Gram Schmidt Process:

Let 'X' be 'n' dimensional inner product space and $\{x_1, x_2 \dots x_n\}$ is Basis for 'X' So by Gram schmidth process we must obtain orthonormal Basis $\{u_1, u_2 \dots u_n\}$

$$u_1 = \frac{y_1}{\|y_1\|} \qquad \text{wher } y_1 = x_1$$

$$u_2 = \frac{y_2}{\|y_2\|} \qquad y_2 = x_2 - \langle x_2, u_1 \rangle u_1$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$u_n = \frac{y_n}{\|y_n\|} \qquad y_n = x_n - \sum_{i=1}^{n-1} \langle x_n, u_i \rangle u_i$$

Example:

Use Gram-Schmidt orthonormalization process to find an othonormal basis for supspace of $\mathbb{R}^4$ Generated by

$$\{(1,1,0,1), (1,-2,0,0), (1,0,-1,2)\}$$

~~Proof:~~ Solution:

$x_1 = (1,1,0,-1)$, $x_2 = (1,-2,0,0)$

$x_3 = (1,0,-1,2)$

Since $\{x_1, x_2, x_3\}$ is Basis For X so

$\{x_1, x_2, x_3\}$ is linearly independent

$$\|x_1\|^2 = \langle x_1, x_2 \rangle = \langle (1,1,0,1), (1,1,0,1) \rangle$$

$$= 1.1 + 1.1 + 0.0 + 1.1$$

$$\|x_1\|^2 = 3$$

Let

$$y_1 = x_1$$

we find 'u_2' For $y_2 = ?$

$$\boxed{u_1 = \frac{x_1}{\|x_1\|} = \frac{(1,1,0,1)}{\sqrt{3}}}$$

$$y_2 = x_2 - \langle x_2, u_1 \rangle u_1$$

$$= (1,-2,0,0) - \left\langle (1,-2,0,0), \frac{(1,1,0,1)}{\sqrt{3}} \right\rangle \frac{(1,1,0,1)}{\sqrt{3}}$$

$$= (1,-2,0,0) - \left(\frac{1 \cdot 1 + (-2)(1) + (0)(0) + (0)(1)}{\sqrt{3}} \right) \frac{(1,1,0,1)}{\sqrt{3}}$$

$$= (1,-2,0,0) - \frac{1}{\sqrt{3}} (1 - 2 + 0 + 0) \frac{(1,1,0,1)}{\sqrt{3}}$$

$$(1,-2,0,0) - \frac{1}{3}(1,1,0,1) = \frac{1}{3}[(3,-6,0,0) - (1,1,0,1)]$$

$$y_2 = \frac{1}{3}\{(4, -8, 0, 1)\}$$

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we define $u_2 = \dfrac{y_2}{\|y_2\|}$

$$\|y_2\|^2 = \langle y_2, y_2 \rangle = \langle \tfrac{1}{3}(4,-5,0,1), \tfrac{1}{3}(4,-5,0,1) \rangle$$

$$= \tfrac{1}{3}\cdot\tfrac{1}{3}(4.4 + -5.-5 + 0.0 + 1.1) = \tfrac{42}{9}$$

$$\|y_2\| = \frac{\sqrt{42}}{3}$$

$$u_2 = \frac{\frac{1}{3}(4,-5,0,1)}{\frac{\sqrt{42}}{3}} = \frac{1}{\sqrt{42}}(4,-5,0,1)$$

Similarly

$$y_3 = x_3 - \langle x_3, u_2 \rangle u_2 - \langle x_3, u_1 \rangle u_1$$

$$y_3 = \frac{1}{7}(-4,-2,-7,6)$$

$$u_3 = \frac{y_3}{\|y_3\|}$$

$$\|y_3\|^2 = \langle y_3, y_3 \rangle = \frac{105}{49}$$

$$\|y_3\| = \frac{\sqrt{105}}{7}$$

$$u_3 = \frac{\frac{1}{7}(-4,-2,-7,6)}{\frac{\sqrt{105}}{7}} = \frac{1}{\sqrt{105}}(-4,-2,-7,6)$$

here $\{u_1, u_2, u_3\}$ is orthonormal Basis For subspace Generated by $\{x_1, x_2, x_3\}$

Assignment

In $\mathbb{R}^3$, Show that vectors

$\{x_1, x_2, x_3\} = \{(1,2,2), (2,1,-2), (2,-2,1)\}$

are orthogonal and Find the Corresponding set of vectors

Example:

let X be three diamensional inner product space spanned by $\{x_1, x_2, x_3\}$ with

$$\langle f, g \rangle = \int_{-1}^{1} f g \, dx$$

in interval $[-1, 1]$, use Grame-schmidt orthogonalization process to construct an orthonormal basis.

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