	-	-	+	-	+
BC	+	+	-	-	-	-	+	+
ABC	-	+	+	-	+	-	-	+

- **Yates Method**

2^2 Factorial Experiment			
1	a+1	ab+b+a+1	Grand effect
a	ab+b	ab-b+a-1	Effect of factor A
b	a-1	ab+b-a-1	Effect of factor B
ab	ab-b	ab-b-a+1	Interaction effect

2^3 Factorial Experiment			
1	a+1	ab+b+a+1	abc+bc+ac+c+ab+b+a+1
a	ab+b	abc+bc+ac+c	abc-bc+ac-c+ab-b+a-1
b	ac+c	ab-b+a-1	abc+bc-ac-c+ab+b-a-1
ab	abc+bc	abc-bc+ac-c	abc-bc-ac+c+ab-b-a+1
c	a-1	ab+b-a-1	abc+bc+ac+c-ab-b-a-1
ac	ab-b	abc+bc-ac-c	abc-bc+ac-c-ab+b-a+1
bc	ac-c	ab-b-a+1	abc+bc-ac-c-ab-b+a+1
abc	abc-bc	abc-bc-ac+c	abc-bc-ac+c-ab+b+a-1

- **Algebraic Method**

2^2 Factorial Experiment

$$\text{Effect of factor A} = (a-1)(b+1) = ab-b+a-1$$

$$\text{Effect of factor B} = (a+1)(b-1) = ab+b-a-1$$

$$\text{Interaction effect(AB)} = (a-1)(b-1) = ab-b-a+1$$

2^3 Factorial Experiment

$$\text{Effect of factor A} = (a-1)(b+1)(c+1) = abc-bc+ac-c+ab-b+a-1$$

$$\text{Effect of factor B} = (a+1)(b-1)(c+1) = abc+bc-ac-c+ab+b-a-1$$

$$\text{Effect of AB} = (a-1)(b-1)(c+1) = abc-bc-ac+c+ab-b-a+1$$

$$\text{Effect of factor C} = (a+1)(b+1)(c-1) = abc+bc+ac+c-ab-b-a-1$$

$$\text{Effect of AC} = (a-1)(b+1)(c-1) = abc-bc+ac-c-ab+b-a+1$$

$$\text{Effect of BC} = (a+1)(b-1)(c-1) = abc+bc-ac-c-ab-b+a+1$$

$$\text{Effect of ABC} = (a-1)(b-1)(c-1) = abc-bc-ac+c-ab+b+a-1$$

Advantages of Factorials

- They are more efficient than one-factor-at-a-time experiments.
- A factorial design is necessary when interactions may be present to avoid misleading conclusions.
- Factorial designs allow the effects of a factor to be estimated at several levels of the other factors, yielding conclusions that are valid over a range of experimental conditions.