Curl OF Vector:

Let V be The Vector Valued ftn of C^m $\forall\, m \in N$ Then The quantity for curl is defiened for the twist in the system of Particles for:

Day:

$$\nabla \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ U & V & W \end{vmatrix}$$ for $V = U\hat{i} + V\hat{j} + Z\hat{k}$

Similarly for the cylindrical/spherical object, the system of particles must actually twist along V_r, V_θ and V_z/V_ϕ (V_ϕ)

Let $V = W \times R$ be a linear ~~vector~~ velocity of the particles at any point $P(x,y,z)$ with specified radius vector R then it's twist maybe defined for

$$\text{curl } V = \nabla \times V = \nabla \times (W \times R) \text{ — (1)}$$

from the identities.

$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B) \text{ — (2)}$$

So (1) in terms of (2) gives

$$\text{curl } V = W(\nabla \cdot R) - R(\nabla \cdot W) \text{ — (3)}$$

If W is a constant vector representing the uniform angular ~~velocity~~ motion than from

BX-29 we have

$\nabla \cdot R = 3$, and

$$R(\nabla \cdot W) = (W \cdot \nabla) R$$

implies for:

$$(W \cdot \nabla) R = \left\{ W_x \frac{\partial}{\partial x} + W_y \frac{\partial}{\partial y} + W_z \frac{\partial}{\partial z} \right\}$$

$\frac{1}{2}\nabla\times(\hat{i}x+\hat{j}y+\hat{k}z)$

$= \hat{i}w_x + \hat{j}w_y + \hat{k}w_z = \omega$

So eq ③ becomes after substituting the identies for,

$\text{Curl } V = 3\omega - \omega = 2\omega$

or

$\nabla \times V = 2\omega$

or

$\omega = \frac{1}{2}\nabla \times V$

That gives the twice in the rotational impact!

$\Rightarrow$ syste of Partics is irrotational if $\nabla \times V = 0$

and if $\nabla \times V \neq 0$ — " " Rotational

If $\nabla \times V = 0$

$\Rightarrow 2\omega = 0$

$\Rightarrow \omega = 0$ and $2 \neq 0$

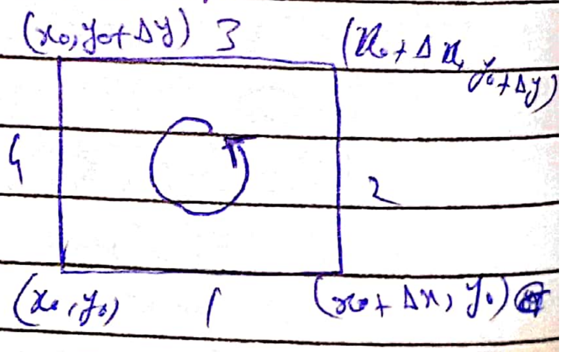
Idh? tk!

Circulations:

let V be a vector valued ftn of c^m $\forall\, m \in N$

Consider a differential loop in xy-Plane

Than,



circulation = $\int_1 V_x(x,y)\,dx + \int_2 V_y(x,y)\,dy - \int_3 V_x(x,y)\,dx - \int_4 V_y(x,y)\,dy$

After using Taylor's expansions in the displacement of the line segments 3 from 1 and 2 from 4 gives,

$$\Gamma_2 = \text{circulation}_{1234} = \left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right] dx\,dy + \frac{d}{dz}(\phi w)$$

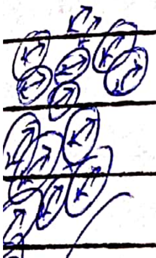
$$\frac{\partial \phi}{\partial y} + w\frac{\partial \phi}{\partial z}.$$

or

$$\text{circulation}/\text{unit Area} = \frac{\Gamma}{2} = \nabla \times V \Big|_z$$

as we have circulation in div impact for

good but not good.



$$\Gamma_1 = \oint V \cdot ds$$

if $\nabla \cdot V = 0 \Rightarrow$ incompressible (since flow Manners in the array of Particles.)

if $\nabla \cdot V \neq 0 \Rightarrow$ compressible (flow)

if diff b/w Particle could be short ... is called shrinking

if experiencing the compression.

Best Examples
water (incompressible)
gas (compressible)

Physical significances of curl of vector field:

(i) Curl of a ~~vortex~~ vector field implies for a circulation or vortex motion in twist form or rotation. If the closed path is taken for $\oint A \cdot dr = 0$ Than the field is conservative

(ii) If E represents the electrostatic field Than it will be conserved, for curl $E = 0$ or $\nabla \times E = 0$

(iii) For the elastic Medium if the displacement is irrotational. i.e

$\nabla \times U = 0$ then Plane wave (Spherical waves at large distance) becomes longitudinal wave if V is Solinoidal i.e $\nabla \cdot U = 0$ then the Plane wave becomes transverse wave.

If the displacement is resolved as a combination of solinoidal and irrotational then the plane wave becomes earthquake wave where longitudinal wave is taken to be Primary and Solinoidal is taken for Secondary wave.

iv) If V be the linear velocity describing the motion of a rigid body rotating with angular velocity ω with the relation for curl $V = 2\omega$ describing for the twice impact of rotational

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If A and B are irrotational than

$$\nabla \times A = 0 \text{ and } \nabla \times B = 0$$

or

$$B.\nabla \times A = 0 \text{ and } A.\nabla \times B = 0$$

Subtracting these eq's gives

$$B.(\nabla \times A) - A.(\nabla \times B) = 0 \quad -①$$

By using the identity in eq 28

$$\nabla.(\phi f) = \phi \nabla.f + f.\nabla\phi \quad -②$$

In sense of ② L.H.S of ① shows that

$$\nabla.(A \times B) = 0$$ that shows for

$A \times B$ to be solinoidal

Ex/36/

A central field A in space given by $A = rF(r)$ than $F(r)$, for solinoidal force $F(r)$,

$$\nabla.A = 0 \Rightarrow \nabla.rF(r) = 0$$

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Since this is class so,

$$\frac{\partial}{\partial x}[x F(r)] + \frac{\partial}{\partial y}[y F(r)] + \frac{\partial}{\partial z}[z F(r)] = 0$$

$$\Rightarrow F + x\frac{\partial F}{\partial x} + F + y\frac{\partial F}{\partial y} + F + z\frac{\partial F}{\partial z} = 0$$

$$\Rightarrow 3F(r) + x\left(\frac{\partial F}{\partial r}\right)\cdot\frac{x}{r} + y\left(\frac{\partial F}{\partial r}\right)\frac{y}{r} + z\left(\frac{\partial F}{\partial r}\right)\frac{z}{r} = 0$$

$$\Rightarrow 3F(r) + \left(\frac{\partial F}{\partial r}\right)\left[\frac{x^2+y^2+z^2}{r}\right] = 0 \quad \text{—— (1)}$$

If $r^2 = x^2+y^2+z^2$

Then (1) gives,

$$\frac{\partial F}{F} = -3\frac{\partial r}{r} \quad \text{—— (2)}$$

where (2) shows a separable Eq

Solving (2) by integrating both sides gives

$\ln F = -3\ln r + \ln c$; c is arbitrary constant

using log properties gives

$$\ln F = \ln c - \ln r^3 = \ln\left(\frac{c}{r^3}\right)$$

taking exp on both sides

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$$\Rightarrow \boxed{F = \frac{c}{r^3}}$$

This is specified solinoidal force for solinoidal field given for A i.e

$$\boxed{A = \frac{r}{r^3}}$$ (inverse sq law)

Assignment (atleast one of each case for all members.)

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

$\nabla\cdot(A+B) = \nabla\cdot A + \nabla\cdot B$

$\nabla\cdot(\phi A) = (\nabla\phi)\cdot A + \phi(\nabla\cdot A)$

$\nabla\times(A+B) = \nabla\times A + \nabla\times B$

$\nabla\times(\phi A) = (\nabla\phi)\times A + \phi(\nabla\times A)$

$\nabla\cdot(A\times B) = B\cdot(\nabla\times A) - A\cdot(\nabla\times B)$

$\nabla\times(A\times B) = (B\cdot\nabla)A - B(\nabla\cdot A) - (A\cdot\nabla)B + A(\nabla\cdot B)$

$\nabla(A\cdot B) = (B\cdot\nabla)A + (A\cdot\nabla)B + B\times(\nabla\times A) + A\times(\nabla\times B)$

$\nabla\cdot(\nabla\phi) = \text{div grad}\,\phi$

$= \text{Laplacian}\,\phi$

$= \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2})$

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11) $\nabla \times (\nabla \phi) = 0 =$ curl of grad ϕ

12) $\nabla \cdot (\nabla \times A) = 0$
(Div of curl A is zero)

13) $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

14) $\nabla \cdot (\nabla \phi + \nabla \psi) = 0$

15) $\nabla \cdot r = 3$

16) $\nabla \times r = 0$

17) $(A \cdot \nabla) r = A$