Chapter 1

Foundations of Mechanics

In this chapter we will discuss the fundamental concepts, principles and la of mechanics and point out how on the basis of some basic concepts a postulates (based on experiments) a coherent and comprehensive theory motion has been developed over the centuries.

1.1 On the Nature of Mechanics and its Divisior

1.1.1 Mechanics as a natural science

The subject of mechanics is a branch of physics which in turn is a natu ral science. All natural sciences study different parts and aspects of th universe or Nature or natural world. The universe or Nature consists all objects which are susceptible or observable through direct or indirec observation or experiment/experience.

Natural sciences are divided into two broad classes:

(I) Life sciences

(II) Sciences of the non-living matter.

Life sciences consist of medicine and associated subjects, zoology, botany physiology and anatomy, etc. They deal with living bodies.

Sciences of non-living matter consist of physics, chemistry, geology, astron omy, etc. They deal with non-living forms of matter.

Any natural science is based on the following fundamental assumptions (o beliefs):

(i) The Universe exists. This means that all those which we ol directly through sense organs or indirectly through observations do exist. Some individuals may suffer hallucination or inability to observ example blindness or other clinical defect), but when a number of trained people observe the same thing then it means that the thing exists.

(ii) The Universe is understandable. This means that different e taking place in the universe, such as sunrise and sunset, day and n motions of the Moon and planets, bodies falling toward the earth ra away from it, eclipses, earthquakes, formation of mountains and ri diseases, flow of rivers and waves on the surface of water, twinklin stars, and a host of other phenomena can all be understood on a rati basis.

(iii) In order to gain information about any aspect of an event taking p anywhere in the universe, there must be a physical source connecting observer with the thing being observed. In other words an experimente observer must interact with the object through a physical source.

1.1.2 What is mechanics?

Mechanics is the science of motion. It studies states of rest and motion a the laws governing rest, equilibrium and motion. Since material bod exist in the form of solids, liquids and gases so there are correspondi types of mechanics to deal with them.

A broad division of mechanics is made through the terms *classical mecha ics* and *quantum mechanics*. Classical mechanics deals with macroscop objects i.e. those bodies which we encounter in everyday life. Such bodi may be in the form of billiard balls, parts of machinery or astronomic bodies. When we have to study the behaviour of atomic and subatom particles, molecules and groups of molecules, classical mechanics based o Newton's laws fails and we have to rely on quantum mechanics. Thus qua tum mechanics is the mechanics of microscopic objects. Its basic concept principles and laws are entirely different from those of classical mechanic

Classical mechanics itself has two versions based on methodology and ap proach. They are

(i) *Newtonian mechanics* or *vector mechanics*, wherein vector quantities such as position vector, velocity, momentum, acceleration and force appea

as basic physical entities. This is directly based on Newton's laws of moti
and many other results follow from these laws.

(ii) *Analytical mechanics* which can also be called *scalar mechanics* becaus
the scalar quantity *energy* occupies a central position in this version
mechanics.

Mechanics being a physical and applied science is of special interest
physicists and engineers. However it is also studied by mathematici
partly because being a deductive science it can be developed in an
iomatic manner and partly because it is here that the strength and succ
of mathematical models of natural processes can be tested most directl

Some of the topmost mathematicians of all times have made contributi
to the development of the science of mechanics. The list of their na
includes Newton, Euler, Lagrange, Laplace, Jacobi, Poincare, Hami
and, in modern times, Birkhoff and Banach. (Detailed account of histo
development of mechanics is given towards the end of this chapter, sec
1.5).

The theoretical structure of mechanics which is mathematically very so
has served as an ideal paradigm for the development of later branche
physics and applied mathematics such as fluid mechanics, solid mecha
electromagnetic theory, relativity theory, quantum mechanics and st
tical mechanics. Three dimensional Euclidean space is the space use
mechanics wherein the bodies are placed and move.

Classical mechanics itself has two broad divisions in the form of
relativistic and relativistic mechanics. Non-relativistic mechanics, base
the laws of Newton is concerned with bodies moving at speeds and ve
ties negligibly small as compared to the speed of light c, $c = 3.0\times 10$
per second. Relativistic mechanics, developed by the renowned theore
and mathematical physicist Albert Einstein (1879–1955) is indispen
for particles and bodies moving with speeds comparable to the spe
light. Although conceptually quite different from Newtonian mechanic
results tally with those of the latter in the limit of low velocities.

In this book we will confine ourselves to non-relativistic classical mech

1.1.3 Mechanics of a particle and of a rigid body

...mechanics are the following:

law. Basic concepts and terms are space, time and mass; particle and body; velocity, momentum and acceleration; force and energy.

(ii) **Mechanics of fluids:** It is also based on Newton's laws and their extension and deals with the behaviour of fluids (liquids and gases) in motion. Its two well-known branches are*Hydrodynamics*(for liquids) and *Aerodynamics*(for gases).

(iii)**Mechanics of elastic solids:** It deals with the behaviour of solids when they undergo deformation under forces.

1.2 The Newtonian Mechanics

1.2.1 Divisions of mechanics

The study of mechanics is usually carried out in the sequence of mechanics of a particle, mechanics of a system of a particles and mechanics of a rigid body. Because of certain conceptual and other difficulties in the formulation of Newtonian mechanics, an equivalent but mathematically more rigorous formulation in the form of Lagrangian and Hamiltonian Mechanics has been developed. Apart from its other merits, the analytical mechanics of Lagrange and Hamilton provides a more powerful calculational tool.

Two most fundamental concepts of mechanics are are those of*particle*and *rigid body.*

The particle is a point mass i.e. a piece of matter having a definite mass but no size i.e. a geometrical point. It is clear from this definition that the particle is an abstraction or idealization. In actual practice, any body whose size is very small as compared to the sizes of other bodies being studied along with it cab be considered a particle.

A rigid body is defined as a collection of particles such that distance between every pair of its constituent particles remains unchanged whatever the forces acting an it.

It is clear from this definition that like particle rigid body is also an abstraction or idealization. Since every body undergoes changes when subjected to sufficiently large forces, strictly speaking there rigid bodies do not exist in real life. However in actual practice bodies which undergo negligibly small changes under the forces applied to them may be regarded as rigid.

The defining condition of a rigid body is called the *constraint of rigidity.*

It can be expressed as

$$(\mathbf{r}_i - \mathbf{r}_j)^2 = (\mathbf{r}_i - \mathbf{r}_j)\cdot(\mathbf{r}_i - \mathbf{r}_j) = c_{ij}$$

where $\mathbf{r}_i$ is the position vector of the i th particle and c_{ij} is a constant. This definition implies that a rigid body will not undergo any deformation (in the form of extension, compression, bending or breaking) however large force is applied to it. In actual life this is not possible; every rigid body suffers some deformation when subjected to a sufficiently large force. Thus the concept of a rigid body like that of a particle is an idealization. It has been found to be useful in theoretical study of approximately rigid bodies. All those bodies which resist any deformation under forces can be regarded as approximately rigid. For example solid bodies such as stones, metallic balls, rods and pieces of machinery can be treated as rigid. On the other hand bodies which offer no resistance to deforming forces such as gases, liquids and solid bodies such as a sheet of a paper are not rigid.

The basic concepts in mechanics are space, time, mass (matter or inertia) and force. These concepts cannot be strictly defined. They are supposed to be known to us intuitively. However their significance must be clearly understood.

The concept of space is connected with the notion of length or distance, which in turn is used to specify the position of an object (particle or body). The position of a particle P (in Euclidean space) is uniquely defined by three coordinates, which require*three measurementsofdistanceorlength.*
In practice concept of absoluteness of space reduces to the invariance of dis-
tance under transformation from one frame of reference to another. Like-
wise.the concept of time is equivalent to measurement of time interval
between two events. In Newtonian mechanics, time is supposed to be abso-
lute. This means that the time interval between two events is independent
of the state of the observer i.e. it does not matter whether the observers
are at rest or in motion relative to the event. In other words, the interval
will be the same in all frames of reference.

The concept of mass is used to describe and compare bodies on the basis of
certain fundamental mechanical experiments. Two bodies having the same
mass will experience the same force of attraction due to the Earth. In other
words, they will be subject to the same force and offer the same resistance
to change in their state of rest or motion (i.e. have the same inertia).

A force is intuitively related to muscular effort and is measured by the
weight of bodies in the gravitational field of the Earth. It is completely
characterized by magnitude, direction and line of application.

In Newtonian mechanics, space, time and mass are fundamental concepts

ton's laws of motion and law of gravitation

on's three well-known laws are directly based on experiment and a to be applicable to a very wide region of experience. All everyda ns such as those of parts of machinery, of vehicles such as cars, m trains and aeroplanes; those of the moon and other heavenly bodi overned by these laws. As mentioned earlier bodies moving at spee arable to that of light are governed by relativistic mechanics. The la vitation is another fundamental law of Newtonian mechanics which for bodies moving under weak gravitational fields such as the gravita l field of the earth. Motion in strong gravitational fields such as tha e sun is studied with the help of Einstein's general theory of relativity

e are three conservation principles that are considered to be universall in mechanics. They are principles of conservation of linear momentum ervation of angular momentum and conservation energy. They are ied in detail in chapter 2.

3 Frames of reference and inertial frames

hanics is based on Newton's laws of motion. These laws are usually ed without explicitly discussing the role of frames of reference. It is e emphasized that Newton's laws are not valid w.r.t. every frame of ence e.g. they are not true in a rotating frame or coordinate system. y hold only in a restricted class of frames, the so-called *inertial frames eference* or (*Newtonian frames*). Such a frame is assumed to be unlerated i.e. neither moving in a straight line with variable velocity nor ting. In other words, it is supposed to be absolutely at rest (or moving uniform velocity w.r.t. an absolutely-at-rest frame). We can easily see if S is an inertial frame, then any other frame S' which is in uniform ion relative to S is also an inertial frame.

To all observers in inertial frames, the force acting on a particle will he same i.e. the law of motion will be invariant under transformations necting inertial fram... In fact all laws of mechanics are the same in rames of reference. This statement is called *Newtonian* or *Galilean*

1.3 Mathematical Modelling and Mechanics

For mathematical description of a physical system we construct a mathematical model of the system. This consists of the following steps.

(i) We start with a concrete physical situation such as the motion of a pendulum or projectile in space, vibrations of a string, temperature distribution in a heated bar, motion of the Moon about the Earth, and then pass on to an *idealized* physical model by making certain simplifying assumptions. For example, in case of a projectile, we assume that the projectile is a point mass, the air resistance is negligible and motion takes place in a vertical plane. In case of motion of a pendulum, we assume that the bob is a particle, there is no friction at the point where the string is fixed, the string is perfectly elastic and weightless, and that the oscillations are very small.

Similarly in the case of the motion of the Moon about the Earth, both are supposed to be point-masses and the perturbations due to gravitational effects of all other bodies are neglected.

(ii) Using the idealized physical model, we construct a mathematical model. This is often difficult and requires much practice. The mathematical model of a real-life problem may consist of a single equation, or so complex that it may consist of several differential equations along with conditions on the unknown function(s).

(iii) The next step is to use available mathematical methods and techniques to explore properties of the model and answer specific questions. If such questions can be answered then we are able to compare the mathematical model with physical reality as observed by experiment. In this way we indirectly test the validity of the laws assumed in making the physical approximation.

It is important not to confuse the model with reality. A model is most useful when it imitates reality closely, but there will be aspects of reality which it does not reproduce, and it will predict events that do not in fact occur. The skill of the scientist lies in knowing how far, and in what context, to use a particular mathematical model.

1.3.1 A model of projectile motion

1. Projectile are particles and as a result are subject to no resistance forces which depend upon their size. All resulting motion will be translational.

For real problems, modelling a body as a particle is a major assum
Any body of finite size will rotate, and that rotational motion may a
to have an effect on the path of the projectile. The effects of resis
can be large, but will be minimized if the timc of flight is kept shor
velocity is not large, and the body's dimensions are small. A careful an
of these assumptions provides a useful feed back to the projectile pro

2. The acceleration due to gravity is a constant. This is a reaso
assumption.

3. The motion is confined to vertical plane. This is not always the ca
real cases of projectile motion, as any golfer or footballer will know. T
projectile will usually move sideways.

4. The space in which the projectile travels is a vacuum. This assump
removes many of the problems of resistance already mentioned. The eff
of wind and air currents would not be experienced in a vacuum.

1.3.2 A model of simple harmonic motion

As another example we consider the motion of a particle lying on a smo
horizontal table and joined to one end of the spring, which is fixed at
other end. The system is illustrated in figure 1.2. We set up a model

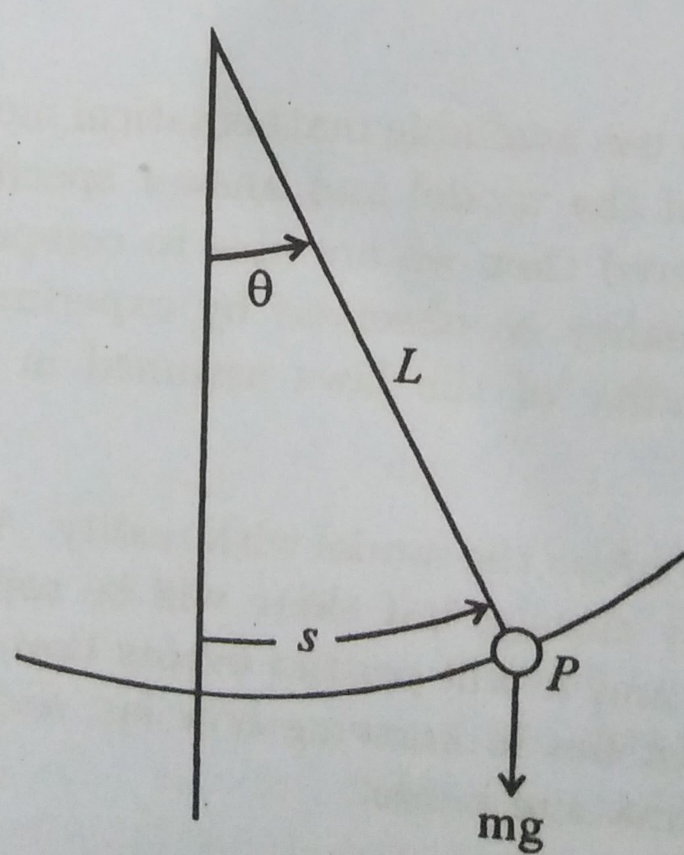

Figure 1.2: Model of h

(ii) the force acting on the particle and its used are governed by Newton's second law of motion.

If$x=x(t)$ is the extension at anytime as measured from the equilibrium position, then $T=\lambda x$, and$T=-md^2x/dt^2$.

On eliminating $T$from these equations, we obtain the differential equation

$$\frac{d^2x}{dt^2}+\frac{\lambda}{m}x=0$$

This DE predicts that the motion of the particle is simple harmonic with a definite period. If the calculated value of the period agrees with experimentally observed value, we conclude that the model is good. But if the observed motion of the particle does not correlate with the predicted results, we return to our model and examine its underlying assumptions. We may find that same important feature such as friction has been neglected. On including it the resulting DE will be found to be more complicated, but a better description of reality.

1.4 Other Fundamental Concepts of Mechanics

As pointed out earlier the concepts of space, time, matter (mass) and force are fundamental in mechanics. But to develop a complete theory of motion many other concepts and terms have also been derived. In this section we will discuss the concepts of work, energy and conservation of mass and energy.

1.4.1 Role of collisions in mechanics

The study of simple collisions has played a central role in establishing of the mechanics. It was thought that collisions play key role in many physical phenomena. Many of the most prominent scientists and thinkers of the seventeenth century studied interactions between balls made of steel, glass, cork, wax and wool looking for an underlying pattern of behaviour, that is, a physical law. In1644 Descartes introduced the concept of conservation, arguing that a certain *quantity of motion*had been given to matter by God, and could not be changed by human action. Early attempts to verify Descartes hypothesis gave contradictory results. The newly-established Royal Society London issued an appeal for papers to clarify the physics ...tician John Wallis, one of