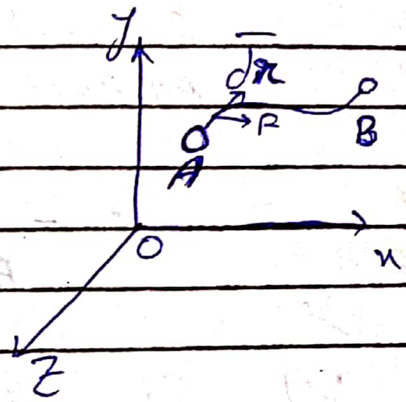
## Line Integral:-

let us consider a system of particles with an entire masses under certain laws of Mechanics in an inertial frame then if the trajectory is evaluated along the curve that describes the evaluation of line integral if $\phi$ and $A$ be the scalar and vector valued ftn's than line integral describes for.

(a) $\int_C \phi\, dr$ , $\int_C A \cdot dr$ ,(c) $\int_C A \times dr$

Note that $\phi, A, r$ be taken for $c^m$ ∀ $m \in R$. where (a) and (c) resulted for vector while (b) gives a net scalar quantity.
Also if the system of particles is located in 2-D or 3-D that gives for multiple integrals extended for double and tripple integrals representing the surface and volume integrals.

$\oint$ Let us consider a particle moving in inertial frame than for infinitecimal changes or displacements, the total work done by applied force is calculated for:




$$dw = F \cdot dr \quad \text{—} \quad (1)$$

where for the sake of sol of Eq. (1) if we integrate on both sides then it will be reffered to ordinary integral fn for the applied force.

Example of line Integral.

i) The line integral $\oint_C F \cdot dr$ represents the work done by the force along the regular curve c for different values of $m \in \mathbb{R}$

ii) The quantitative relationship b/w current $i$ and magnetic field $B$ is given by Ampere's Law:

$$\oint B \cdot dl = \mu_0 i$$

iii) The Potential difference b/w two Points A and B is related to The electric field by line integral

$$-\int_A^B E \cdot dl = V_B - V_A$$

v) If $A = \text{grad}\, \phi$ then $\int_A^B A \cdot dr$

depends ~~along~~ only on the initial and boundry values of $\phi$ depending on the Path Also

$$\oint A \cdot dr = 0$$

Conversely if $\oint A \cdot dr = 0$

Then $\exists$ a scalar ftn $\phi$ s.t (?)

$A = \text{grad}\, \phi$

vector said conservative.

Then The vector field is ~~Said~~ Said to be Conservative white $\oint$ closed or line integrals. ~ is closed integral! little bit!

Date: 27-02-2020

Day:

So,

v) Magnetostatics is also very important example of line integral of a loop of wire C carrying I in magnetic field B then

$$dF = I\, dr \times B$$

So the entire force is defined for

$$F = I \oint_C dr \times B.$$

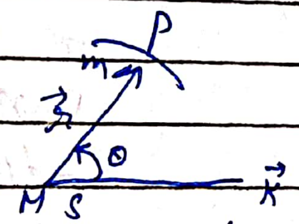
Hey!

~~[illegible]~~ — what wh

02-03-2020 Monday,

# Kepler's Problem:-

Let us consider a gravitational system in which particles of specified mass are moving in an inertial frame. Also if M be the mass of sun, K be the arbitrary fixed vector along an arbitrary reference line in polar axis then the force acting on the planet by the sun is




$$F = -\left(\frac{GmM}{r^3}\right)\vec{r} \quad \text{—} \ (1)$$

By Newton's II law for

$$F = ma$$

or

$$F = m\frac{dV}{dt} \quad — ②$$

From ① and ②

$$\frac{GmM}{r^3}\vec{r} = m\frac{dV}{dt}$$

or

$$\frac{dV}{dt} = -\frac{GM}{r^3}\vec{r} \quad — ②^*$$

Now,

$$\frac{d}{dt}(r \times V) = r \times \frac{dV}{dt} + \frac{dr}{dt} \times V \quad ③$$

$$\therefore \frac{d}{dt}(r \times V) = r \times \frac{dV}{dt} \quad — ④$$

It is noted that IInd term on the R.H.S of ③ vanishes because $\parallel$ vector's so after using $②^*$ in ④

$$\frac{d}{dt}(r \times V) = r \times \left(-\frac{GM}{r^3} r\right) = 0$$

and $r \times V = h =$ Const. Vector

or,

$$r \times \frac{dr}{dt} = h \text{ for } V = \frac{dr}{dt}$$

Since,

$$\left| r \times \frac{dr}{dt} \right| = \text{twice the area of sector.}$$

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That we get

$$2\left|\frac{dA}{dt}\right| = |h| \quad (5)$$

This eq (5) is the Kepler's IInd law of Planetary Motion. If two vectors in scalar triple product are identical then the product is zero.

$$r\cdot\left(r\times\frac{dr}{dt}\right) = r\cdot h = 0 \quad (6)$$

where (6) shows that $r \perp h$ and the motion of Planet. Taking cross product of (4)

$$\frac{dr}{dt}\times h = -\frac{GMr}{r^3}\times h$$

$$= -\frac{GMr}{r^3}\times(r\times V) \quad (7)$$

Now,

$r = \vec{r}\,\hat{e}_r$ for $\hat{e}_r$ be unit vector

So,

$$V = \frac{dr}{dt} = \frac{r\,de_r}{dt} + \frac{dr}{dt}e_r$$

So (7) reduces to,

$$\frac{d}{dt}(V\times h) = -\frac{GM}{r^3}r\times\left(r\times r\frac{de_r}{dt}\right)$$

$$= -GMe_r\times\left(e_r\times\frac{de_r}{dt}\right)$$

or,

$$\frac{d}{dt}(V\times h) = -GM\left[\left(e_r\cdot\frac{de_r}{dt}\right)e_r - (e_r\cdot e_r)\frac{de_r}{dt}\right]$$

Using the vector identities, we have,

$$\frac{d}{dt}(v\times h) = GM\frac{de_r}{dt}$$, on integrating gives

$$\int \frac{d}{dt}(v\times h) = \int GM\frac{de_r}{dt}$$

$$\Rightarrow \quad v\times h = GMe_r$$

if

$$r\cdot(v\times h) = r\cdot(GMe_r + K)$$

Then implies for

$$r\cdot(v\times h) = GM(r\cdot e_r) + r\cdot K$$

from the polar curves,

$$r\cdot(v\times h) = GMr + rK\cos\theta$$

where $K$ is arbitrary const with specified magnitude, $\theta$ is angle b/w $K$ and $e_r$. so using relation for 3-vectors in BX. 9 gives,

$$r\cdot(v\times h) = (r\times v)\cdot h = h\cdot h = h^2$$

for $r\times v = h$.

$$\Rightarrow \boxed{r = \frac{h^2}{GM + K\cos\theta}} \rightarrow (8)$$

This is taken as the Kepler's first law that is responsible for the motions of the planets around sun to show that curl is elliptic in nature for a closed curve c.

## Surface Integrals:-

under the classical forces is taken for the double of intersected area about a closed curve/path gives for,

(a) $\iint_s \phi \, ds$ (b) $\iint_s A \cdot ds$ (c) $\iint_s A \times ds$

OR,

$\oiint_s \phi \, ds$ or $\oiint_s A \cdot ds$ or $\oiint_s A \times ds$.