1 The masses $m, m, 2m$ are placed at the vertices of an equilateral triangle of side a. Find c.m. of the combined system.

- Show that the ratio of the distances of two mas m_1, m_2 from their centre of mass is in inver ratio of their masses.

- Find centre of mass of a uniform solid hemisphere of radius a.

S Linear momentum and angular momentum and their conservation laws.

If m is mass of a particle moving with velocity $\vec{v}$, then $P = mv$ is called linear momentum.

where as angular momentum is defined as the moment of linear momentum.

i.e Angular Momentum $= J = r \times p$

or $J = r \times mv$

Now using Newton's law of motion, if F is applied force,

then $F = ma$ or

$$F = m\frac{dv}{dt}$$

$$F = \frac{d}{dt}(mv) = \frac{dP}{dt}$$

In the absence of external force

i.e if $F = 0 \Rightarrow \frac{dP}{dt} = 0$

$\Rightarrow P = \text{Constant}$

i.e If total force F acting on a particle is zero, then linear momentum is conserved. This is known as law of conservation of momentum

ii To prove the law of conservation of angular momentum, we have from def

$$J = \vec{r} \times \vec{P}$$

$$\Rightarrow J = \vec{r} \times m\vec{v}$$

$$\Rightarrow \frac{dJ}{dt} = \frac{d}{dt}(\vec{r} \times m\vec{v})$$

$$= \frac{d\vec{r}}{dt} \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$= m(\vec{v} \times \vec{v}) + m\,\vec{r} \times \frac{d\vec{v}}{dt}$$

$$= 0 + \vec{r} \times m\vec{a}$$

$$= \vec{r} \times \vec{F}$$

$$= T \quad (\text{torque of the body})$$

Now if $F = 0$ or F is a central force, then line of action of F is along $\vec{r}$

i.e $\vec{F} \parallel \vec{r} \Rightarrow \vec{r} \times \vec{F} = 0$

Hence in case of the absence of the external force or in case of central force,

$$\frac{dJ}{dt} = 0 \Rightarrow J = \text{constant}.$$

i.e angular momentum is conserved.

N.B For a single body (or a particle)

$$\underline{p} = m\underline{v} \quad \text{and} \quad J = \underline{r} \times \underline{p}$$

In case of a system of particles, we have

$$P = \sum_{i=1}^{n} p_i = \sum_{i=1}^{n} m_i v_i$$

and $J = \sum_{i=1}^{n} r_i \times p_i$

§ Angular momentum of a system of particles in terms of centre of mass of the systems.

Let r_i is the position vector of the particle of mass m_i (any ith particle from system of n particles $m_1, m_2, \dots, m_n$)

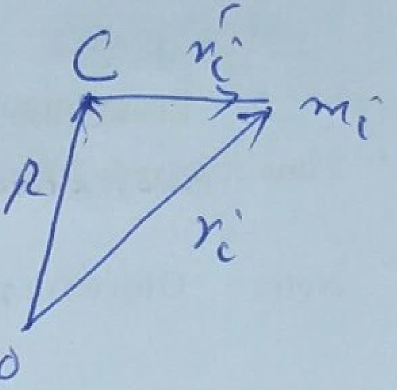
If $\sum_{i=1}^{n} m_i = M$, let R is p.v. of the ~~whole system~~ c.m. of C of the system.

If r_i' is position vector of the mass m_i with respect to c.m. C

Then

$$R + r_i' = r_i$$

or $r_i' = r_i - R$



If P_i' is linear momentum of the mass m_i w.r.t. centre of mass, then

$$P_i' = m_i v_i'$$

$$P_i' = m_i \dot{r}_i' \qquad \because \; v_i' = \frac{dr_i'}{dt} = \dot{r}_i'$$

$$= m(\dot{r}_i - \dot{R})$$

$$= m\dot{r}_i - m\dot{R} = m_i v_i - m_i V_{cm}$$

$$\Rightarrow \quad P_i' = P_i - m_i V_{cm}$$

So angular momentum about c.m. is

$$J' = \sum r_i' \times P_i'$$

$$= \sum (r_i - R) \times (P_i - m_i V_{cm})$$

$$= \sum (r_i - R) \times (m_i v_i - m_i V_{cm})$$

$$= \sum m_i (r_i - R) \times (v_i - V_{cm})$$

J_{cM}

$$= \sum m_i r_i \times v_i - \left(\sum m_i r_i\right) \times V_{cm} - \sum m_i R \times v_i + \sum m_i R \times V_{cm}$$

J^4

J'

$$= \sum r_i \times m_i v_i - M\left(\frac{\sum m_j r_j}{M}\right) \times V_{cm} - + \left(\sum m_i\right) R \times \dot{R}$$

$$= \sum r_j \times P_j - M \cancel{R} \times \dot{R} - R \times \sum m_j v_j + M \cancel{R} \times \dot{R}$$

$$J' = \sum r_j \times P_i - R \times P = J - J_{cm}$$