chapter

7

First-Order Circuits

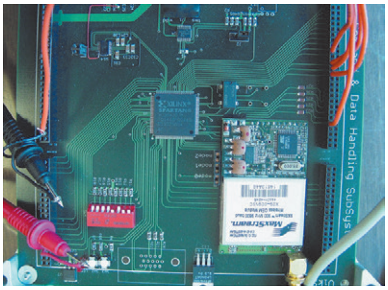
We live in deeds, not years; in thoughts, not breaths; in feelings, not in figures on a dial. We should count time in heart-throbs. He most lives who thinks most, feels the noblest, acts the best.

—P. J. Bailey

Enhancing Your Career

Careers in Computer Engineering

Electrical engineering education has gone through drastic changes in recent decades. Most departments have come to be known as Department of Electrical and Computer Engineering, emphasizing the rapid changes due to computers. Computers occupy a prominent place in modern society and education. They have become commonplace and are helping to change the face of research, development, production, business, and entertainment. The scientist, engineer, doctor, attorney, teacher, airline pilot, businessperson—almost anyone benefits from a computer's abilities to store large amounts of information and to process that information in very short periods of time. The internet, a computer communication network, is essential in business, education, and library science. Computer usage continues to grow by leaps and bounds.



Computer design of very large scale integrated (VLSI) circuits.
Courtesy Brian Fast, Cleveland State University

An education in computer engineering should provide breadth in software, hardware design, and basic modeling techniques. It should include courses in data structures, digital systems, computer architecture, microprocessors, interfacing, software engineering, and operating systems.

Electrical engineers who specialize in computer engineering find jobs in computer industries and in numerous fields where computers are being used. Companies that produce software are growing rapidly in number and size and providing employment for those who are skilled in programming. An excellent way to advance one's knowledge of computers is to join the IEEE Computer Society, which sponsors diverse magazines, journals, and conferences.

7.1 Introduction

Now that we have considered the three passive elements (resistors, capacitors, and inductors) and one active element (the op amp) individually, we are prepared to consider circuits that contain various combinations of two or three of the passive elements. In this chapter, we shall examine two types of simple circuits: a circuit comprising a resistor and capacitor and a circuit comprising a resistor and an inductor. These are called *RC* and *RL* circuits, respectively. As simple as these circuits are, they find continual applications in electronics, communications, and control systems, as we shall see.

We carry out the analysis of *RC* and *RL* circuits by applying Kirchhoff's laws, as we did for resistive circuits. The only difference is that applying Kirchhoff's laws to purely resistive circuits results in algebraic equations, while applying the laws to *RC* and *RL* circuits produces differential equations, which are more difficult to solve than algebraic equations. The differential equations resulting from analyzing *RC* and *RL* circuits are of the first order. Hence, the circuits are collectively known as *first-order* circuits.

> A **first-order** circuit is characterized by a first-order differential equation.

In addition to there being two types of first-order circuits (*RC* and *RL*), there are two ways to excite the circuits. The first way is by initial conditions of the storage elements in the circuits. In these so-called *source-free circuits*, we assume that energy is initially stored in the capacitive or inductive element. The energy causes current to flow in the circuit and is gradually dissipated in the resistors. Although source-free circuits are by definition free of independent sources, they may have dependent sources. The second way of exciting first-order circuits is by independent sources. In this chapter, the independent sources we will consider are dc sources. (In later chapters, we shall consider sinusoidal and exponential sources.) The two types of first-order circuits and the two ways of exciting them add up to the four possible situations we will study in this chapter.

Finally, we consider four typical applications of *RC* and *RL* circuits: delay and relay circuits, a photoflash unit, and an automobile ignition circuit.

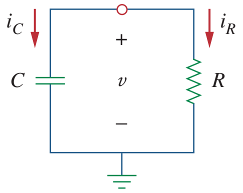


Figure 7.1
A source-free *RC* circuit.

7.2 The Source-Free *RC* Circuit

A source-free *RC* circuit occurs when its dc source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.

> A circuit response is the manner in which the circuit reacts to an excitation.

Consider a series combination of a resistor and an initially charged capacitor, as shown in Fig. 7.1. (The resistor and capacitor may be the equivalent resistance and equivalent capacitance of combinations of resistors and capacitors.) Our objective is to determine the circuit response, which, for pedagogic reasons, we assume to be the voltage

$v(t)$ across the capacitor. Since the capacitor is initially charged, we can assume that at time $t = 0$, the initial voltage is

$$v(0) = V_0 \tag{7.1}$$

with the corresponding value of the energy stored as

$$w(0) = \frac{1}{2}CV_0^2 \tag{7.2}$$

Applying KCL at the top node of the circuit in Fig. 7.1 yields

$$i_C + i_R = 0 \tag{7.3}$$

By definition, $i_C = C\,dv/dt$ and $i_R = v/R$. Thus,

$$C\frac{dv}{dt} + \frac{v}{R} = 0 \tag{7.4a}$$

or

$$\frac{dv}{dt} + \frac{v}{RC} = 0 \tag{7.4b}$$

This is a *first-order differential equation*, since only the first derivative of v is involved. To solve it, we rearrange the terms as

$$\frac{dv}{v} = -\frac{1}{RC}dt \tag{7.5}$$

Integrating both sides, we get

$$\ln v = -\frac{t}{RC} + \ln A$$

where $\ln A$ is the integration constant. Thus,

$$\ln\frac{v}{A} = -\frac{t}{RC} \tag{7.6}$$

Taking powers of e produces

$$v(t) = Ae^{-t/RC}$$

But from the initial conditions, $v(0) = A = V_0$. Hence,

$$v(t) = V_0 e^{-t/RC} \tag{7.7}$$

This shows that the voltage response of the *RC* circuit is an exponential decay of the initial voltage. Since the response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external voltage or current source, it is called the *natural response* of the circuit.

> The **natural response** of a circuit refers to the behavior (in terms of voltages and currents) of the circuit itself, with no external sources of excitation.

The natural response is illustrated graphically in Fig. 7.2. Note that at $t = 0$, we have the correct initial condition as in Eq. (7.1). As t increases, the voltage decreases toward zero. The rapidity with which

The natural response depends on the nature of the circuit alone, with no external sources. In fact, the circuit has a response only because of the energy initially stored in the capacitor.

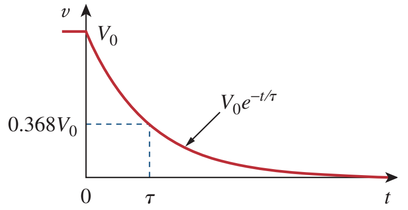


Figure 7.2
The voltage response of the RC circuit.

the voltage decreases is expressed in terms of the *time constant*, denoted by τ, the lowercase Greek letter tau.

> The **time constant** of a circuit is the time required for the response to decay to a factor of $1/e$ or 36.8 percent of its initial value.[1]

This implies that at $t = \tau$, Eq. (7.7) becomes

$$V_0e^{-\tau/RC} = V_0e^{-1} = 0.368V_0$$

or

$$\boxed{\tau = RC} \tag{7.8}$$

In terms of the time constant, Eq. (7.7) can be written as

$$\boxed{v(t) = V_0e^{-t/\tau}} \tag{7.9}$$

TABLE 7.1

Values of $v(t)/V_0 = e^{-t/\tau}$.

t	$v(t)/V_0$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674

With a calculator it is easy to show that the value of $v(t)/V_0$ is as shown in Table 7.1. It is evident from Table 7.1 that the voltage $v(t)$ is less than 1 percent of V_0 after 5τ (five time constants). Thus, it is customary to assume that the capacitor is fully discharged (or charged) after five time constants. In other words, it takes 5τ for the circuit to reach its final state or steady state when no changes take place with time. Notice that for every time interval of τ, the voltage is reduced by 36.8 percent of its previous value, $v(t + \tau) = v(t)/e = 0.368v(t)$, regardless of the value of t.

Observe from Eq. (7.8) that the smaller the time constant, the more rapidly the voltage decreases, that is, the faster the response. This is illustrated in Fig. 7.4. A circuit with a small time constant gives a fast response in that it reaches the steady state (or final state) quickly due to quick dissipation of energy stored, whereas a circuit with a large time constant gives a slow response because it takes longer to reach steady state. At any rate, whether the time constant is small or large, the circuit reaches steady state in five time constants.

With the voltage $v(t)$ in Eq. (7.9), we can find the current $i_R(t)$,

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R}e^{-t/\tau} \tag{7.10}$$

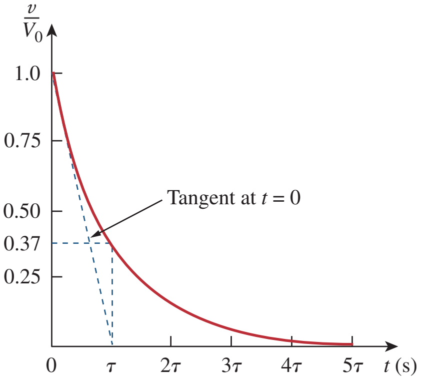


Figure 7.3
Graphical determination of the time constant τ from the response curve.

[1] The time constant may be viewed from another perspective. Evaluating the derivative of $v(t)$ in Eq. (7.7) at $t = 0$, we obtain

$$\frac{d}{dt}\left(\frac{v}{V_0}\right)\bigg|_{t=0} = -\frac{1}{\tau}e^{-t/\tau}\bigg|_{t=0} = -\frac{1}{\tau}$$

Thus, the time constant is the initial rate of decay, or the time taken for v/V_0 to decay from unity to zero, assuming a constant rate of decay. This initial slope interpretation of the time constant is often used in the laboratory to find τ graphically from the response curve displayed on an oscilloscope. To find τ from the response curve, draw the tangent to the curve at $t = 0$, as shown in Fig. 7.3. The tangent intercepts with the time axis at $t = \tau$.

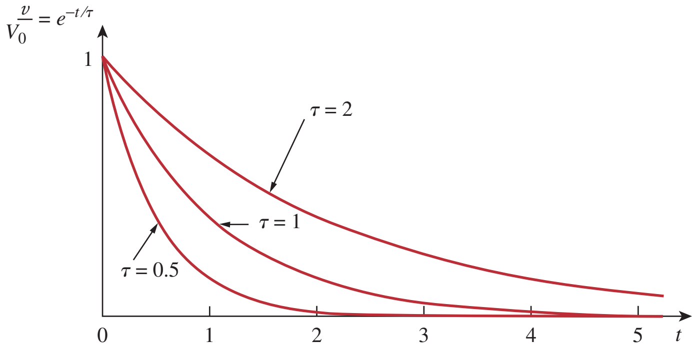


Figure 7.4
Plot of $v/V_0 = e^{-t/\tau}$ for various values of the time constant.

The power dissipated in the resistor is

$$p(t) = vi_R = \frac{V_0^2}{R}e^{-2t/\tau} \tag{7.11}$$

The energy absorbed by the resistor up to time t is

$$\begin{aligned} w_R(t) &= \int_0^t p(\lambda)d\lambda = \int_0^t \frac{V_0^2}{R}e^{-2\lambda/\tau}d\lambda \\ &= -\frac{\tau V_0^2}{2R}e^{-2\lambda/\tau}\Big|_0^t = \frac{1}{2}CV_0^2(1 - e^{-2t/\tau}), \qquad \tau = RC \end{aligned} \tag{7.12}$$

Notice that as $t \to \infty$, $w_R(\infty) \to \frac{1}{2}CV_0^2$, which is the same as $w_C(0)$, the energy initially stored in the capacitor. The energy that was initially stored in the capacitor is eventually dissipated in the resistor.

In summary:

The Key to Working with a Source-Free *RC* Circuit Is Finding:

1. The initial voltage $v(0) = V_0$ across the capacitor.
2. The time constant τ.

The time constant is the same regardless of what the output is defined to be.

With these two items, we obtain the response as the capacitor voltage $v_C(t) = v(t) = v(0)e^{-t/\tau}$. Once the capacitor voltage is first obtained, other variables (capacitor current i_C, resistor voltage v_R, and resistor current i_R) can be determined. In finding the time constant $\tau = RC$, R is often the Thevenin equivalent resistance at the terminals of the capacitor; that is, we take out the capacitor C and find $R = R_{Th}$ at its terminals.

When a circuit contains a single capacitor and several resistors and dependent sources, the Thevenin equivalent can be found at the terminals of the capacitor to form a simple *RC* circuit. Also, one can use Thevenin's theorem when several capacitors can be combined to form a single equivalent capacitor.

Example 7.1

In Fig. 7.5, let $v_C(0) = 15$ V. Find v_C, v_x, and i_x for $t > 0$.

Solution:

We first need to make the circuit in Fig. 7.5 conform with the standard *RC* circuit in Fig. 7.1. We find the equivalent resistance or the Thevenin

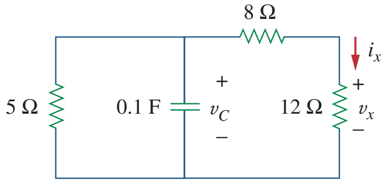


Figure 7.5
For Example 7.1.

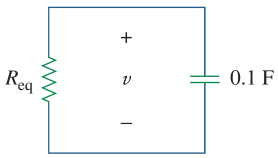


Figure 7.6
Equivalent circuit for the circuit in Fig. 7.5.

resistance at the capacitor terminals. Our objective is always to first obtain capacitor voltage v_C. From this, we can determine v_x and i_x.

The 8-Ω and 12-Ω resistors in series can be combined to give a 20-Ω resistor. This 20-Ω resistor in parallel with the 5-Ω resistor can be combined so that the equivalent resistance is

$$R_{eq} = \frac{20 \times 5}{20 + 5} = 4\ \Omega$$

Hence, the equivalent circuit is as shown in Fig. 7.6, which is analogous to Fig. 7.1. The time constant is

$$\tau = R_{eq}C = 4(0.1) = 0.4\ \text{s}$$

Thus,

$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\ \text{V}, \qquad v_C = v = 15e^{-2.5t}\ \text{V}$$

From Fig. 7.5, we can use voltage division to get v_x; so

$$v_x = \frac{12}{12 + 8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\ \text{V}$$

Finally,

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\ \text{A}$$

Practice Problem 7.1

Refer to the circuit in Fig. 7.7. Let $v_C(0) = 60$ V. Determine v_C, v_x, and i_o for $t \geq 0$.

Answer: $60e^{-0.25t}$ V, $20e^{-0.25t}$ V, $-5e^{-0.25t}$ A.

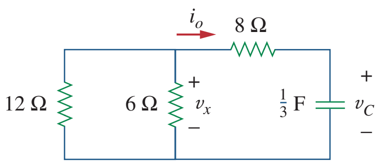


Figure 7.7
For Practice Prob. 7.1.

Example 7.2

The switch in the circuit in Fig. 7.8 has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.

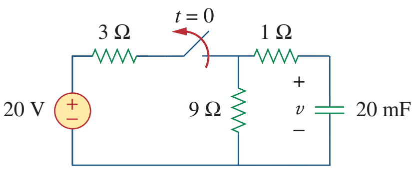


Figure 7.8
For Example 7.2.

Solution:
For $t < 0$, the switch is closed; the capacitor is an open circuit to dc, as represented in Fig. 7.9(a). Using voltage division

$$v_C(t) = \frac{9}{9 + 3}(20) = 15\ \text{V}, \qquad t < 0$$

Since the voltage across a capacitor cannot change instantaneously, the voltage across the capacitor at $t = 0^-$ is the same at $t = 0$, or

$$v_C(0) = V_0 = 15\ \text{V}$$

For $t > 0$, the switch is opened, and we have the *RC* circuit shown in Fig. 7.9(b). [Notice that the *RC* circuit in Fig. 7.9(b) is source free; the independent source in Fig. 7.8 is needed to provide V_0 or the initial energy in the capacitor.] The 1-Ω and 9-Ω resistors in series give

$$R_{eq} = 1 + 9 = 10\ \Omega$$

The time constant is

$$\tau = R_{eq}C = 10 \times 20 \times 10^{-3} = 0.2 \text{ s}$$

Thus, the voltage across the capacitor for $t \geq 0$ is

$$v(t) = v_C(0)e^{-t/\tau} = 15e^{-t/0.2} \text{ V}$$

or

$$v(t) = 15e^{-5t} \text{ V}$$

The initial energy stored in the capacitor is

$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25 \text{ J}$$

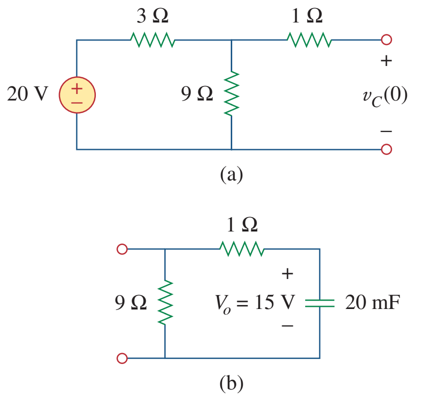


Figure 7.9
For Example 7.2: (a) $t < 0$, (b) $t > 0$.

Practice Problem 7.2

If the switch in Fig. 7.10 opens at $t = 0$, find $v(t)$ for $t \geq 0$ and $w_C(0)$.

Answer: $8e^{-2t}$ V, 5.333 J.

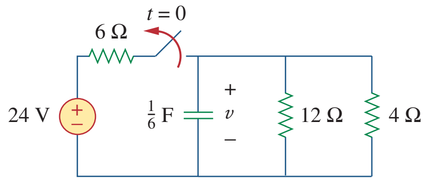


Figure 7.10
For Practice Prob. 7.2.

7.3 The Source-Free *RL* Circuit

Consider the series connection of a resistor and an inductor, as shown in Fig. 7.11. Our goal is to determine the circuit response, which we will assume to be the current $i(t)$ through the inductor. We select the inductor current as the response in order to take advantage of the idea that the inductor current cannot change instantaneously. At $t = 0$, we assume that the inductor has an initial current I_0, or

$$i(0) = I_0 \tag{7.13}$$

with the corresponding energy stored in the inductor as

$$w(0) = \frac{1}{2}L\,I_0^2 \tag{7.14}$$

Applying KVL around the loop in Fig. 7.11,

$$v_L + v_R = 0 \tag{7.15}$$

But $v_L = L\,di/dt$ and $v_R = iR$. Thus,

$$L\frac{di}{dt} + Ri = 0$$

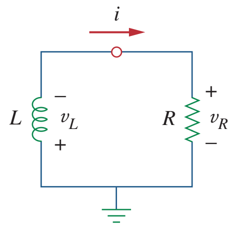


Figure 7.11
A source-free *RL* circuit.

or

$$\frac{di}{dt} + \frac{R}{L}i = 0 \tag{7.16}$$

Rearranging terms and integrating gives

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L}dt$$

$$\ln i\Big|_{I_0}^{i(t)} = -\frac{Rt}{L}\Big|_0^t \quad \Rightarrow \quad \ln i(t) - \ln I_0 = -\frac{Rt}{L} + 0$$

or

$$\ln \frac{i(t)}{I_0} = -\frac{Rt}{L} \tag{7.17}$$

Taking the powers of e, we have

$$i(t) = I_0 e^{-Rt/L} \tag{7.18}$$

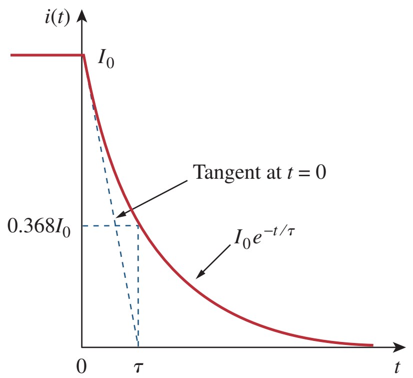


Figure 7.12
The current response of the RL circuit.

This shows that the natural response of the RL circuit is an exponential decay of the initial current. The current response is shown in Fig. 7.12. It is evident from Eq. (7.18) that the time constant for the RL circuit is

$$\boxed{\tau = \frac{L}{R}} \tag{7.19}$$

with τ again having the unit of seconds. Thus, Eq. (7.18) may be written as

$$\boxed{i(t) = I_0 e^{-t/\tau}} \tag{7.20}$$

The smaller the time constant τ of a circuit, the faster the rate of decay of the response. The larger the time constant, the slower the rate of decay of the response. At any rate, the response decays to less than 1 percent of its initial value (i.e., reaches steady state) after 5τ.

With the current in Eq. (7.20), we can find the voltage across the resistor as

$$v_R(t) = iR = I_0 R e^{-t/\tau} \tag{7.21}$$

The power dissipated in the resistor is

$$p = v_R i = I_0^2 R e^{-2t/\tau} \tag{7.22}$$

The energy absorbed by the resistor is

$$w_R(t) = \int_0^t p(\lambda)d\lambda = \int_0^t I_0^2 e^{-2\lambda/\tau}\, d\lambda = -\frac{\tau}{2} I_0^2 R e^{-2\lambda/\tau}\Big|_0^t, \qquad \tau = \frac{L}{R}$$

or

$$w_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau}) \tag{7.23}$$

Note that as $t \rightarrow \infty$, $w_R(\infty) \rightarrow \frac{1}{2} L I_0^2$, which is the same as $w_L(0)$, the initial energy stored in the inductor as in Eq. (7.14). Again, the energy initially stored in the inductor is eventually dissipated in the resistor.

Figure 7.12 shows an initial slope interpretation may be given to τ.

In summary:

The Key to Working with a Source-Free *RL* Circuit Is to Find:

1. The initial current $i(0) = I_0$ through the inductor.
2. The time constant τ of the circuit.

When a circuit has a single inductor and several resistors and dependent sources, the Thevenin equivalent can be found at the terminals of the inductor to form a simple *RL* circuit. Also, one can use Thevenin's theorem when several inductors can be combined to form a single equivalent inductor.

With the two items, we obtain the response as the inductor current $i_L(t) = i(t) = i(0)e^{-t/\tau}$. Once we determine the inductor current i_L, other variables (inductor voltage v_L, resistor voltage v_R, and resistor current i_R) can be obtained. Note that in general, R in Eq. (7.19) is the Thevenin resistance at the terminals of the inductor.

Example 7.3

Assuming that $i(0) = 10$ A, calculate $i(t)$ and $i_x(t)$ in the circuit of Fig. 7.13.

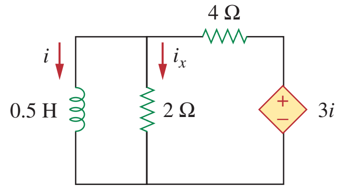


Figure 7.13
For Example 7.3.

Solution:
There are two ways we can solve this problem. One way is to obtain the equivalent resistance at the inductor terminals and then use Eq. (7.20). The other way is to start from scratch by using Kirchhoff's voltage law. Whichever approach is taken, it is always better to first obtain the inductor current.

METHOD 1 The equivalent resistance is the same as the Thevenin resistance at the inductor terminals. Because of the dependent source, we insert a voltage source with $v_o = 1$ V at the inductor terminals *a*-*b*, as in Fig. 7.14(a). (We could also insert a 1-A current source at the terminals.) Applying KVL to the two loops results in

$$2(i_1 - i_2) + 1 = 0 \quad \Rightarrow \quad i_1 - i_2 = -\frac{1}{2} \tag{7.3.1}$$

$$6i_2 - 2i_1 - 3i_1 = 0 \quad \Rightarrow \quad i_2 = \frac{5}{6}i_1 \tag{7.3.2}$$

Substituting Eq. (7.3.2) into Eq. (7.3.1) gives

$$i_1 = -3 \text{ A}, \qquad i_o = -i_1 = 3 \text{ A}$$

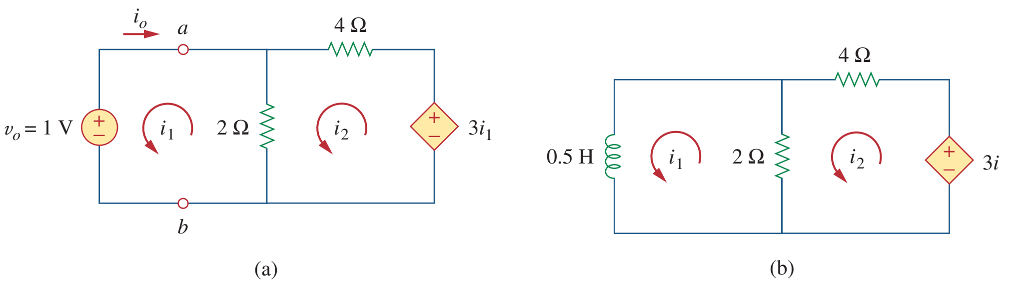


Figure 7.14
Solving the circuit in Fig. 7.13.

Hence,

$$R_{eq} = R_{Th} = \frac{v_o}{i_o} = \frac{1}{3}\,\Omega$$

The time constant is

$$\tau = \frac{L}{R_{eq}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}\text{ s}$$

Thus, the current through the inductor is

$$i(t) = i(0)e^{-t/\tau} = 10e^{-(2/3)t}\text{ A}, \qquad t > 0$$

METHOD 2 We may directly apply KVL to the circuit as in Fig. 7.14(b). For loop 1,

$$\frac{1}{2}\frac{di_1}{dt} + 2(i_1 - i_2) = 0$$

or

$$\frac{di_1}{dt} + 4i_1 - 4i_2 = 0 \tag{7.3.3}$$

For loop 2,

$$6i_2 - 2i_1 - 3i_1 = 0 \qquad \Rightarrow \qquad i_2 = \frac{5}{6}i_1 \tag{7.3.4}$$

Substituting Eq. (7.3.4) into Eq. (7.3.3) gives

$$\frac{di_1}{dt} + \frac{2}{3}i_1 = 0$$

Rearranging terms,

$$\frac{di_1}{i_1} = -\frac{2}{3}dt$$

Since $i_1 = i$, we may replace i_1 with i and integrate:

$$\ln i\Big|_{i(0)}^{i(t)} = -\frac{2}{3}t\Big|_0^t$$

or

$$\ln\frac{i(t)}{i(0)} = -\frac{2}{3}t$$

Taking the powers of e, we finally obtain

$$i(t) = i(0)e^{-(2/3)t} = 10e^{-(2/3)t}\text{ A}, \qquad t > 0$$

which is the same as by Method 1.

The voltage across the inductor is

$$v = L\frac{di}{dt} = 0.5(10)\left(-\frac{2}{3}\right)e^{-(2/3)t} = -\frac{10}{3}e^{-(2/3)t}\text{ V}$$

Since the inductor and the 2-Ω resistor are in parallel,

$$i_x(t) = \frac{v}{2} = -1.6667e^{-(2/3)t}\ \text{A}, \qquad t > 0$$

Practice Problem 7.3

Find i and v_x in the circuit of Fig. 7.15. Let $i(0) = 12$ A.

Answer: $12e^{-2t}$ A, $-12e^{-2t}$ V, $t > 0$.

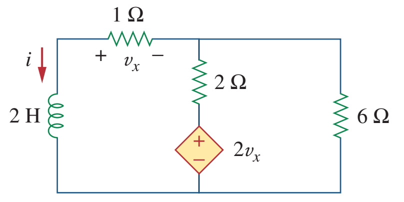


Figure 7.15
For Practice Prob. 7.3.

Example 7.4

The switch in the circuit of Fig. 7.16 has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.

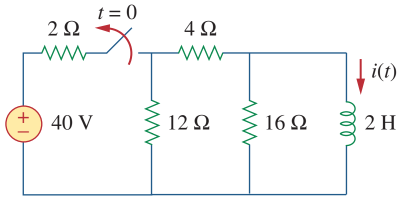


Figure 7.16
For Example 7.4.

Solution:
When $t < 0$, the switch is closed, and the inductor acts as a short circuit to dc. The 16-Ω resistor is short-circuited; the resulting circuit is shown in Fig. 7.17(a). To get i_1 in Fig. 7.17(a), we combine the 4-Ω and 12-Ω resistors in parallel to get

$$\frac{4 \times 12}{4 + 12} = 3\ \Omega$$

Hence,

$$i_1 = \frac{40}{2 + 3} = 8\ \text{A}$$

We obtain $i(t)$ from i_1 in Fig. 7.17(a) using current division, by writing

$$i(t) = \frac{12}{12 + 4} i_1 = 6\ \text{A}, \qquad t < 0$$

Since the current through an inductor cannot change instantaneously,

$$i(0) = i(0^-) = 6\ \text{A}$$

When $t > 0$, the switch is open and the voltage source is disconnected. We now have the source-free *RL* circuit in Fig. 7.17(b). Combining the resistors, we have

$$R_{eq} = (12 + 4) \parallel 16 = 8\ \Omega$$

The time constant is

$$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\ \text{s}$$

Thus,

$$i(t) = i(0)e^{-t/\tau} = 6e^{-4t}\ \text{A}$$

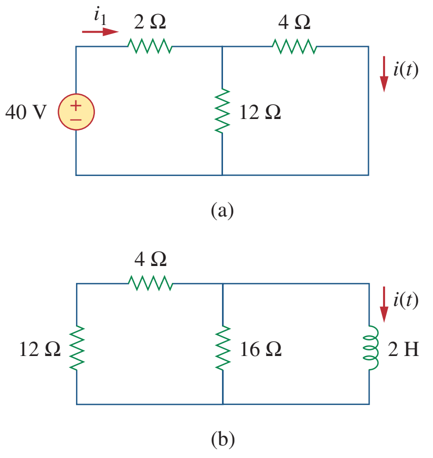


Figure 7.17
Solving the circuit of Fig. 7.16: (a) for $t < 0$, (b) for $t > 0$.

Practice Problem 7.4

For the circuit in Fig. 7.18, find $i(t)$ for $t > 0$.

Answer: $5e^{-2t}$ A, $t > 0$.

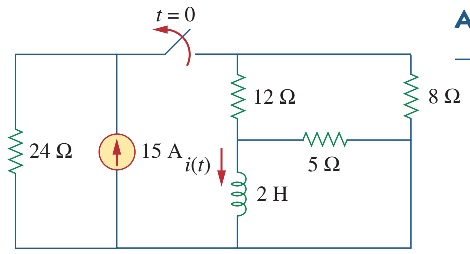


Figure 7.18
For Practice Prob. 7.4.

Example 7.5

In the circuit shown in Fig. 7.19, find i_o, v_o, and i for all time, assuming that the switch was open for a long time.

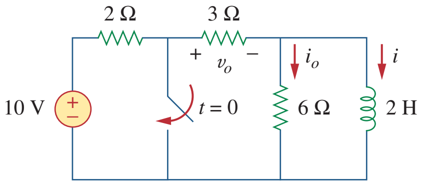


Figure 7.19
For Example 7.5.

Solution:
It is better to first find the inductor current i and then obtain other quantities from it.

For $t < 0$, the switch is open. Since the inductor acts like a short circuit to dc, the 6-Ω resistor is short-circuited, so that we have the circuit shown in Fig. 7.20(a). Hence, $i_o = 0$, and

$$i(t) = \frac{10}{2 + 3} = 2 \text{ A}, \qquad t < 0$$

$$v_o(t) = 3i(t) = 6 \text{ V}, \qquad t < 0$$

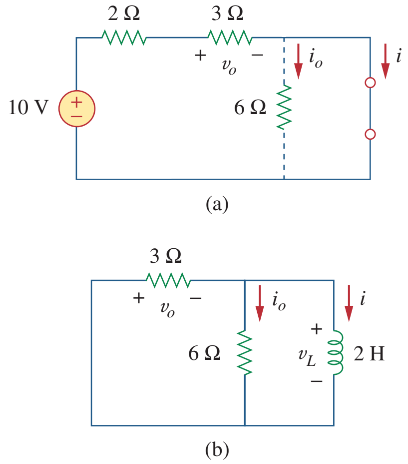


Figure 7.20
The circuit in Fig. 7.19 for: (a) $t < 0$, (b) $t > 0$.

Thus, $i(0) = 2$.

For $t > 0$, the switch is closed, so that the voltage source is short-circuited. We now have a source-free RL circuit as shown in Fig. 7.20(b). At the inductor terminals,

$$R_{\text{Th}} = 3 \parallel 6 = 2 \ \Omega$$

so that the time constant is

$$\tau = \frac{L}{R_{\text{Th}}} = 1 \text{ s}$$

Hence,

$$i(t) = i(0)e^{-t/\tau} = 2e^{-t} \text{ A}, \qquad t > 0$$

Since the inductor is in parallel with the 6-Ω and 3-Ω resistors,

$$v_o(t) = -v_L = -L\frac{di}{dt} = -2(-2e^{-t}) = 4e^{-t} \text{ V}, \qquad t > 0$$

and

$$i_o(t) = \frac{v_L}{6} = -\frac{2}{3}e^{-t} \text{ A}, \qquad t > 0$$

Thus, for all time,

$$i_o(t) = \begin{cases} 0 \text{ A}, & t < 0 \\ -\dfrac{2}{3}e^{-t} \text{ A}, & t > 0 \end{cases}, \qquad v_o(t) = \begin{cases} 6 \text{ V}, & t < 0 \\ 4e^{-t} \text{ V}, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2 \text{ A}, & t < 0 \\ 2e^{-t} \text{ A}, & t \geq 0 \end{cases}$$

We notice that the inductor current is continuous at $t = 0$, while the current through the 6-Ω resistor drops from 0 to $-2/3$ at $t = 0$, and the voltage across the 3-Ω resistor drops from 6 to 4 at $t = 0$. We also notice that the time constant is the same regardless of what the output is defined to be. Figure 7.21 plots i and i_o.

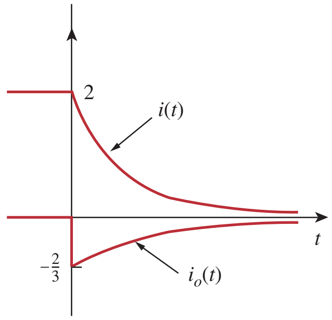


Figure 7.21
A plot of i and i_o.

Practice Problem 7.5

Determine i, i_o, and v_o for all t in the circuit shown in Fig. 7.22. Assume that the switch was closed for a long time. It should be noted that opening a switch in series with an ideal current source creates an infinite voltage at the current source terminals. Clearly this is impossible. For the purposes of problem solving, we can place a shunt resistor in parallel with the source (which now makes it a voltage source in series with a resistor). In more practical circuits, devices that act like current sources are, for the most part, electronic circuits. These circuits will allow the source to act like an ideal current source over its operating range but voltage-limit it when the load resistor becomes too large (as in an open circuit).

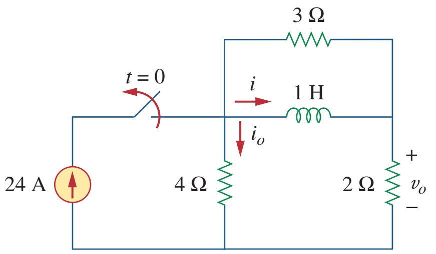


Figure 7.22
For Practice Prob. 7.5.

Answer:

$$i = \begin{cases} 16 \text{ A}, & t < 0 \\ 16e^{-2t} \text{ A}, & t \geq 0 \end{cases}, \qquad i_o = \begin{cases} 8 \text{ A}, & t < 0 \\ -5.333e^{-2t} \text{ A}, & t > 0 \end{cases},$$

$$v_o = \begin{cases} 32 \text{ V}, & t < 0 \\ 10.667e^{-2t} \text{ V}, & t > 0 \end{cases}$$

7.4 Singularity Functions

Before going on with the second half of this chapter, we need to digress and consider some mathematical concepts that will aid our understanding of transient analysis. A basic understanding of singularity functions will help us make sense of the response of first-order circuits to a sudden application of an independent dc voltage or current source.

Singularity functions (also called *switching functions*) are very useful in circuit analysis. They serve as good approximations to the switching signals that arise in circuits with switching operations. They are helpful in the neat, compact description of some circuit phenomena, especially the step response of *RC* or *RL* circuits to be discussed in the next sections. By definition,

> Singularity functions are functions that either are discontinuous or have discontinuous derivatives.