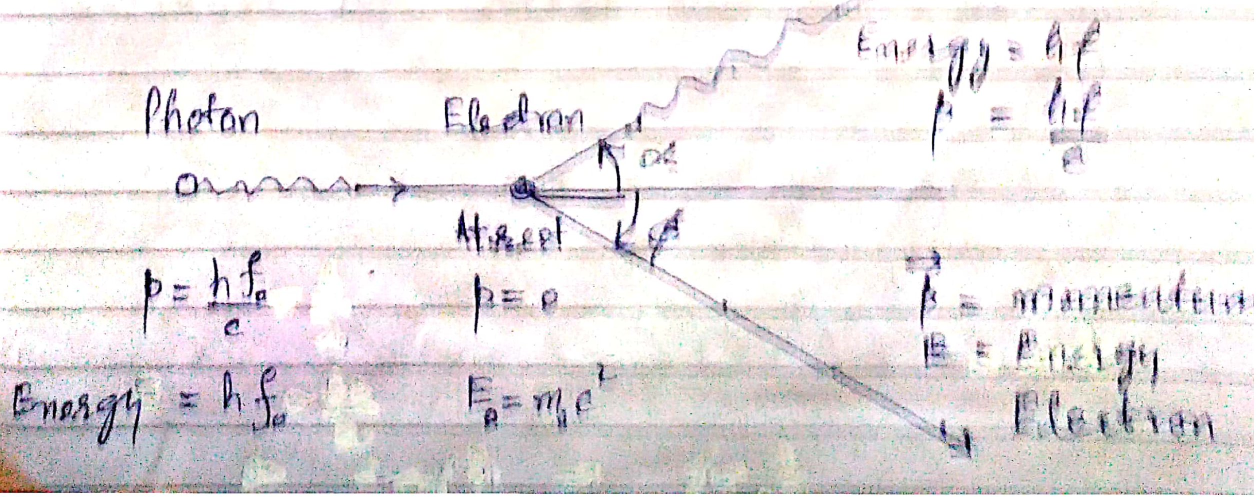
Compton Effect.

When electromagnetic radiation of high frequency and energy is incident of a light element in which the electrons are loosely bound to the nucleus and can be treated as free, the Scattered rays are found to have smaller frequency as that of original frequency. This is known as the Compton effect

The change in frequency of the incident radiation is independent of its initial frequency and depends only upon the angle of Scattering.

Consider a photon of frequency f_0 strike with an electron which is at rest. After Scattering both particles moves in different direction. Photon makes an angle α and electron makes an angle ϕ with the initial direction of photon as shown



The Compton effect can be treated as a problem of collision between a photon and a stationary free electron.

According to the law of conservation of energy

Before collision   After collision

$$hf_0 + m_0c^2 = hf + E$$

$$hf_0 - hf = E - m_0c^2 \quad \text{---}$$

Squaring both sides we get

$$h^2f_0^2 + h^2f^2 - 2h^2f_0f = E^2 + m_0^2c^4 - 2Em_0c^2 \quad \text{---}$$

By law of conservation of momentum

Before collision   After collision

vector form $\frac{hf_0}{c}\hat{n} + 0 = \frac{hf}{c}\hat{n}' + \vec{p}$

where $\hat{n}$ unit vector for incident photon, $\hat{n}'$ unit vector for scattered photon

$$hf_0\hat{n} - hf\hat{n}' = \vec{p}c$$

Squaring both sides

$$h^2f_0^2 + h^2f^2 - 2h^2f_0f\cos\phi = p^2c^2 \quad \text{---}$$

using momentum energy relation

$$E^2 = p^2c^2 + m_o^2c^4$$

$$p^2c^2 = E^2 - m_o^2c^4$$

(iii) becomes

$$h^2f_o^2 + h^2f^2 - 2h^2ff_o\cos\alpha = E^2 - m_o^2c^4 \quad \text{--- (IV)}$$

Subtracting (ii) from (IV) we get

$$2h^2ff_o(1-\cos\alpha) = 2Em_oc^2 - 2m_o^2c^4$$

$$2h^2ff_o(1-\cos\alpha) = 2m_oc^2(E - m_oc^2)$$

$$h^2ff_o(1-\cos\alpha) = m_oc^2(hf_o - hf) \quad \text{using (i)}$$

$$\frac{h^2}{m_oc^2}(1-\cos\alpha) = h\frac{(f_o - f)}{ff_o}$$

$$\frac{h}{m_oc^2}(1-\cos\alpha) = \frac{1}{f} - \frac{1}{f_o}$$

using $c = f\lambda$

$$= \frac{\lambda}{c} - \frac{\lambda_o}{c}$$

$$\Rightarrow \lambda - \lambda_o = \frac{h}{m_oc}(1-\cos\alpha)$$

the quantity

$$\frac{h}{m_oc} = 0.0024\,nm$$

known as compton wave length

Q: Show that it is impossible for photon to transfer all its energy to free electron.

Sol. Suppose a photon of energy E and momentum p transfer all its energy to an electron of mass m velocity V using energy momentum relation

$$E = T = mc^2 - m_0c^2$$

$$E = m_0\gamma c^2 - m_0c^2$$

$$E = (\gamma - 1)m_0c^2$$

$$\frac{E}{c} = (\gamma - 1)m_0c \quad \text{--- i)}$$

Momentum of a photon $p = \frac{E}{c}$

Momentum of a electron

$$p = mv = \gamma m_0 V$$

$$\Rightarrow \frac{E}{c} = \gamma m_0 V \quad \text{--- ii)}$$

From i) & ii)

$$\gamma m_0 V = (\gamma - 1)m_0 c$$

$$\frac{V}{c} = 1 - \frac{1}{\gamma}$$

$$= 1 - \left(1 - \frac{V^2}{c^2}\right)^{\frac{1}{2}}$$

$$\left(1 - \frac{V^2}{c^2}\right)^{1/2} = 1 - \frac{V}{c}$$

Sq

$$1 - \frac{V^2}{c^2} = 1 + \frac{V^2}{c^2} - 2\frac{V}{c}$$

$$-2\frac{V^2}{c^2} = -2\frac{V}{c}$$

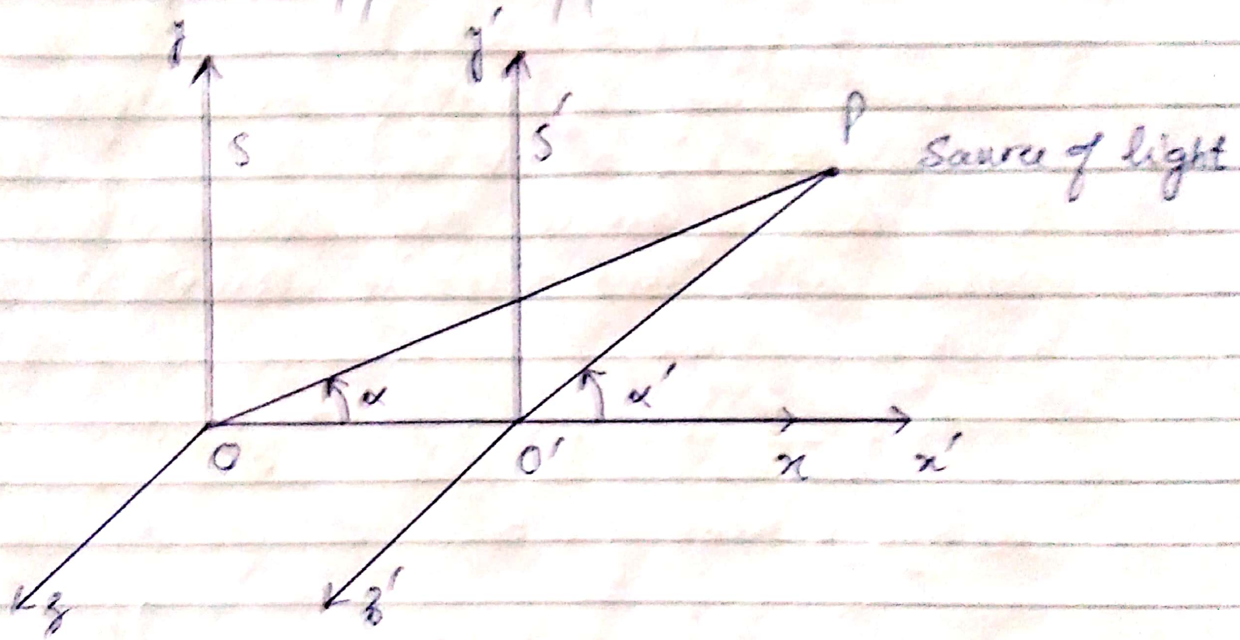
$$\Rightarrow \frac{V}{c} = 1 \Rightarrow V = c$$

which is impossible as no material particle can move with velocity of light

DOPPLER EFFECT

The change in the frequency of light radiation emitted by a source due to the relative motion of the source and the observer is called the Doppler effect.

We will use the transformation laws of energy and momentum to derive the relativistic formula for the Doppler effect.



Let S and S' be two frames of reference such that S' is moving relative to S with a velocity V along the positive direction of the x-axis.

Suppose that a source P of a monochromatic radiation is at rest in S' and is emitting radiation of frequency f' as measured by an observer at the origin O' in S'.

Then according to this observer the energy and magnitude of the

momentum of the photon are given by
$E' = hf'$ $\quad p' = \frac{hf'}{c}$
If f is the frequency of the same radiation as measured by an observer at the origin O of S, then energy and momentum of photon
$E = hf$ $\quad p = \frac{hf}{c}$
Transformation law for energy
$$E' = \gamma(E - v p_x)$$
Inverse transformation for energy
$$E = \gamma(E' + v p_x') \quad \text{---- (i)}$$
If α' is the angle the line $\overrightarrow{OP}$ makes with the positive direction of x-axis, then
$$p_x' = \frac{hf'}{c}\cos(\pi - \alpha') = -\frac{hf'}{c}\cos\alpha'$$
using values of E' and p_x' in (i)
$$hf = \gamma\left[hf' + v\left(-\frac{hf'}{c}\cos\alpha'\right)\right]$$
$$f = \gamma f'\left[1 - \frac{v}{c}\cos\alpha'\right] \text{---- (ii)}$$
where f' is proper frequency of light. The inverse transformation for Doppler effect is given by
$$f' = \gamma f\left[1 + \frac{v}{c}\cos\alpha\right] \text{---- (iii)}$$
where α is the angle which the line $\overrightarrow{OP}$ makes with x-axis

We shall now consider two special cases

1. Longitudinal Doppler Effect

When the source P is receding directly away from the observer O $\alpha' = 0$ (ii) becomes

$$f = \gamma f' \left(1 - \frac{v}{c}\right)$$

$$= f' \frac{\left(1 - \frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$f = f' \sqrt{\left(1 - \frac{v}{c}\right) / \left(1 + \frac{v}{c}\right)}$$

Since $\sqrt{\left(1 - v/c\right) / \left(1 + \frac{v}{c}\right)} < 1$

$$\Rightarrow \quad f < f'$$

It shows that when the source moves away from the observer the frequency of light radiation is smaller than it's proper frequency frequency of light is shifted towards the red end of the spectrum.

When source is moving directly towards the observer $\alpha' = \pi$ (ii) becomes

$$f = \gamma f' \left(1 + \frac{v}{c}\right)$$

$$f = f'\sqrt{(1+v/c)/(1-\frac{v}{c})}$$

$$\Rightarrow \quad f > f'$$

As $\sqrt{(1+\frac{v}{c})/(1-\frac{v}{c})} > 1$

f is greater the f'
It shows that light is shifted towards violet colour.

2- Transvorse Doppler Effect.

If $\alpha' = 90°$

$$f = \gamma(f' - 0)$$

$$f = \gamma f'$$

In classical mechanics

$\gamma = 1 \qquad f = f'$

In relativity

$\gamma > 1 \qquad f > f'$

The frequency of light changed when the source moving perpendicular to the direction of motion of light