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Divergence

Let $\vec{V}$ be the vector valued function of C^m $\forall m \in N$. Then its divergence can be defined for

$$\operatorname{div} \vec{V} = \nabla \cdot \vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(V_1\hat{i} + V_2\hat{j} + V_3\hat{k}\right)$$

$$\operatorname{div} \vec{V} = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z} \longrightarrow (1)$$

$\therefore$ (1) is taken as the divergence of $\vec{V}$ for the vector valued function.

Note that

$$\nabla \cdot V \neq V \cdot \nabla$$

Most importantly Div (V) represents the streaming of the flow in the mechanical impact of the applied forces under classical mechanics rules.

Note that if

$$\nabla \cdot V = 0 \quad \text{or} \longrightarrow (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{for} \quad V = u\hat{i} + v\hat{j} + w\hat{k}$$

where u, v & w are the velocity components along x, y & z in the flux identities parallel to the the x, y & z axes respectively representing 1D, 2D or 3D objects moving in an inertial frame obeying law of inertia representing continuous flux.

Now if ψ be the specified function that is responsible for the streaming of the particles under certain forces then

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x}$$

represents the velocity components in a Cartesian plane.

Similarly

For the cylindrical object, it may be represented for

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial r} \quad , \quad V_\theta = \frac{\partial \psi}{\partial \theta}$$

This is important to note that ψ must be satisfied in the co-ordinates of Equation of Continuity given in eq ②

Also the streaming may also be defined for,

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{\omega} \rightarrow ③$$

implies for 2D objects in the certain plane having system of particles is

$$\frac{dx}{u} = \frac{dy}{v}$$

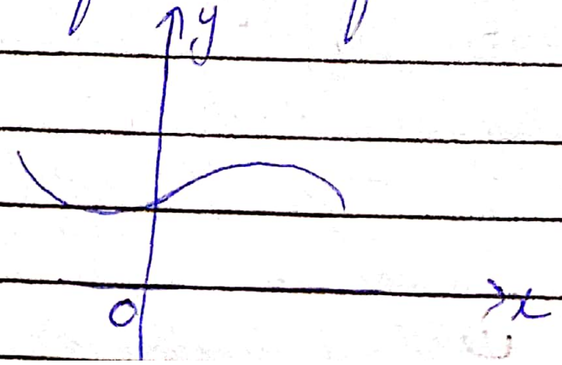
$$\rightarrow \quad v\,dx = u\,dy$$

fully separated in x & y respectively that is simply a separable eq. in ODE which may be solved just by integrating on both sides leads for a specified function say $y = f(x)$

escape plane

escape plane

escape plane



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Generally if densities of the particles are involved then ② is given for

$$\nabla . V + \frac{\partial \rho}{\partial t} = 0$$

That is an important equation in classical nature for the entire change in system of particles.

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Velocity Potential

Good but not good

(Physical Significance of Divergence/Del Operator):

Let us consider a system of particles with specified mass m_i, $i \in N$ & velocity vectors $v_i \in C^m$ $\forall$ $m \in N$, then its potential generated in the system of particles is given by

$$V = -\nabla\phi$$

where ϕ is differentiable form of C^m $\forall$ $m \in N$ as well.

For a twist generated in the system of particles

$$\nabla \times V = 0$$

$$\Rightarrow \nabla \times (-\nabla\phi) = -\nabla^2\phi = 0$$

that represents a Laplace Equation represented for

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad \text{for a 3D System}$$

So the velocity vector field is represented for

$$* \; u = \frac{-\partial\phi}{\partial x}, \; v = \frac{-\partial\phi}{\partial x} \longrightarrow ①$$

for a 2D system while velocity components ~~are~~

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in streaming are denoted for

$$u = \frac{\partial \psi}{\partial y} \quad \& \quad v = -\frac{\partial \psi}{\partial x} \longrightarrow ②$$

for rotating particles i.e ψ

$$\nabla \times V = 0$$

$$\Rightarrow \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \longrightarrow ③$$

Using ② in ③ gives

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \longrightarrow ④$$

thats also representation of Laplace equation.

If we use ① in eq of Continuity i.e

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \longrightarrow ⑤$$

So both of the functions ϕ & ψ represent Laplace equations.

Note that ϕ & ψ are orthogonal.

Let

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\Rightarrow u\,dy - v\,dx = 0 \longrightarrow Ⓐ$$

if $u = \frac{\partial \psi}{\partial y}$ & $v = -\frac{\partial \psi}{\partial x}$

then Ⓐ gives

$$\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0 \longrightarrow Ⓑ$$

If $\psi = \psi(x,y)$ then from Calculus:

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy \longrightarrow Ⓒ$$

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From (B) & (C):

$$d\psi = 0$$

that shows ψ is constant.

So tangential component i.e slope of ψ

$$\left.\frac{dy}{dx}\right|_{\psi} = \frac{v}{u} = \frac{-\frac{\partial\psi}{\partial x}}{\frac{\partial\psi}{\partial y}} = m_1 \rightarrow (D)$$

Now ϕ is also constant.

i.e $d\phi = 0$

$$\Rightarrow \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy = 0$$

So slope of ϕ:

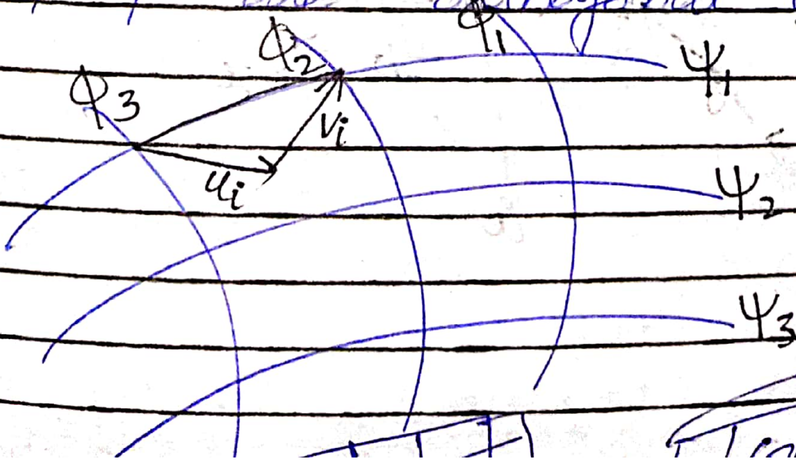
$$\left.\frac{dy}{dx}\right|_{\phi} = \frac{v}{u} = \frac{-\frac{\partial\phi}{\partial x}}{+\frac{\partial\phi}{\partial y}} = m_2 \rightarrow (E)$$

From (D) & (E)

$$m_1 \cdot m_2 = \left.\frac{dy}{dx}\right|_{\psi} \cdot \left.\frac{dy}{dx}\right|_{\phi} = -1.$$

condition of $\perp$.

So ψ & ϕ are orthogonal to each other.



Good!! Good!!