chapter 2

Linear Kinematics

Describing Objects in Linear Motion

objectives

When you finish this chapter, you should be able to do the following:

- Distinguish between linear, angular, and general motion
- Define distance traveled and displacement and distinguish between the two
- Define average speed and average velocity and distinguish between the two
- Define instantaneous speed and instantaneous velocity
- Define average acceleration
- Define instantaneous acceleration
- Name the units of measurement for distance traveled and displacement, speed and velocity, and acceleration
- Use the equations of projectile motion to determine the vertical or horizontal position of a projectile given the initial velocities and time

The world's best female sprinters are lined up at the starting line in the 100 m dash finals at the Olympic Games. The winner will earn the title of world's fastest woman. The starter's pistol goes off, and Shelly-Ann jumps into an early lead. At 50 m she has a 1 m lead on the other runners. But during the last 40 m of the race, Carmelita slowly reduces that lead. At the finish, Shelly-Ann finishes less than 1 m ahead of Carmelita and wins the race. Shelly-Ann wins the title of world's fastest woman, but was her top speed really faster than Carmelita's? Whose acceleration was greater? Were both athletes accelerating during the entire race? Did either athlete decelerate? What performance parameters could be used to account for the last 40 m of the race? These questions concern the kinematic measures of performance covered in this chapter.

This chapter is about the subbranch of mechanics called kinematics. Dynamics is the branch of rigid-body mechanics concerned with the mechanics of moving objects. Kinematics, the topic of this chapter, is the branch of dynamics concerned with the description of motion. The outcomes of many sporting events are kinematic measures, so an understanding of these measures is important. Some of the kinematic terminology introduced in this chapter may sound familiar to you (speed, velocity, acceleration, and so forth). You may believe that you already know all about these terms, but we will be using them in specific ways. The precise mechanical definitions may not agree with the meanings you associate with the terms, and there will be misunderstandings unless our definitions agree. With that in mind, let's begin.

➲ Kinematics is the branch of dynamics concerned with the description of motion.

Motion

What is motion? Can you define it? We might define motion as the action or process of a change in position. Movement is a change in position. Moving involves a change in position from one point to another. Two things are necessary for motion to occur: space and time—space to move in and time during which to move. To make the study of movement easier, we classify movements as linear, angular, or both (general).

Linear Motion

Linear motion is also referred to as translation. It occurs when all points on a body or object move the same distance, in the same direction, and at the same time. This can happen in two ways: rectilinear translation or curvilinear translation.

Rectilinear translation is the motion you probably would think of as linear motion. Rectilinear translation occurs when all points on a body or object move in a straight line so that the direction of motion does not change, the orientation of the object does not change, and all points on the object move the same distance.

Curvilinear translation is very similar to rectilinear translation. Curvilinear translation occurs when all points on a body or object move so that the orientation of the object does not change and all points on the object move the same distance. The difference between rectilinear and curvilinear translation is that the paths followed by the points on an object in curvilinear translation are curved, so the direction of motion of the object is constantly changing, even though the orientation of the object does not change.

Try to think of some examples of linear motion in sports or human movement. What about a figure skater gliding across the ice in a static position? Is her motion rectilinear or curvilinear? What about a sailboarder zipping across the lake in a steady breeze? Is it possible for the sailboarder's motion to be rectilinear? What about a bicyclist coasting along a flat section of the road? (In each of these examples, it is possible for the athletes to achieve rectilinear motion.) Can you think of any examples of curvilinear motion? Can a gymnast on a trampoline experience linear motion? How? What about a diver? A ski jumper? A skateboarder rolling along a flat section of concrete? An in-line skater? (It's possible for the gymnast, diver, and ski jumper to achieve curvilinear motion. The gymnast, diver, skateboarder, and in-line skater can achieve both rectilinear and curvilinear motion. The ski jumper can achieve rectilinear motion during the in-run to the jump, and curvilinear motion during the flight phase of the jump.)

To determine whether a motion is linear, imagine two points on the object in question. Now imagine a straight line connecting these two points. As the object moves, does the line keep its same orientation; that is, does the line point in the same direction throughout the movement? Does the line stay the same length during the movement? If both of these conditions are true throughout the movement, the motion is linear. If both points on the imaginary line move in parallel straight lines during the motion, the motion is rectilinear. If both points on the imaginary line move in parallel lines that are not straight, the motion is curvilinear. Now try to think of more examples of linear motion in sport. Would you classify the motions you thought of as rectilinear or curvilinear?

Angular Motion

Angular motion is also referred to as rotary motion or rotation. It occurs when all points on a body or object move in circles (or parts of circles) about the same fixed central line or axis. Angular motion can occur about an axis within the body or outside of the body. A child on a swing is an example of angular motion about an axis of rotation external to the body. An ice-skater in a spin is an example of angular motion about an axis of rotation within the body. To determine whether or not a motion is angular, imagine any two points on the object in question. As the object moves, are the paths that each of these points follow circular? Do these two circular paths have the same center or axis? If you imagine a line connecting the two imaginary points, does this line continuously change orientation as the object moves? Does the line continuously change the direction in which it points? If these conditions are true, the object is rotating.

Examples of angular motion in sports and human movement are more numerous than examples of linear motion. What about a giant swing on the horizontal bar? Are parts of this motion rotary? What about individual movements of our limbs? Almost all of our limb movements (if they are isolated) are examples of angular motion. Hold your right arm at your side. Keeping your upper arm still, flex and extend your forearm at the elbow joint. This is an example of angular motion. Your forearm rotated about a fixed axis (your elbow joint). During the flexing and extending, your wrist moved in a circular path about your elbow joint. Every point on your forearm and wrist moved in a circular path about your elbow joint. Consider each limb and the movements it can make when movement about only one joint is involved. Are these movements rotary—that is, do all the points on the limb move in circular paths about the joint?

Let's consider motion about more than one joint. Is the limb's motion still angular? Extend your knee and hip at the same time. Was the movement of your foot angular? Did your foot move in a circular path? Was the motion of your foot linear?

General Motion

Combining the angular motions of our limbs can produce linear motions of one or more body parts. When both the knee and hip joints extend, you can produce a linear motion of your foot. Similarly, extension at the elbow and horizontal adduction at the shoulder can produce a linear motion of the hand. **General motion** is a combination of linear and angular motions. Try self-experiment 2.1.

Self-Experiment 2.1

Grab hold of a pencil that is lying flat on a desk or a table. While keeping the pencil flat on the table, try to move the pencil rectilinearly across the table. Can you do it? You produced that motion by combining angular motions of your hand, forearm, and upper arm. The total motion of our limbs is called general or mixed motion.

General motion is the most common type of motion exhibited in sports and human movement. Running and walking are good examples of general motion. In these activities, the trunk often moves linearly as a result of the angular motions of the legs and arms. Bicycling is another example of general motion. Think of various human movements in sports and consider how you would classify them.

Classifying motion as linear, angular, or general motion makes the mechanical analysis of movements easier. If a motion can be broken down into linear components and angular components, the linear components can be analyzed using the mechanical laws that govern linear motion. Similarly, the angular components can be analyzed using the mechanical laws that govern angular motion. The linear and angular analyses can then be combined to understand the general motion of the object.

Classifying motion as linear, angular, or general motion makes the mechanical analysis of movements easier.

Linear Kinematics

Now let's examine linear motion in more detail. Linear kinematics is concerned with the description of linear motion. Questions about speed, distance, and direction are all inquiries about the linear kinematics of an object.

Try self-experiment 2.2 to identify some of the characteristics of linear motion.

Self-Experiment 2.2

How would you describe something that is moving? Roll a ball across the floor. Describe its movement. What words do you use? You might describe how fast or slow it is going, mention whether it is speeding up or slowing down, and note that it is rolling and not sliding. You also might say something about where it started and where it might end up. Or you might describe its direction: "It's moving diagonally across the room," or "It's moving toward the wall or toward the door." After it stops, you might say how far it traveled and how long it took to get to where it went. All of the terms you used to describe the motion of the ball are words that concern the kinematics of linear motion.

Position

The first kinematic characteristic we might describe about an object is its position. Our definition of motion—the action or process of change in position—refers to position. Mechanically, **position** is defined as location in space. Where is an object in space at the beginning of its movement or at the end of its movement or at some time during its movement? This may not seem like such an important characteristic at first, but consider the importance of the positions of players on the field or court in sports such as football, tennis, racquetball, squash, soccer, field hockey, ice hockey, and rugby. The strategies employed often depend on where the players on each team are positioned.

Let's start with a simple example. Consider a runner competing in a 100 m dash (see figure 2.1). How would you go about describing the runner's position during the race? You might describe the runner's position relative to the starting line: "She's 40 m from the start." Or you might describe the runner's position relative to the finish line: "She's 60 m from the finish." In both cases, you have used a measure of length to identify the runner's position relative to some fixed, nonmoving reference.* The references were the starting line and the finish line. Some concept of direction was also implied by your description and the event itself. When you say the runner is 40 m from the start, this is usually interpreted to mean that the runner is 40 m in front of the start and toward the finish line. Mechanically, if we used the starting line as our reference, we would say that the runner is at +40 m. If the runner was on the other side of the starting line, we would describe the runner's position as –40 m. We use the positive and negative signs to indicate which side of the starting line the runner is on.

This example of the 100 m dash is only one-dimensional. We were concerned about only one dimension—the dimension from the starting line to the finish line. Only one number was required to identify the position of the runner in the race. Now let's consider a two-dimensional situation. Imagine you are watching a game of American football. A running back has broken out of the backfield and is running toward the goal line. He is on the opposing team's 20 yd line. To describe his position, you would say he is 20 yd from the goal line. But to fully describe his position, you would also have to give information about his location relative to the sidelines. Using the left sideline as a reference, you could then describe his position as 20 yd from the goal line and 15 yd from the left sideline. This is shown in figure 2.2*a*.

In this situation, it might be helpful to set up a **Cartesian coordinate system** to help identify the location of the runner. Cartesian coordinates are named after René Descartes (1596-1650), a French philosopher and mathematician who is credited with inventing analytic geometry. You may remember this type of coordinate system from high school mathematics. First, we would need to locate a fixed reference point for our coordinate system. This fixed point is called the origin, because all our position measurements originate from it. Let's put the origin for this system at the intersection of the left sideline and the running back's goal line. We could put the origin at any fixed point; we chose the intersection of the goal line and the sideline because it was convenient. Imagine the *x*-axis lying along the goal line with zero at the origin and positive numbers to the right on the playing field. Imagine the *y*-axis lying along the left sideline with zero at the origin and positive numbers increasing as you move toward the opposite goal. With this system, we could identify the running back's position with two numbers corresponding to his *x*- and *y*-coordinates in yards as follows: (15, 80). This situation is shown in figure 2.2*b*. The *x*-coordinate of 15 indicates that he is 15 yd from the left sideline on the field, and the *y*-coordinate of 80 indicates that he is 80 yd from his goal line or 20 yd from scoring, because we know that the goal lines are 100 yd apart.

In three dimensions, we would need three numbers to describe the position of an object in space. For example, how would you describe the position of the ball during a game of racquetball? We might set up a three-dimensional Cartesian coordinate with one axis in the vertical direction and two axes in the horizontal plane. If we put the

*Are these really fixed, nonmoving references? Relative to the surface of the earth they are, but the earth itself is moving around the sun in the solar system. And the solar system is moving in the galaxy. And the galaxy is moving in the universe. So, it is difficult to define a position in terms of an absolute nonmoving reference frame. For our purposes, however, we will consider anything that doesn't move relative to the earth's surface a fixed reference.

Figure 2.1 How would you describe a runner's position in a 100 m dash?

point of reference or origin in the lower left front corner of the court (where the front wall, left side wall, and floor intersect), the *x*-axis would be the line along the intersection of the front wall and floor. The *y*-axis would be the line along the intersection of the front wall and the left side wall, and the *z*-axis would be the line along the intersection of the left side wall and the floor. This is shown in figure 2.3. If the ball were 3 m to the right of the left side wall, 2 m above the floor, and 4 m away from the front wall, its *x*-, *y*-, and *z*-coordinates in meters would be (3, 2, 4).

In three dimensions, we would need three numbers to describe the position of an object in space.

To describe the position of something in space, we need to identify a fixed reference point to serve as the origin of our coordinate system. For our purposes, any point fixed relative to the earth will do. Then we set up a Cartesian coordinate system. If we are describing the position of objects in only one dimension, only one axis is needed; for two dimensions, two axes are needed; and for three dimensions, three axes are needed. The axes of this system may point in any direction that is convenient, as long as they are at right angles to each other if we are describing the position of something in two or three dimensions. Typically, one axis will be oriented vertically (the *y*-axis), and the other axis (the *x*-axis) or axes (the *x*- and *z*-axes) will be oriented horizontally. Each of these axes will have a positive and negative direction along them. The *x*-coordinate of an object is the distance the object is away from the plane formed by the *y*- and *z*-axes. The *y*-coordinate of an object is the distance the object is away from the plane formed by the *x*- and *z*-axes, and the *z*-coordinate of an object is the distance the object is away from the plane formed by the *x*- and *y*-axes. Units of length are used to describe position.

Distance Traveled and Displacement

Now we have a method of describing and locating the position of an object in space. This is our first task in describing motion. If we remember how we defined motion—the action or process of change in position—our next task will be discovering a way to describe or measure change in position. How would you do this?

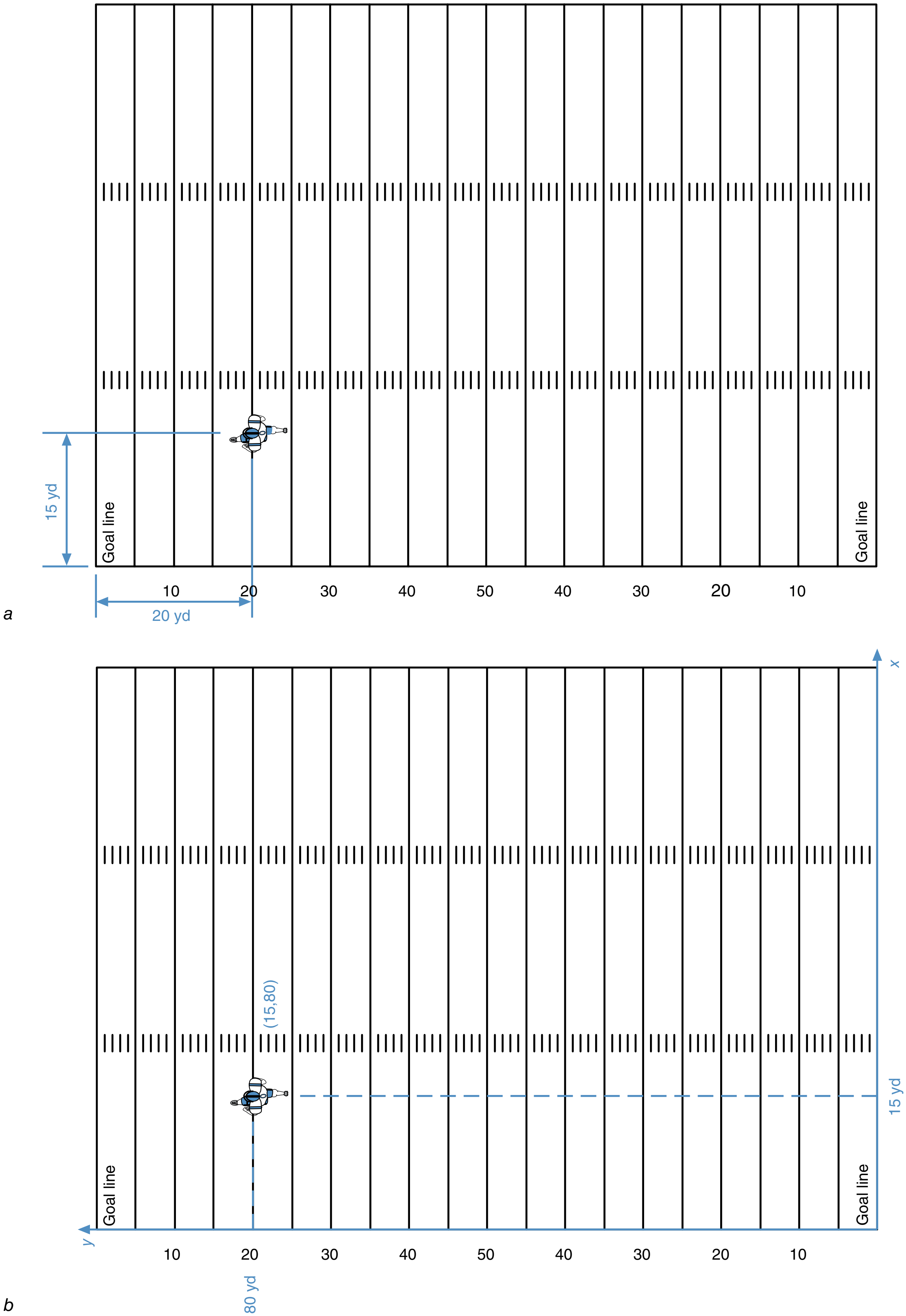


Figure 2.2 The location of a running back on a football field using the sideline and the opponent's goal line as references *(a)* or using a Cartesian coordinate system *(b)*.

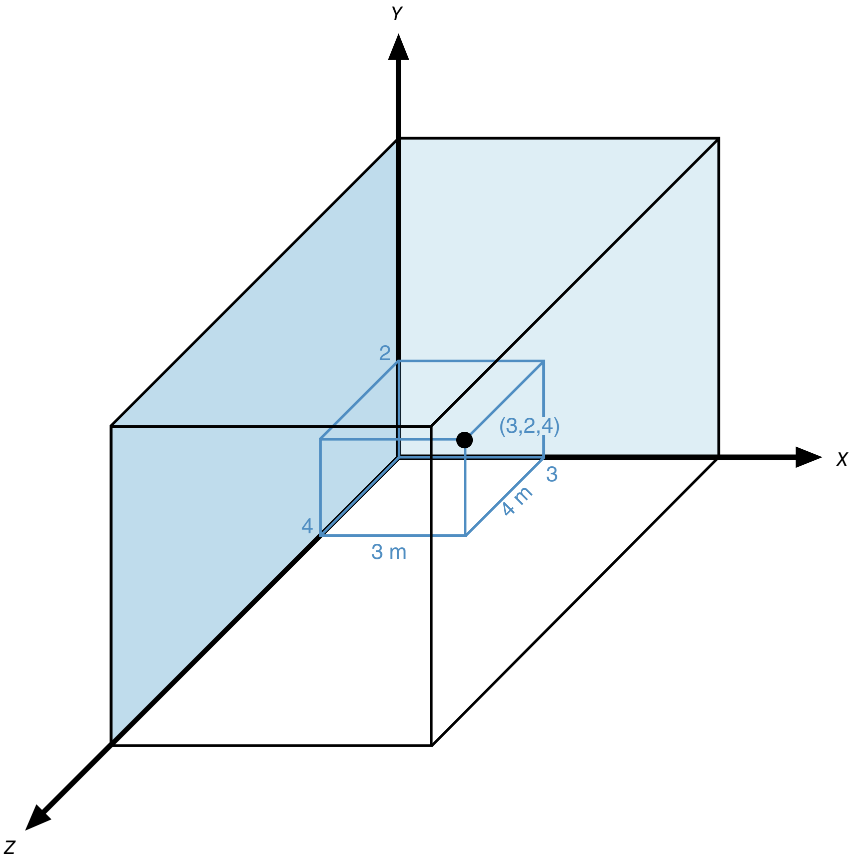


Figure 2.3 The location of a ball in a racquetball court, using Cartesian coordinates.

Distance Traveled

Let's use a football example again. Suppose a football player has received the kickoff at his 5 yd line, 15 yd from the left sideline. His position on the field (using the Cartesian coordinate system we established in the previous section) is (15, 5) when he catches the ball. He runs the ball back following the path shown in figure 2.4*a*. He is finally tackled on his 35 yd line, 5 yd from the left sideline. His position on the field at the end of the play is (5, 35). If we measure the length of the path of his run with the ball, it turns out to be 48 yd. So we might describe this run as a run of 48 yd to gain 30 yd.

Another way of saying this is to say that the runner's displacement was +30 yd in the *y*-direction and −10 yd in the *x*-direction, or a resultant displacement of 31.6 yd toward the left sideline and goal. The distance traveled by the runner was 48 yd. We've used two different terms to describe the runner's progress: **displacement** and **distance traveled**. Distance traveled is easily defined—it's simply a measure of the length of the path followed by the object whose motion is being described, from its starting (initial) position to its ending (final) position. Distance traveled doesn't mean a whole lot in a football game, though, because the direction of travel isn't considered. Displacement does take into account the direction of travel.

Displacement

Displacement is the straight-line distance in a specific direction from initial (starting) position to final (ending) position. The **resultant displacement** is the distance measured in a straight line from the initial position to the final position. Displacement is a vector quantity. If you recall from chapter 1, we said force was also a vector quantity. A vector has a size associated with it as well as a direction. It can be represented graphically as an arrow whose length represents the size of the vector and whose orientation and arrowhead represent the direction of the vector. Representation of displacement with an arrow is appropriate and communicates what displacement means as well. Figure 2.4*b* shows the path of the player in the kick return example. The arrow from the initial position of the player to where he was tackled represents the displacement of the back.

Displacement is the straight-line distance in a specific direction from starting (initial) position to ending (final) position.

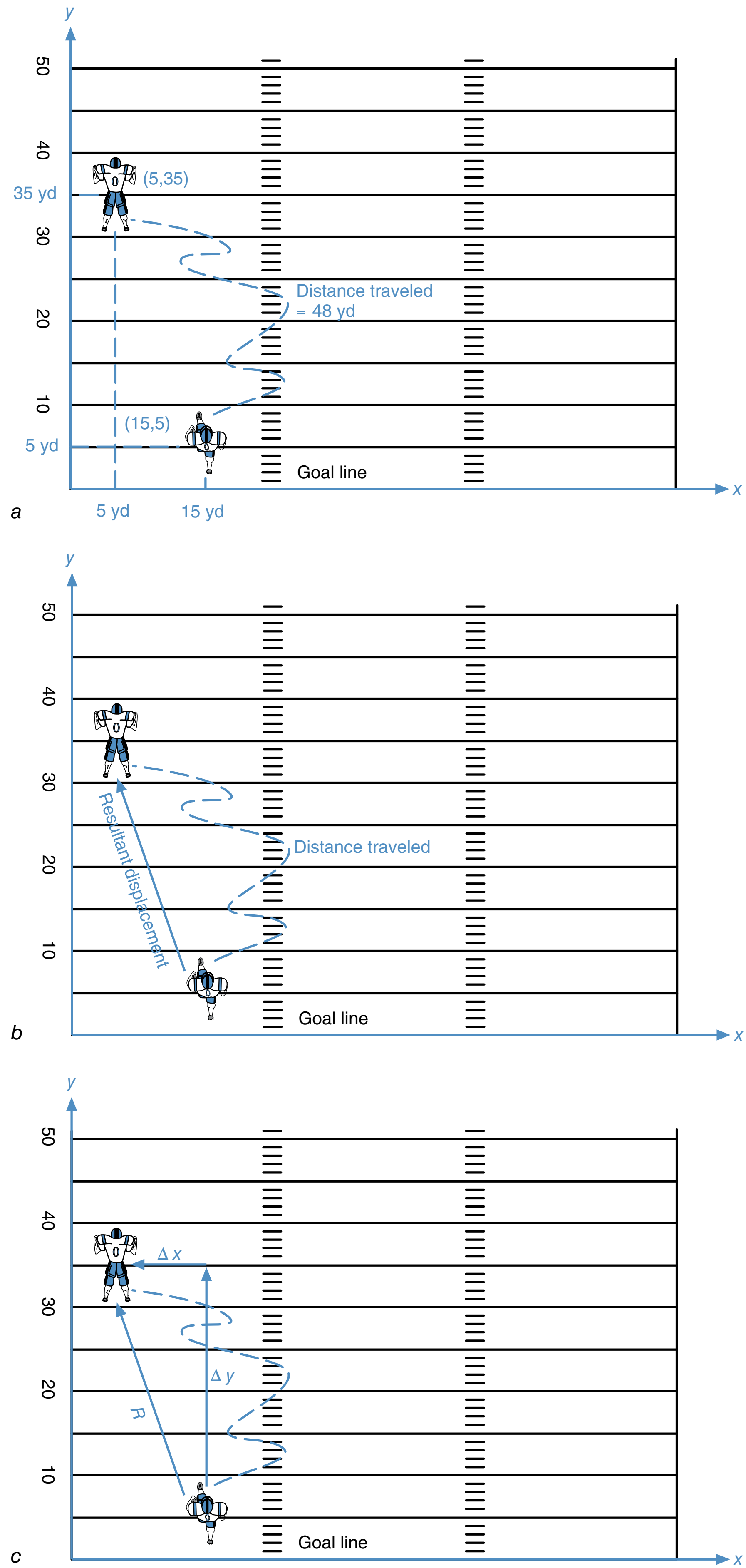


Figure 2.4 The kick return of a running back described with initial and final coordinate positions and distance traveled *(a)*, resultant displacement and distance traveled *(b)*, and resultant displacement and component displacements *(c)*.

If you also recall from chapter 1, vectors can be resolved into components. In the football example, the resultant displacement of the running back does not indicate directly how many yards the back gained. But if we resolve this resultant displacement into components in the x-direction (across the field) and y-direction (down the field toward the goal), we then have a measure of how effective the run was. In this case, the y-displacement of the running back is the measure of importance. His initial y-position was 5 yd and his final y-position was 35 yd. We can find his y-displacement by subtracting his initial position from his final position:

$$d_y = \Delta y = y_f - y_i \tag{2.1}$$

where

d_y = displacement in the y-direction,

Δ = change, so Δy = change in y-position,

y_f = final y-position, and

y_i = initial y-position.

If we put in the initial (5 yd) and final (35 yd) values for y-position, we get the runner's y-displacement:

$$d_y = \Delta y = y_f - y_i = 35\text{ yd} - 5\text{ yd}$$

$$d_y = +30\text{ yd}$$

The runner's y-displacement or displacement down the field was +30 yd. The positive sign indicates that the displacement was in the positive y-direction or toward the goal (a gain in field position in this case). This measure is probably the most important measurement to the coaches, players, and fans because it indicates the effectiveness of the kick return.

We may also be curious about the player's displacement across the field (in the x-direction). We can use the same equation to determine the x-displacement:

$$d_x = \Delta x = x_f - x_i \tag{2.2}$$

where

d_x = displacement in the x-direction,

Δx = change in x-position,

x_f = final x-position, and

x_i = initial x-position.

If we put in the initial (15 yd) and final (5 yd) values for x-position, we get the runner's x-displacement:

$$d_x = \Delta x = x_f - x_i = 5\text{ yd} - 15\text{ yd}$$

$$d_x = -10\text{ yd}$$

The runner's x-displacement or displacement across the field was −10 yd. The negative sign indicates that the displacement was in the negative x-direction or toward the left sideline.

We could find the resultant displacement of the runner similarly to the way we found a resultant force. Graphically, we could do this by drawing the arrows representing the component displacements of the runner in the x- and y-directions. Look at figure 2.4*c*. Put the tail of the x-displacement vector at the tip of the y-displacement vector, and then draw an arrow from the tail of the y-displacement vector to the tip of the x-displacement vector. This arrow represents the resultant displacement.

We could also determine this resultant displacement by starting with the x-displacement vector and then putting the tail of the y-displacement vector at the tip of the x-displacement vector. We would determine the resultant by drawing an arrow from the tail of the x-displacement vector to the tip of the y-displacement vector. We should get the same resultant as determined using the method shown in figure 2.4*c*.

We could determine this resultant displacement in still another way, using trigonometric relationships. The displacement vectors arranged as shown in figure 2.4*c* form a triangle, specifically, a right triangle with the hypotenuse represented by the resultant displacement. As explained in chapter 1, the size of the hypotenuse can be determined as follows. If A and B represent the two sides that make up the right angle and C represents the hypotenuse, then

$$A^2 + B^2 = C^2 \tag{2.3}$$

$$(\Delta x)^2 + (\Delta y)^2 = R^2.$$

For our displacements, then, we can substitute −10 yd for Δx and +30 yd for Δy and then solve for R, which represents the resultant displacement.

$$(-10\text{ yd})^2 + (30\text{ yd})^2 = R^2$$

$$100\text{ yd}^2 + 900\text{ yd}^2 = R^2$$

$$1000\text{ yd}^2 = R^2$$

$$R = \sqrt{1000\text{ yd}^2} = 31.6\text{ yd}$$

To find the direction of this resultant displacement, we can use the relationship between the two sides of the displacement triangle.

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} \tag{2.4}$$

$$\theta = \arctan\left(\frac{\text{opposite side}}{\text{adjacent side}}\right)$$

$$\theta = \arctan\left(\frac{\Delta x}{\Delta y}\right)$$

In these equations, θ, which is pronounced "theta," represents the angle between the resultant displacement vector and the y-displacement vector. To find the value of θ, substitute −10 yd for Δx and +30 yd for Δy.

$$\theta = \arctan\left(\frac{-10\text{ yd}}{30\text{ yd}}\right)$$

To determine the angle θ, we use the inverse of the tangent function or the arctangent. On most scientific calculators, the arctangent function is the second function for the tangent key and is usually abbreviated as $\tan^{-1}$.

$$\theta = \arctan(-.333)$$

$$\theta = -18.4°$$

We can now describe several qualities of movement—initial and final positions, distance traveled, and displacement. Distance traveled can be described by a single number that represents the length of the path followed by the object during its motion. Displacement, however, is a vector quantity, so it is expressed with a length measurement and a direction. The resultant displacement is the length of a straight line from the initial position to the final position in the direction of motion from the initial position to the final position. Components of the resultant displacement may also be used to describe displacement of the object in specific directions. In some situations (such as our football examples), a component displacement is more important than the resultant displacement.

Now let's see if we understand the concept of displacement. Imagine two downhill ski racers, Tamara and Cindy, competing on the same course. They start at the same starting position and finish the race at the same finish point. Tamara takes wider turns than Cindy does, so the length of the path Tamara follows is longer. Who has the greater resultant displacement from start to finish? Because they start at the same spot and finish at the same spot, their resultant displacements are the same. Now consider a 100 m swimming race in a 50 m pool. Which measure (displacement or distance traveled) is more meaningful? In a 100 m swimming race in a 50 m pool, you have to start and finish in the same place, so your displacement is zero! Distance traveled is the more meaningful measure. What about a 400 m running race around a 400 m oval track? Or a 100 m running race on a straight section of track?

Speed and Velocity

We can now describe an object's position, and we have measures (distance traveled and displacement) for describing its change in position, but how do we describe how quickly something changes its position? When we speak of how fast or slow something moves, we are describing its speed or velocity. Both are used to refer to the rate of motion. You have probably used both of these terms, perhaps interchangeably.

Speed

Are speed and velocity the same thing? Mechanically, speed and velocity are different. **Speed** is just rate of motion. More specifically, it is the rate of distance traveled. It is described by a single number only. **Velocity** is rate of motion in a specific direction. More specifically it is the rate of displacement. Since displacement is a vector quantity, so is velocity. Velocity has a magnitude (number) and a direction associated with it.

Speed is rate of motion; velocity is rate of motion in a specific direction.

Average speed of an object is distance traveled divided by the time it took to travel that distance. Mathematically, this can be expressed as

$$\bar{s} = \frac{\ell}{\Delta t} \tag{2.5}$$

where

$\bar{s}$ = average speed,

ℓ = distance traveled, and

Δt = time taken or change in time.

The units for describing speed are a unit of length divided by a unit of time. The SI unit for describing speed is meters per second. You have probably used other units of measurement for speed. If you have driven a car, you are probably more familiar with miles per hour or kilometers per hour. These are also units of measurement for speed.

Average speed is an important descriptor of performance in many sport activities. In some activities, average speed is in fact the measure of success. Consider almost any type of racing event (swimming, running, cycling, and so on). The winner is the person who completes the specified distance in the shortest time. The average speed of the winner is the distance of the race divided by the time. The winner's average speed over the race distance will always be the fastest among all the competitors if everyone raced the same distance.

This one number, average speed, doesn't tell us much about what went on during the race itself, though. It doesn't tell us how fast the racer was moving at any specific instant in the race. It doesn't tell us the maximum speed reached by the racer during the race. It doesn't indicate when the racer was slowing down or speeding up. Average speed for the whole race is just a number indicating that, on average, the competitor was moving that fast. To find out more about the speed of a competitor in a race, a coach or athlete may want to measure more than one average speed.

Let's look at a 100 m dash as an example. At the 12th IAAF World Championships in Athletics in Berlin in 2009, the men's 100 m dash was won by Usain Bolt of Jamaica in an astounding world-record time of 9.58 s. The second-place finisher, Tyson Gay of the United States, finished in 9.71 s, the fastest time ever for a second-place finish. Comparing the average speeds of these two sprinters over the entire 100 m using equation 2.5, we find the following:

$$\text{Average speed} = \bar{s} = \frac{\ell}{\Delta t}$$

Bolt:

$$\bar{s} = \frac{100 \text{ m}}{9.58 \text{ s}}$$

$$\bar{s} = 10.44 \text{ m/s}$$

Gay:

$$\bar{s} = \frac{100 \text{ m}}{9.71 \text{ s}}$$

$$\bar{s} = 10.30 \text{ m/s}$$

To find out more about how the two sprinters ran this race, we might have timed them for the first 50 m of the 100 m as well. Bolt's time for the first 50 m was 5.47 s. Gay's time for the first 50 m was 5.55 s. Their average speeds for the first 50 m of the race were

$$\text{Average speed} = \bar{s}_{0\text{-}50\text{m}} = \frac{\ell}{\Delta t}.$$

Bolt:

$$\bar{s}_{0\text{-}50\text{m}} = \frac{50 \text{ m}}{5.47 \text{ s}}$$

$$\bar{s}_{0\text{-}50\text{m}} = 9.14 \text{ m/s}$$

Gay:

$$\bar{s}_{0\text{-}50\text{m}} = \frac{50 \text{ m}}{5.55 \text{ s}}$$

$$\bar{s}_{0\text{-}50\text{m}} = 9.01 \text{ m/s}$$

Their average speeds from 50 to 100 m also could be determined:

$$\text{Average speed} = \bar{s}_{50\text{-}100\text{m}} = \frac{\ell}{\Delta t} = \frac{100 \text{ m} - 50 \text{ m}}{\Delta t}$$

Bolt:

$$\bar{s}_{50\text{-}100\text{m}} = \frac{100 \text{ m} - 50 \text{ m}}{9.58 \text{ s} - 5.47 \text{ s}}$$

$$\bar{s}_{50\text{-}100\text{m}} = \frac{50 \text{ m}}{4.11 \text{ s}}$$

$$\bar{s}_{50\text{-}100\text{m}} = 12.17 \text{ m/s}$$

Gay:

$$\bar{s}_{50\text{-}100\text{m}} = \frac{100 \text{ m} - 50 \text{ m}}{9.71 \text{ s} - 5.55 \text{ s}}$$

$$\bar{s}_{50\text{-}100\text{m}} = \frac{50 \text{ m}}{4.16 \text{ s}}$$

$$\bar{s}_{50\text{-}100\text{m}} = 12.02 \text{ m/s}$$

With two numbers to describe each runner's speed during the race, we know much more about how each runner ran the race. Usain Bolt took the lead in the first 50 m. His average speed was 0.13 m/s faster than Gay's over this portion of the race. Both sprinters were even faster over the second 50 m, but Bolt's average speed over the second 50 m was 0.15 m/s faster than Gay's.

If we wanted to know which athlete had the fastest top speed in the 100 m, we would have to record split times at more frequent intervals in the race. This would give us even more information about the performance of each sprinter. Sport scientists at the 12th IAAF World Championships in Athletics in Berlin recorded the split times at the 20, 40, 60, and 80 m marks for the finalists in the men's 100 m dashes (IAAF 2009). The split times the scientists recorded were used to estimate the 10 m split times for Usain Bolt and Tyson Gay shown in table 2.1.

These 10 m split times can be used to determine the average speed of each sprinter during each 10 m interval. To do this we divide the distance covered in each interval, 10 m in this case, by the time taken to run that distance, the interval time. Table 2.2 shows the values of each runner's average speed over each 10 m interval.

Now we have much more information about each sprinter's performance. From table 2.2, we can tell that

Table 2.1 Elapsed and Interval Times for Each 10 m Interval for Usain Bolt and Tyson Gay in the Men's 100 m Dash Final at the 12th IAAF World Championships in Athletics in Berlin, 2009

	Usain Bolt		Tyson Gay	
Position (m)	**Elapsed time (s)**	**Interval time (s)**	**Elapsed time (s)**	**Interval time (s)**
0	0		0	
10	1.89	1.89	1.91	1.91
20	2.88	.99	2.92	1.01
30	3.78	.90	3.83	.91
40	4.64	.86	4.70	.87
50	5.47	.83	5.55	.85
60	6.29	.82	6.39	.84
70	7.10	.81	7.20	.81
80	7.92	.82	8.02	.82
90	8.75	.83	8.86	.84
100	9.58	.83	9.71	.85

Table 2.2 Interval Times and Average Speeds for Each 10 m Interval for Usain Bolt and Tyson Gay in the Men's 100 m Dash Final at the 12th IAAF World Championships in Athletics in Berlin, 2009

	Usain Bolt		Tyson Gay	
Interval (m)	Interval time (s)	Average speed (m/s)	Interval time (s)	Average speed (m/s)
0-10	1.89	5.29	1.91	5.24
10-20	.99	10.10	1.01	9.90
20-30	.90	11.11	.91	10.99
30-40	.86	11.63	.87	11.49
40-50	.83	12.05	.85	11.76
50-60	.82	12.20	.84	11.90
60-70	.81	12.35	.81	12.35
70-80	.82	12.20	.82	12.20
80-90	.83	12.05	.84	11.90
90-100	.83	12.05	.85	11.76

Bolt was faster than Gay over every interval up to the 60 to 80 m. During the 60 to 70 m interval, both Bolt and Gay reached their maximum speed, and their average speeds during this interval were the same. After 70 m, both runners slowed down, but Gay slowed down more, especially over the last 20 m of the race, from 80 to 100 m.

By taking more split times during the race, we can determine the runners' average speeds for more intervals and shorter intervals. This procedure also gives us a better idea of what each runner's speeds were at specific instants of time during the race. The speed of an object at a specific instant of time is its **instantaneous speed**. The speed of an object may vary with time, especially in an event such as a 100 m dash. The maximum or top speed a runner achieves during a race is an example of an instantaneous speed. An average speed gives us an estimate of how fast something was moving over only an interval of time—not an instant in time. If we are told what a runner's average speed was for an interval of time, we can correctly assume that the runner's instantaneous speed was faster than the average speed during some parts of that interval and slower than the average speed during other parts of that interval.

Think about your car's speedometer. Does it measure average speed or instantaneous speed? Does it indicate how fast you were going during the past hour? During the past minute? During the past second? The speedometer on your car measures instantaneous speed. It indicates how fast you are going at the instant in time that you are looking at it. Practically speaking, we can think of instantaneous speed as distance traveled divided by the time it took to travel that distance if the time interval used in the measurement is very small. If the word *average* does not precede the word *speed,* you should assume that instantaneous speed is being referred to.

We can think of instantaneous speed as distance traveled divided by the time it took to travel that distance if the time interval used in the measurement is very small.

Velocity

Now let's turn our attention to velocity. **Average velocity** is displacement of an object divided by the time it took for that displacement. Because displacement is a vector, described by a number (magnitude) and a direction, average velocity is also a vector, described by a number (magnitude) and a direction. Mathematically, this can be expressed as

$$\bar{v} = \frac{d}{\Delta t} \tag{2.6}$$

where

$\bar{v}$ = average velocity,

d = displacement, and

Δt = time taken or change in time.

The units for describing velocity are the same as those for describing speed: a unit of length divided by a unit of time. The SI unit for describing velocity is meters per second. To measure the average velocity of an object, you need to know its displacement and the time taken for that displacement.

Sometimes we are interested in the components of velocity. So, just as we were able to resolve force and displacement vectors into components, we can also resolve velocity vectors into components. To resolve a resultant average velocity into components, we could simply determine the components of the resultant displacement. For the football player returning the kickoff in the example used earlier, the player's displacement from the instant he received the ball until he was tackled was −10 yd in the *x*-direction (across the field) and +30 yd in the *y*-direction (down the field). His resultant displacement was 31.6 yd down and across the field (or −71.6° across the field). If this kick return lasted 6 s, his resultant average velocity, using equation 2.6, was

$$\bar{v} = \frac{d}{\Delta t}$$

$$\bar{v} = \frac{31.6 \text{ yd}}{6 \text{ s}}$$

$$\bar{v} = 5.3 \text{ yd/s}$$

This resultant average velocity was in the same direction as the resultant displacement. Similarly, the running back's average velocity across the field (in the x-direction) would be the x-component of his displacement divided by time or

$$\bar{v}_x = \frac{\Delta x}{\Delta t} \tag{2.7}$$

$$\bar{v}_x = \frac{-10 \text{ yd}}{6 \text{ s}}$$

$$\bar{v}_x = -1.7 \text{ yd/s}$$

The running back's average velocity down the field (in the y-direction), which is the most important of all these velocities, would be the y-component of his displacement divided by time or

$$\bar{v}_y = \frac{\Delta y}{\Delta t} \tag{2.8}$$

$$\bar{v}_y = \frac{30 \text{ yd}}{6 \text{ s}}$$

$$\bar{v}_y = 5.0 \text{ yd/s}$$

Just as with the displacements, the resultant average velocity is larger than any of its components. And just as with the displacements, the square of the resultant average velocity should equal the sum of the squares of its components. Let's check, starting with equation 2.3.

$$A^2 + B^2 = C^2$$

$$(\bar{v}_x)^2 + (\bar{v}_y)^2 = \bar{v}^2$$

$$(-1.7 \text{ yd/s})^2 + (5.0 \text{ yd/s})^2 = \bar{v}^2$$

$$2.8 \text{ yd}^2/\text{s}^2 + 25.0 \text{ yd}^2/\text{s}^2 = \bar{v}^2$$

$$5.3 \text{ yd/s} = \sqrt{27.8 \text{ yd}^2/\text{s}^2} = \bar{v}$$

This indeed matches the resultant average velocity of 5.3 yd/s we computed from the resultant displacement and elapsed time.

Average velocity and average speed would both be good descriptors to use for the 100 m dash because it is in a straight line. The runner's speed and the magnitude of the velocity toward the finish line would be identical. In such a case, speed and velocity may be used interchangeably with no problem. Generally, if the motion of the object under analysis is in a straight line and rectilinear, with no change in direction, average speed and average velocity will be identical in magnitude. However, if we are speaking of an activity in which the direction of motion changes, speed and the magnitude of velocity are not synonymous. Imagine a 100 m swimming race in a 50 m pool. If the first-place finisher completes the race in 50 s, we can use equation 2.5 to calculate the swimmer's average speed.

$$\bar{s} = \frac{\ell}{\Delta t}$$

$$\bar{s} = \frac{100 \text{ m}}{50 \text{ s}}$$

$$\bar{s} = 2.0 \text{ m/s}$$

What is the swimmer's average velocity? If the swimmer starts and finishes in the same place, the swimmer's displacement is zero, which means the swimmer's average velocity would also have to be zero. In this case, average velocity and average speed do not mean the same thing, and the average speed measurement is a better descriptor.

If the motion of the object under analysis is in a straight line and rectilinear, with no change in direction, average speed and average velocity will be identical in magnitude.

What about instantaneous speed and instantaneous velocity? We haven't discussed instantaneous velocity yet. It is similar to the concept of instantaneous speed except that direction is included. If we measured average velocity over shorter and shorter intervals of time, practically speaking we would soon have a measure of instantaneous velocity. **Instantaneous velocity** is the velocity of an object at an instant in time. When we speak of the magnitude of the resultant instantaneous velocity of an object, that number is the same as the instantaneous speed of the object.

A resultant instantaneous velocity can also be resolved into components in the direction of interest. For the football player running back the kickoff, we could describe his instantaneous resultant velocity, and we could also describe his instantaneous velocity in the x-direction (across the field) or the y-direction (down the field). If we were concerned about how quickly he was gaining yardage, his instantaneous velocity down the field would be important. Similarly, with the downhill ski racers, it is not their instantaneous resultant velocity that is important; it is the component of this velocity in the direction down the hill that will have the greater effect on the result of the race. Try self-experiment 2.3 to illustrate the difference between speed and velocity.

Self-Experiment 2.3

Imagine that you are in a room with four walls. You are facing the north wall. Let's consider north the direction we are interested in, so north is positive. We are interested only in the component of velocity in the north–south directions. As you begin walking forward, toward the north wall, your velocity north is positive. When you stop, your velocity north is zero. As you begin walking backward, toward the south wall, your velocity north is negative (you are moving in the negative direction). If you walk to your right or left, directly east or west, your velocity north is zero, because you are not getting closer to or farther away from the north wall. If you walk forward toward the north wall and begin turning right toward the east wall, your velocity north is positive and then decreases as you turn. If you are walking east and then turn left toward the north wall, your velocity north is zero and then increases as you turn. During all of these turns, your speed may not even be changing, but if your direction of motion changes, then your velocity changes.

Importance of Speed and Velocity

Now let's make sure we realize the importance of speed and velocity in different sport activities. We've already indicated that in racing events, average speed and average velocity are direct indicators of performance. The athlete with the greatest average speed or greatest average velocity will be the winner. In what other sports is speed or velocity important? How about baseball? A good fastball pitch, which moves with a velocity of 90 to 100 mi/h (145 to 160 km/h), is difficult to hit. Why? The faster the ball is pitched, the less time the batter has to react and decide whether or not to swing at the ball. For instance, in 2010, Ardolis Chapman of the Cincinnati Reds threw a fastball pitch that was clocked at 105.1 mi/h. This is equivalent to 154 ft/s or 47 m/s. The distance from the pitching rubber to home plate is 60 ft 6 in., or 60.5 ft (18.4 m). The ball is released about 2 ft 6 in. in front of the rubber, so the horizontal distance it must travel to reach the plate is only 58 ft (60.5 ft – 2.5 ft) or 17.7 m. Another way to say this is that the horizontal displacement of the ball is 58 ft. How much time does a batter have to react to a fastball pitched at 105.1 mi/h? If we assume that this is the average horizontal velocity of the ball during its flight, then, using equation 2.6,

$$\bar{v} = \frac{d}{\Delta t}$$

$$154 \text{ ft/s} = \frac{58 \text{ ft}}{\Delta t}$$

$$\Delta t = \frac{58 \text{ ft}}{154 \text{ ft/s}}$$

$$\Delta t = 0.38 \text{ s}$$

Wow! A batter only has 0.38 s to decide whether or not to swing his bat, and if he does decide to swing it, he has to do so in the time he has left. No wonder hitting a baseball thrown by a major league pitcher is so difficult. The faster the pitcher can pitch the ball, the less time the batter has to react, and the less likely it is that the batter will hit the ball. In 2003, *USA Today* ranked hitting a baseball thrown at more than 90 mi/h as the most difficult thing to do in sports. Speed and velocity are very important in baseball.

Are speed and velocity important in soccer, lacrosse, ice hockey, field hockey, team handball, or any other sport where a goal is guarded by a goalkeeper? The speed of the ball (or puck) when it is shot toward the goal is very important to the goalkeeper. The faster the shot, the less time the goalkeeper has to react and block it.

Are speed and velocity important in the jumping events in track and field? Yes! Faster long jumpers jump farther. Faster pole-vaulters vault higher. Speed is also related to success in the high jump and triple jump.

Can you think of any sports where speed and velocity are not important? There aren't many. Speed and velocity play an important role in almost every sport. Table 2.3 lists the fastest reported speeds for a variety of balls and implements used in sport. The typical speeds of the balls and implements used in these sports are much slower than those reported in table 2.3.

Acceleration

We now have a large repertoire of motion descriptors: position, distance traveled, displacement, speed, and velocity. In addition, we can use component displacements or velocities to describe an object's motion, because displacement and velocity are vector quantities. Did we use any other descriptors at the beginning of this section to describe the motion of a ball rolling across the floor? Let's try another motion of the ball. Throw the ball up in the air and let it fall back into your hand. How would you describe this motion? You might say that the ball moves upward and slows down on the way up, then begins moving downward and speeds up on the way down. Another way to describe how the ball slows down or speeds up would be to say that it decelerates on the way up and accelerates on the way down. Acceleration is a term you are probably somewhat familiar with, but the mechanical definition of acceleration may differ from yours, so we'd better get some agreement.

Table 2.3 Fastest Reported Speeds for Balls and Implements Used in Various Sports

Ball or implement	Mass (g)	Fastest speed (mi/h)	Fastest speed (m/s)
Golf ball	≤45.93	204	91.2
Jai alai pelota	125-140	188	84.0
Squash ball	23-25	172	76.9
Golf club head	–	163	72.9
Tennis ball	56.0-59.4	156	69.7
Baseball (batted)	142-149	120	53.6
Hockey puck	160-170	110	49.2
Baseball (pitched)	142-149	105	46.9
Softball (12 in.)	178.0-198.4	104	46.5
Lacrosse ball	140-149	100	44.7
Cricket ball (bowled)	156-163	100	44.7
Volleyball	260-280	88	39.3
Soccer ball	410-450	82	36.7
Field hockey ball	156-163	78	34.9
Javelin (men)	800	70	31.3
Team handball (men)	425-475	63	28.2
Water polo ball	400-450	60	26.8

SAMPLE PROBLEM 2.1

The average horizontal velocity of a penalty kick in soccer is 22 m/s. The horizontal displacement of the ball from the kicker's foot to the goal is 11 m. How long does it take for the ball to reach the goal after it is kicked?

Solution:

Step 1: Write down the known quantities.

$v_x = 22$ m/s

$d_x = 11$ m

Step 2: Identify the variable to solve for.

$\Delta t = ?$

Step 3: Review equations and definitions, and identify the appropriate equation with the known quantities and the unknown variable in it.

$$\bar{v} = \frac{d}{\Delta t}$$

Step 4: Substitute values into the equation and solve for the unknown variable. Keep track of the units when doing arithmetic operations.

$$22 \text{ m/s} = \frac{11 \text{ m}}{\Delta t}$$

$$\Delta t = \frac{11 \text{ m}}{22 \text{ m/s}}$$

$$\Delta t = 0.5 \text{ s}$$

Step 5: Check your answer using common sense.
A penalty kick is pretty quick, definitely less than a second. A half second seems reasonable.

Mechanically, **acceleration** is the rate of change in velocity. Because velocity is a vector quantity, with a number and direction associated with it, acceleration is also a vector quantity, with a number and direction associated with it. An object accelerates if the magnitude or direction of its velocity changes.

When an object speeds up, slows down, starts, stops, or changes direction, it is accelerating.

Average acceleration is defined as the change in velocity divided by the time it took for that velocity change to take place. Mathematically, this is

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

$$\bar{a} = \frac{v_f - v_i}{\Delta t} \tag{2.9}$$

where

$\bar{a}$ = average acceleration,

Δv = change in velocity,

v_f = instantaneous velocity at the end of an interval, or final velocity,

v_i = instantaneous velocity at the beginning of an interval, or initial velocity, and

Δt = time taken or change in time.

From this mathematical definition of average acceleration, it is apparent that acceleration can be positive or negative. If the final velocity is less (slower) than the initial velocity, the change in velocity is a negative number, and the resulting average acceleration is negative. This happens if an object slows down in the positive direction. You may have thought of this as a deceleration, but we'll call it a negative acceleration. A negative average acceleration will also result if the initial and final velocities are both negative and if the final velocity is a larger negative number than the initial velocity. This occurs if an object is speeding up in the negative direction.

The units for describing acceleration are a unit of length divided by a unit of time divided by a unit of time. The SI units for describing acceleration are meters per second per second or meters per second squared. You may have seen car ads that tout the acceleration capabilities of the car. An ad may say that a car can accelerate from 0 to 60 in 7 s. Using equation 2.9, this would represent an average acceleration for the car of

$$\bar{a} = \frac{v_f - v_i}{\Delta t}$$

$$\bar{a} = \frac{60 \text{ mi/h} - 0 \text{ mi/h}}{7\text{s}}$$

$$= 8.6 \text{ mi/h/s}$$

This acceleration can be interpreted as follows: In 1 s, the car's velocity increases (the car speeds up) by 8.6 mi/h. If the car is accelerating at 8.6 mi/h/s and moving at 30 mi/h, 1 s later the car will be traveling 8.6 mi/h faster or 38.6 mi/h. Two seconds later the car will be traveling two times 8.6 mi/h faster (17.2 mi/h) or 47.2 mi/h (=30 mi/h + 17.2 mi/h), and so on.

If we measured average acceleration over shorter and shorter time intervals, practically speaking we would soon have a measure of instantaneous acceleration. **Instantaneous acceleration** is the acceleration of an object at an instant in time. Instantaneous acceleration indicates the rate of change of velocity at that instant in time.

Because acceleration is a vector (as are force, displacement, and velocity), it can also be resolved into component accelerations. This is true for both average and instantaneous accelerations. But how is the direction of an acceleration determined? Try self-experiment 2.4 to get a better understanding of acceleration direction. One of the difficulties of understanding acceleration is that it is not directly observed as displacement and velocity are. The direction of motion is not necessarily the same as the direction of the acceleration.

Self-Experiment 2.4

Let's go back to the example of walking around the room with four walls. You are facing the north wall. Again, consider north to be the direction we are interested in, so north is positive. We are interested in describing the motion only in the north–south directions. As you begin walking forward, toward the north wall, your velocity north is positive; and since you speed up in the northerly direction, your acceleration north is positive (your velocity and acceleration are north). When you slow down and stop, your velocity north decreases to zero, and your acceleration north must be negative since you are slowing down in the positive direction. This could also be described as an acceleration in the southerly direction. This is where you may be confused—you were moving north, but your acceleration was south! This is correct, however, since acceleration indicates your change in motion. As you begin walking backward, you speed up toward the south wall; your velocity north is negative and increasing (you are moving in the negative direction), and your acceleration is also negative (or an acceleration in the southerly direction). If you walk to your right or left, directly east or west, your velocity north is zero because you are not getting closer to or farther away from the north wall. Your acceleration is also zero since you are not speeding up or slowing down toward the north wall. If you walk forward toward the north wall and begin turning right toward the east wall, your velocity north is positive and decreases as you turn, so your acceleration north is negative as you turn. If you are walking east and then turn left toward the north wall, your velocity north is zero and then increases as you turn, so your acceleration north is positive as you turn. During all of these turns, your speed may not even be changing, but if your direction of motion changes, then your velocity changes and you are accelerating. Figure 2.5 illustrates the directions of motion and acceleration for various motions in one dimension (along a line).

The direction of motion does not indicate the direction of the acceleration.

Let's summarize some things about acceleration. If you are speeding up, your acceleration is in the direction of your motion. If you are slowing down, your acceleration is in the opposite direction of your motion. If we assign positive and negative signs to the directions along a line, then the acceleration direction along that line is determined as follows. If something speeds up in the positive direction, its acceleration is positive (it accelerates in the positive direction). Think of this as a double positive (+ +), which results in a positive (+). If it slows down in the positive direction, its acceleration is negative (it accelerates in the negative direction). Think of this as a negative positive (– +), which results in a negative (–). If something speeds up in the negative direction, its acceleration is negative (it accelerates in the negative direction). Think of this as a positive negative (+ –), which results in a negative (–). If something slows down in the negative direction, its acceleration is positive (it accelerates in the positive direction). Think of this as a double negative (– –), which results in a positive (+). Remember, though, the algebraic signs + and – are only symbols we use to indicate directions in the real world. Before analyzing a problem, first establish which direction you will identify as +.

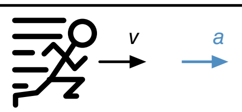
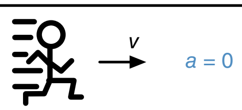
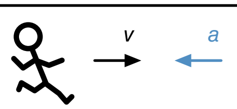
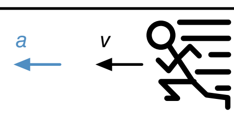
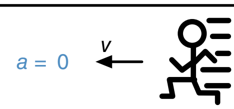
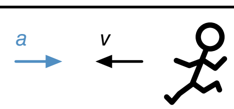
– Direction +		*v* (Direction of motion)	Change in motion (Speeding up +; slowing down –)	*a* (Direction of acceleration)
Speeding up	 *v* *a*	+	+	+
Not changing	 *v* *a* = 0	+	0 (Constant velocity)	0
Slowing down	 *v* *a*	+	–	–
Speeding up	 *a* *v*	–	+	–
Not changing	 *a* = 0 *v*	–	0 (Constant velocity)	0
Slowing down	 *a* *v*	–	–	+

Figure 2.5 The direction of motion and direction of acceleration are the same when the object is speeding up, but opposite to each other when the object is slowing down.

Uniform Acceleration and Projectile Motion

In certain situations, the acceleration of an object is constant—it doesn't change. This is an example of **uniform acceleration**. It occurs when the net external force acting on an object is constant and unchanging. If this is the case, then the acceleration of the object is also constant and unchanging. The motion of such an object can then be described by equations relating time with velocity, position, or acceleration. Using these equations, we can predict the future! If an object undergoes uniform acceleration, its position and velocity at any future instant in time can be predicted. Wow! Can you think of any situations in which the net external force acting on an object is constant and thus the resulting acceleration is uniform? Try self-experiment 2.5 and see if this is an example of uniform acceleration.

Self-Experiment 2.5

Throw a ball straight up into the air and try to describe its motion. Let's use the terms we have learned—displacement, velocity, and acceleration. If we set up a coordinate system with the *x*-axis oriented horizontally in the direction of the horizontal motion of the ball and the *y*-axis oriented vertically, how would you describe the vertical motion of the ball? Let's consider the positive direction along the *y*-axis (vertical axis) as upward. As the ball leaves your hand, it is moving in the upward direction, so its velocity is positive. Is the ball speeding up or slowing down as it goes up? The ball is slowing down in the upward direction, so its acceleration is negative or in the downward direction. When the ball reaches the peak of its flight, its velocity is changing from positive to negative (or from upward to downward), so it is still accelerating downward. After the ball is past its peak, it falls downward, so its velocity is negative (downward). Since the ball speeds up in the downward direction, its acceleration is still negative (downward). Despite the changes in the direction the ball was moving in, its vertical acceleration was always downward while it was in the air. The direction of its acceleration was constant. Was the magnitude of the acceleration constant as well? What forces acted on the ball while it was in the air? If air resistance can be ignored, then the only force acting on the ball was the force of gravity or the weight of the ball. Since the ball's weight does not change while it's in the air, the net external force acting on the ball is constant and equal to the weight of the ball. This means that the acceleration of the ball is constant as well.

Vertical Motion of a Projectile

In self-experiment 2.5, the ball you threw up in the air was a projectile. A **projectile** is an object that has been projected into the air or dropped and is acted on only by the forces of gravity and air resistance. If air resistance is too small to measure, and the only force acting on a projectile is the force of the earth's gravity, then the force of gravity will accelerate the projectile. In the previous chapter we learned that this acceleration, the acceleration due to gravity or g, is 9.81 m/s^2 downward. This is a uniform acceleration. Now let's see if we can come up with the equations that describe the vertical motion of a projectile such as the ball in self-experiment 2.5.

Since the vertical acceleration of the ball is constant, we already have one equation to describe this kinematic variable. If we define upward as the positive vertical direction, then

$$a = \mathrm{g} = -9.81 \text{ m/s}^2. \tag{2.10}$$

The negative sign indicates that the acceleration due to gravity is in the downward direction.

We know what the vertical acceleration of the ball is; perhaps we can use this knowledge to determine its velocity from equation 2.9, which relates acceleration to velocity.

$$\bar{a} = \frac{v_f - v_i}{\Delta t}$$

The acceleration in equation 2.9 is an average acceleration, but in our case we know the acceleration of the ball at any instant in time—it's 9.81 m/s^2 downward. But since the acceleration is constant, 9.81 m/s^2 is also the average acceleration. So, we can substitute g for average acceleration, $\bar{a}$, in equation 2.9 and solve for final velocity, v_f:

$$\bar{a} = \frac{v_f - v_i}{\Delta t} = \mathrm{g}$$

$$v_f - v_i = \mathrm{g}\Delta t$$

$$v_f = v_i + \mathrm{g}\Delta t \tag{2.11}$$

Equation 2.9 gives us a means of determining the instantaneous vertical velocity of the ball (v_f) at the end of some time interval (Δt) if we know its initial vertical velocity (v_i) and the length of the time interval. We can predict the future! Look closer at this equation. If you remember your high school algebra, you might recognize this as the equation for a line:

$$y = mx + b \tag{2.12}$$

where

y = dependent variable (plotted on vertical axis),

x = independent variable (plotted on horizontal axis),

m = slope of line = $\dfrac{\Delta y}{\Delta x}$, and

b = intercept.

In equation 2.11,

$v_f = v_i + g\Delta t$,

v_f is the dependent variable, y,

Δt is the independent variable, x,

g is the slope, m, and

v_i is the intercept, b.

The vertical velocity of the ball changes linearly with changes in time—the vertical velocity of the ball is directly proportional to the time that the ball has been in the air.

What about the vertical position of the ball? Perhaps we can use the definition for average velocity from equation 2.8.

$$\bar{v}_y = \frac{\Delta y}{\Delta t}$$

$$\bar{v}_y = \frac{y_f - y_i}{\Delta t}$$

Since velocity is linearly proportional to time (it's defined by a linear equation), the average velocity over a time interval is equal to the velocity midway between the initial and final velocities. This velocity is the average of the initial and final velocities:

$$\bar{v}_y = \frac{v_f + v_i}{2}$$

$$\bar{v}_y = \frac{v_f + v_i}{2} = \frac{y_f - y_i}{\Delta t} \qquad \textbf{(2.13)}$$

If we use the expression from equation 2.11,

$v_f = v_i + g\Delta t$,

and substitute it for v_f in equation 2.13,

$$\frac{v_f + v_i}{2} = \frac{y_f - y_i}{\Delta t},$$

we can solve for y_f.

$$\frac{(v_i + g\Delta t) + v_i}{2} = \frac{y_f - y_i}{\Delta t}$$

$$\frac{(2v_i + g\,\Delta t)\Delta t}{2} = \frac{y_f - y_i}{\Delta t}$$

$$\frac{(2v_i + g\Delta t)\Delta t}{2} = y_f - y_i$$

$$\frac{2v_i\Delta t + g(\Delta t)^2}{2} = y_f - y_i$$

$$v_i\Delta t + \frac{1}{2}g(\Delta t)^2 = y_f - y_i$$

$$y_f = y_i + v_i\Delta t + \frac{1}{2}g(\Delta t)^2 \qquad \textbf{(2.14)}$$

If you couldn't follow the derivation of equation 2.14, don't worry about it. The result is what is important for our understanding of the motion of the ball. Equation 2.14 gives us a means of determining the vertical position of the ball (y_f) at the end of a time interval (Δt) if we know its initial vertical velocity (v_i) and the length of the time interval.

There is one more equation that describes vertical velocity of the ball as a function of its vertical displacement and initial vertical velocity. The equation is presented here, but we'll have to wait until chapter 4 (see p. 127) for the derivation of this equation.

$$v^2 = v^2 + 2g\Delta y \qquad \textbf{(2.15)}$$

Using equations 2.11 and 2.14 (or 2.15), we can now predict not only how fast the ball will be moving vertically, but where it will be as well. We now have four equations to describe the vertical motion of a projectile.

Vertical position of projectile (equation 2.14):

$$y_f = y_i + v_i\Delta t + \frac{1}{2}g(\Delta t)^2$$

Vertical velocity of projectile (equation 2.11 and 2.15):

$$v_f = v_i + g\Delta t$$

$$v^2 = v^2 + 2g\Delta y$$

Vertical acceleration of projectile (equation 2.10):

$$a = g = -9.81 \text{ m/s}^2$$

where

y_i = initial vertical position,

y_f = final vertical position,

$\Delta y = y_f - y_i$ = vertical displacement,

Δt = change in time,

v_i = initial vertical velocity,

v_f = final vertical velocity, and

g = acceleration due to gravity = -9.81 m/s^2.

If we are analyzing the motion of something that is dropped, the equations are simplified. For a dropped object, $v_f = 0$. If we set the vertical scale to zero at the position the object was dropped from, then $y_i = 0$ as well. For a dropped object, the equations become the following:

Vertical position of falling object:

$$y_f = \frac{1}{2}g(\Delta t)^2 \qquad (2.16)$$

Vertical velocity of falling object:

$$v_f = g\Delta t \qquad (2.17)$$

$$v^2 = 2g\Delta y \qquad (2.18)$$

Imagine that you could safely drop a ball from the top of some tall building and that air resistance is not significant. When you let go of the ball, its vertical velocity is zero. According to equation 2.17, after it has fallen for 1 s, its velocity would be 9.81 m/s downward. According to equation 2.16, its position would be 4.91 m below you. After 2 s, its velocity would be another 9.81 m/s faster, or -19.62 m/s, and its position would be 19.62 m below you. After 3 s its velocity would be another 9.81 m/s faster, or -29.43 m/s, and its position would be 44.15 m below you. Notice that the ball's velocity is just increasing by the same amount (9.81 m/s) during each 1 s time interval, but the ball's position changes by a larger and larger amount during each second it falls (see figure 2.6).

Some other observations about the vertical motion of projectiles may simplify things further. Throw a ball straight up in the air again. How fast is the vertical velocity of the ball at the instant it reaches its peak height? Hmmm. Just before it reached its peak height, it had a small positive velocity (it was going upward slowly). Just after it reached its peak height, it had a small negative velocity (it was going downward slowly). Its vertical velocity went from positive to negative. What number is between positive and negative numbers? How fast is it moving if it's not moving up anymore and hasn't started

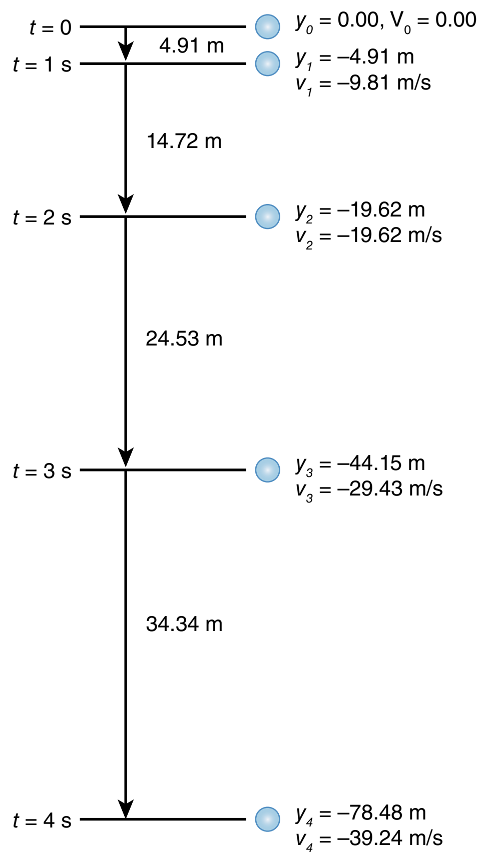


Figure 2.6 Vertical position of a dropped ball at each 1 s interval.

moving downward yet? The ball's vertical velocity at the peak of its flight is zero.

$$v_{peak} = 0 \qquad (2.19)$$

A useful application of this is in the sport of tennis. When you serve a tennis ball, you want to toss it up in the air just high enough that your racket hits it when it is at or near the peak of its flight. Small errors in the timing of your serve won't significantly affect where on the racket the ball hits, because at the peak of its flight the ball's velocity is zero, so it will be near this position for a longer time. If you toss the ball up too high, however, the time during which the ball is in the hitting zone of the racket will be shorter, since the ball is moving much faster as it falls through the hitting zone.

The symmetry of the flight of a projectile is the source of more simplification for our analyses. Toss a ball up again and try to determine which is longer—the time it takes for the ball to reach its peak height or the time it takes for the ball to fall back down from its peak height to its initial height. Wow, those time intervals are close to the same. In fact, they are the same.

$\Delta t_{up} = \Delta t_{down}$ if the initial and final y-positions are the same **(2.20)**

or,

$\Delta t_{flight} = 2\Delta t_{up}$ if the initial and final y-positions are the same **(2.21)**

Similarly, the upward velocity of the ball as it passes any height on the way up is the same as the downward velocity of the ball when it passes that same height on the way down. The time it takes for the ball's upward velocity to slow down to zero is the same as the time it takes for the ball's downward velocity to speed up from zero to the same size velocity downward. If you throw a ball upward with an initial vertical velocity of 5 m/s, when you catch it on the way down its velocity will also be 5 m/s but downward.

Horizontal Motion of a Projectile

Now we can describe the vertical motion of a projectile—at least a projectile that is moving only up and down. What about the horizontal motion of a projectile? Try self-experiment 2.6.

SAMPLE PROBLEM 2.2

A volleyball player sets the ball for the spiker. When the ball leaves the setter's fingers, it is 2 m high and has a vertical velocity of 5 m/s upward. How high will the ball go?

Solution:

Step 1: Write down the known quantities and any quantities that can be inferred from the problem

$y_i = 2$ m

$v_i = 5$ m/s

$v_f = v_{peak} = 0$

Step 2: Identify the variable to solve for.

$h = y_f = ?$

Step 3: Review equations and definitions, and identify the appropriate equation with the known quantities and the unknown variable in it (equation 2.15).

$v^2 = v^2 + 2g\Delta y$

Step 4: Substitute values into the equation and solve for the unknown variable. Keep track of the units when doing arithmetic operations.

$v^2 = v^2 + 2g\Delta y$

$0 = (5\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)\Delta y$

$$\Delta y = \frac{(5\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 1.27\text{ m}$$

$\Delta y = y_f - y_i$

$1.27\text{ m} = y_f - 2\text{ m}$

$y_f = h = 2\text{ m} + 1.27\text{ m} = 3.27\text{ m}$

Step 5: Check your answer using common sense.
The answer, 3.27 m, is almost 11 feet, which seems about right for a set in volleyball.

Self-Experiment 2.6

Throw a ball in the air from one hand to the other so the ball has both vertical motion and horizontal motion. What forces act on the ball? If we resolve the motion of the ball into horizontal *(x)* and vertical *(y)* components, we know that gravity is an external force that acts in the vertical direction and pulls downward on the ball. What about horizontally (sideways)? Are there any external forces pulling or pushing sideways against the ball to change its horizontal motion once it leaves your hand? The only thing that could exert a horizontal force on the ball is the air through which the ball moves. This force will probably be very small in most cases, and its effect will be too small to notice. If air resistance is negligible, the horizontal velocity of the ball should not change from the time it leaves your hand until it contacts your other hand or another object, since no horizontal forces act on the ball. Try to observe only the horizontal motion of the ball. The ball continues to move in the direction you projected it. It does not swerve right or left. Its horizontal velocity is positive. Does the ball accelerate horizontally while it is in the air? Does it speed up or slow down horizontally? No. Does it change its direction horizontally? No. If the ball doesn't speed up or slow down or change direction, it is not accelerating in the horizontal direction.

SAMPLE PROBLEM 2.3

A punter punts the football. The football leaves the punter's foot with a vertical velocity of 20 m/s and a horizontal velocity of 15 m/s. What is the hang time of the football (how long is it in the air)? (Assume that air resistance has no effect and that the height at landing and at release are the same.)

Solution:

Step 1: Write down the known quantities and any quantities that can be inferred from the problem.

$$y_i = y_f$$

$$v_i = 20 \text{ m/s}$$

$$v_x = 15 \text{ m/s}$$

$$v_{peak} = 0$$

$$\Delta t_{up} = \Delta t_{down}$$

Step 2: Identify the variable to solve for.

$$\Delta t = ?$$

Step 3: Review equations and definitions, and identify the appropriate equation with the known quantities and the unknown variable in it (equation 2.11).

$$\Delta t = \Delta t_{up} + \Delta t_{down} = 2\Delta t_{up}$$

$$v_f = v_i + g\Delta t$$

Step 4: Substitute values into the equation and solve for the unknown variable. Keep track of the units when doing arithmetic operations.

$$v_f = v_i + g\Delta t$$

$$0 = 20 \text{ m/s} + (-9.81 \text{ m/s}^2)(\Delta t_{up})$$

$$\Delta t_{up} = \frac{(-20 \text{ m/s})}{(-9.81 \text{ m/s})} = 2.04 \text{ s}$$

$$\Delta t = 2\,\Delta t_{up} = 2(2.04 \text{ s}) = 4.08 \text{ s}$$

Step 5: Check your answer using common sense.
Four seconds seems like a reasonable hang time for a punt.

It is difficult to examine or observe the horizontal motion of a projectile separately from its vertical motion, though, because when you observe a projectile, you see the horizontal and vertical motions simultaneously as one motion. How can we view a projectile, such as the ball in self-experiment 2.6, so that we isolate only its horizontal motion? What if we watched the projectile from above? Imagine yourself perched on the catwalk of a gymnasium watching a basketball game. Better yet, imagine watching a football game from the Goodyear blimp. How would the motions of the football or basketball appear to you from these vantage points? If your depth perception was hindered (if you closed one eye), could you see the vertical motion of the football during a kickoff? Could you detect the vertical motion of the basketball during a free throw? The answer is no in both cases. All you see is the horizontal motion of the balls. Does the basketball slow down, speed up, or change direction horizontally as you view it from above? How about the football? If we tried to represent the motion of the basketball as viewed from above in a single picture, it might look something like figure 2.7.

To represent the motion, we show the position of the basketball at four instants in time, each 0.10 s apart. Notice that the images line up along a straight line, so the motion of the ball is in a straight line. Also notice that the displacement of the ball over each interval of time is the same, so the velocity of the ball is constant. The horizontal velocity of a projectile is constant, and its horizontal motion is in a straight line.

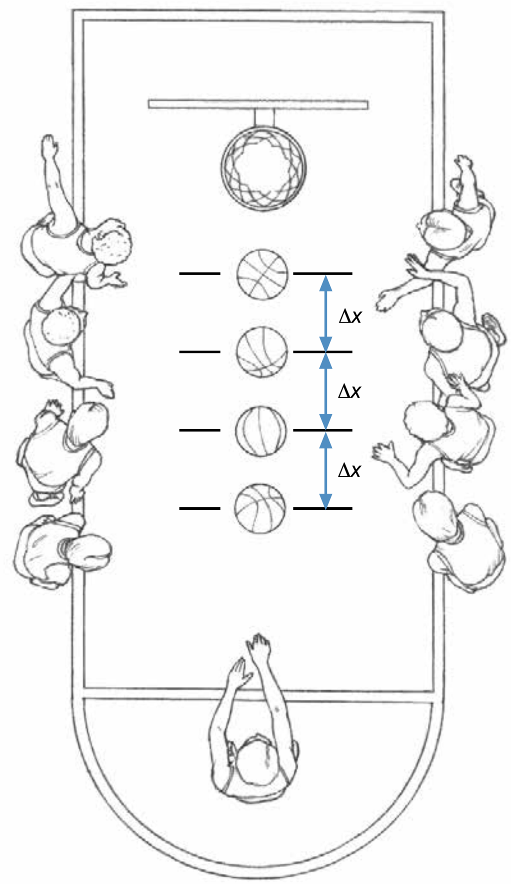


Figure 2.7 An overhead view of a basketball free throw shows that the horizontal displacement, Δx, for each 0.10 s time interval is the same.

The horizontal velocity of a projectile is constant and its horizontal motion is in a straight line.

We derived equations describing the vertical position, velocity, and acceleration of a projectile. Now we can do the same for the horizontal position, velocity, and acceleration of a projectile. We start with the fact that the horizontal velocity of a projectile is constant.

$$v = v_f = v_i = \text{constant} \tag{2.22}$$

If the horizontal velocity is constant, that means there is no change in horizontal velocity. If horizontal velocity doesn't change, then horizontal acceleration must be zero, since acceleration was defined as the rate of change in velocity.

$$a = 0 \tag{2.23}$$

Also, if the horizontal velocity is constant, then the average horizontal velocity of the projectile is the same as its instantaneous horizontal velocity. Since average velocity is displacement divided by time, displacement is equal to velocity times time (equation 2.6).

$$\bar{v} = \frac{d}{\Delta t}$$

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v\Delta t \tag{2.24}$$

$$x_f - x_i = v\Delta t$$

$$x_f = x_i + v\Delta t \tag{2.25}$$

If our measuring system is set up so that the initial horizontal position (x_i) is zero, then equation 2.25 simplifies to

$$x = v\Delta t. \tag{2.26}$$

Using equations 2.22 and 2.26 (or 2.25), we can now predict not only how fast a projectile will be moving horizontally but where it will be as well. We now have the equations to describe the horizontal motion of a projectile.

- Horizontal position of projectile (equations 2.25 and 2.26):

$$x_f = x_i + v\Delta t$$

$$x = v\Delta t \qquad \text{if initial position is zero}$$

- Horizontal velocity of projectile (equation 2.22):

$$v = v_f = v_i = \text{constant}$$

- Horizontal acceleration of projectile (equation 2.23):

$$a = 0$$

where

x_i = initial horizontal position,

x_f = final horizontal position,

Δt = change in time,

v_i = initial horizontal velocity, and

v_f = final horizontal velocity.

Combined Horizontal and Vertical Motions of a Projectile

We have now developed equations that describe the motion of a projectile in terms of its vertical and horizontal components. Does the vertical motion of a projectile affect its horizontal motion or vice versa? Try self-experiment 2.7.

Self-Experiment 2.7

Put a coin on the edge of a tabletop. Place another same-denomination coin on the end of a ruler or other long, flat object. Place the ruler with the coin on it on the table next to the other coin so that the end of the ruler with the coin on it overhangs the tabletop. Strike the ruler with your hand so that it in turn strikes the coin on the table and knocks the coin off of the table. Simultaneously, the movement of the ruler will dislodge the coin off the end of the ruler. Figure 2.8 shows the setup of the demonstration.

Which coin will strike the floor first? Try it several times to see. The two coins hit the floor at the same time. The coin that is knocked off of the table has a horizontal velocity as it begins to fall, while the coin that slips off of the ruler does not. The two coins fall the same vertical distance, and neither of them has a vertical velocity when it begins to fall. What force pulls the coins toward the earth? The force of gravity pulls the coins downward and accelerates both downward at the same rate of 9.81 m/s^2. Does the fact that one coin has a horizontal velocity affect how the force of gravity acts on that coin, and thus affect the vertical acceleration of that coin? No, the force of gravity has the same effect on the coin knocked off the table as it does on the coin that slid off of the ruler.

The vertical and horizontal motions of a projectile are independent of each other. In other words, a projectile continues to accelerate downward at 9.81 m/s^2 with or without horizontal motion, and the horizontal velocity of a projectile remains constant even though the projectile is accelerating downward at 9.81 m/s^2. Although the motions of a projectile are independent of each other, an equation can be derived to describe the path of a projectile in two dimensions. Take equation 2.26 and solve for Δt.

$$\Delta t = \frac{x}{v_x}$$

Now substitute this expression for Δt in equation 2.14.

$$y_f = y_i + v_i\Delta t + \frac{1}{2}g(\Delta t)^2$$

$$y_f = y_i + v_{y_i}\left(\frac{x}{v_x}\right) + \frac{1}{2}g\left(\frac{x}{v_x}\right)^2 \tag{2.27}$$

Equation 2.27 is the equation of a parabola. It describes the vertical *(y)* and horizontal *(x)* coordinates of a projectile during its flight based solely on the initial vertical position and vertical and horizontal velocities. Figure 2.9 shows the parabolic path followed by a ball thrown in the air with an initial vertical velocity of 6.95 m/s and an initial horizontal velocity of 4.87 m/s. The ball was

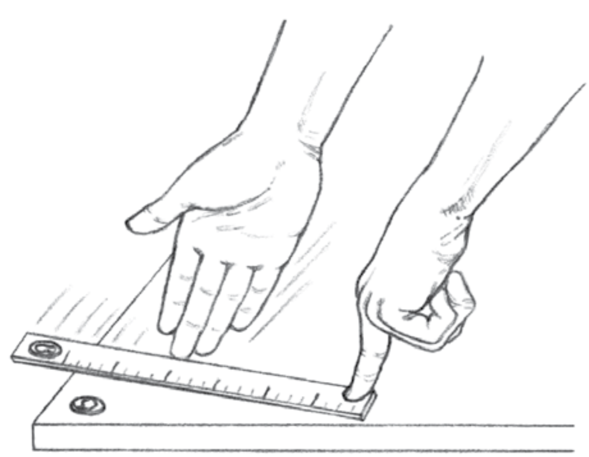


Figure 2.8 The coin experiment demonstrates the independence of the horizontal and vertical components of projectile motion.

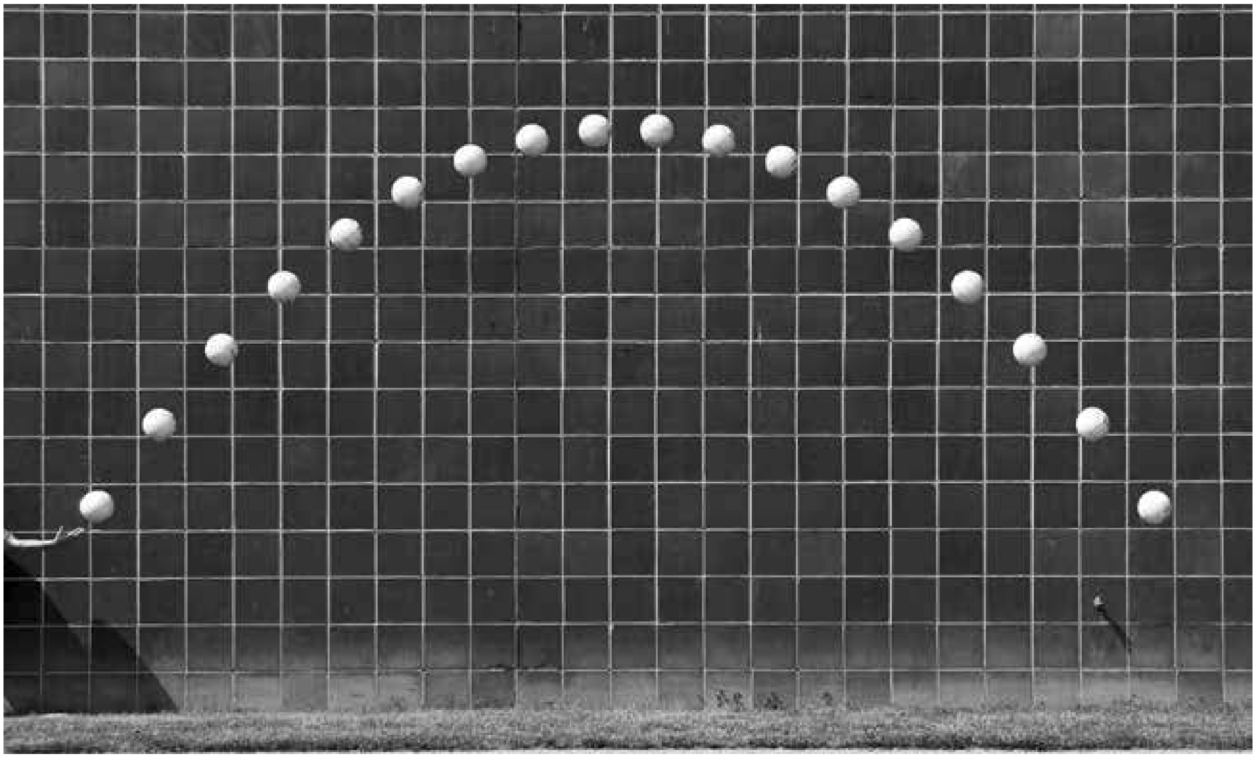


Figure 2.9 Stroboscopic photos of a ball in flight taken at equally spaced time intervals. Note the parabolic trajectory.

photographed at a rate of 12 frames per second, so the position of the ball at each 0.0833 s interval is shown in the figure. Notice that the horizontal displacements over each time interval are the same and that the path is symmetrical on either side of the peak. The peak height actually occurs between the ninth and 10th ball images as counted from the left.

Several of the equations that describe projectile motion can be written with only three variables. These equations (2.11, 2.15, and 2.24) are

$$v_f = v_i + g\Delta t$$

$$v^2 = v^2 + 2g\Delta y$$

$$\Delta x = v\Delta t$$

Equation 2.14 has four variables, but it can be modified by substituting Δy for y_i and y_f as shown to produce equation 2.28 with only three variables.

$$y_f = y_i + v_i\Delta t + \frac{1}{2}g(\Delta t)^2$$

$$y_f - y_i = v_i\Delta t + \frac{1}{2}g(\Delta t)^2$$

$$\Delta y = v_i\Delta t + \frac{1}{2}g(\Delta t)^2 \quad (2.28)$$

We now have four equations, and each has only three variables. In each of these equations, if two of the variables are known, the equation can be solved for the third variable. Table 2.4 lists these equations and their variables. You can use this table as an aid to help you solve projectile problems by following these steps. First identify the unknown variable that you are trying to determine. Look in the first column, labeled “Unknown variable” in table 2.4, to see if the variable is in the table. If it appears in a row in this column, look to the right in that row to see if you know the values for the two variables listed in the “Known variables” column. If you know the values for those two variables, look to the right in the “Equation” column and plug the values into the equation and solve for the unknown variable. Remember, you might have to solve two or more equations before you get to the equation that has the unknown variable that interests you.

Projectiles in Sport

Examples of projectiles in sports and human movement are numerous. Can you name a few? Here are some examples of projectiles: a shot in flight during a shot put, a basketball in flight, a hammer in flight during a hammer throw, a volleyball in flight, a squash ball in flight, a lacrosse ball in flight, a football in flight, a rugby ball in flight . . . just about any ball used in sport becomes a projectile once it is thrown, released, or hit,

Table 2.4 Solution Guide for Solving Projectile Problems If Two Variables Are Known

Identify the unknown variable in the first column set. Find the two known variables in the second column set that match the row of the unknown variable. Solve for the unknown variable using the equation in the rightmost column of that row.

	Unknown variable *If you want to find this . . .*						**Known variables** *. . . and you know these,*		**Equation** *. . . use this equation to find the unknown variable.*
y (vertical)	Δy						v_i	Δt	$\Delta y = v_i\Delta t + \frac{1}{2}g(\Delta t)^2$
		v_i					Δt	Δy	
				Δt			Δy	v_i	
		v_i					v_f	Δt	$v_f = v_i + g\Delta t$
			v_f				v_i	Δt	
				Δt			v_f	v_i	
	Δy						v_f	v_i	$v_f^2 = v_i^2 + 2g\Delta y$
		v_i					v_f	Δy	
			v_f				v_i	Δy	
x (horizontal)					Δx		v_x	Δt	$\Delta x = v_x\Delta t$
						v_x	Δt	Δx	
				Δt			Δx	v_x	

Variable definitions:

Δt = time

$\Delta y = y_f - y_i$ = vertical displacement

y_i = initial vertical position

y_f = final vertical position

v_i = initial vertical velocity

v_f = final vertical velocity

g = acceleration due to gravity = -9.81 m/s^2

$\Delta x = x_f - x_i$ = horizontal displacement

x_i = initial horizontal position

x_f = final horizontal position

v_x = horizontal velocity

if air resistance is negligible. So in ball sports, the path of the ball cannot be changed in flight if air resistance is negligible. Its path is determined by equation 2.27. Vertically, the ball is constantly accelerating downward, and horizontally it won't slow down or speed up. Once a ball has left our hands and is in flight, our actions and antics cannot change its predetermined course or velocity.

It seems pretty obvious that the balls used in sport are projectiles, but what if we ourselves are the projectiles? Can the human body be a projectile? Are there situations in which the only external force acting on you is the force of gravity? Yes, of course there are! Think of some examples in sport in which the human body is a projectile. How about in running? High jumping? Long jumping? Diving? Pole vaulting? Volleyball? Basketball? Soccer? Football? In each of these sports, there are situations in which the athlete is airborne and the only force acting on her is gravity. Do the projectile equations govern the motion of an athlete in these situations? Yes! This means that, once an athlete's body has left the ground and become a projectile, the athlete cannot change her path. Once a volleyball player has jumped up to the left to block a shot, the path of her body motion cannot be changed; in other words, once she has jumped up to the left, she won't be able to change direction and block a shot to the right. And once a pole-vaulter releases the pole, he cannot change his motion. Once he has let go of the pole, he no longer has control over where he falls. Once a long jumper leaves the takeoff board and becomes a projectile, her actions while in the air will not affect the velocity of her body. She cannot speed up her horizontal velocity to increase the distance of the jump after leaving the ground. Nor can she turn off gravity to stay in the air longer.

In projectile activities, the initial conditions (the initial position and initial velocity) of the projectile determine the motion that the projectile will have. In sports involving projectiles, the athlete's objective when throwing, kicking, striking, shooting, or hitting the projectile usually concerns one of three things: time of flight, peak height reached by the projectile, or horizontal displacement.

Time of flight of a projectile is dependent on two things: initial vertical velocity and initial vertical position. We can use the equations to mathematically demonstrate this, or we can just make some simple observations. Drop a ball to the floor first from waist height, then from shoulder height, and then from over your head. Which one took the shortest time to reach the floor? Which one took the longest time to reach the floor? The higher the initial height of the projectile, the longer it stays in the air. The shorter the initial height of the projectile, the shorter the time it stays in the air.

Now, rather than dropping the ball, throw it upward. Throw it upward again, but harder this time, and try to release it at the same height. Now throw it down, and again, try to release it at the same height. What should you do if you want the ball to stay in the air longer? The faster the initial upward velocity of the projectile, the longer it stays in the air. The slower the initial upward velocity (or the faster its initial downward velocity), the shorter the time it stays in the air.

Maximizing time in the air is desirable in certain situations in sport such as a football punt or a lob in tennis. Gymnasts and divers also need sufficient time in the air to complete stunts. In these situations, the initial vertical velocity of the projectile is relatively large (compared to the horizontal velocity), and the angle of projection is above 45°. The optimal angle of projection to achieve maximum height and time of flight is 90° or straight up.

In some sport activities, minimizing the projectile's time in the air is important. Examples of these activities include a spike in volleyball, an overhead smash in tennis, throws in baseball, and a penalty kick in soccer. In these situations, the initial upward vertical velocity of the ball is minimized or the ball may even have an initial downward velocity. The projection angle is relatively small—less than 45°—and in some cases even less than zero.

The peak height reached by a projectile is also dependent on its initial height and initial vertical velocity. The higher a projectile is at release and the faster it is moving upward at release, the higher it will go. Maximizing peak height is important in sports such as volleyball and basketball, where the players themselves are the projectiles. Another obvious example of a sport in which maximal peak height is desired is high jumping. Again, the athlete is the projectile. In these activities, the angle of projection is large, above 45°.

Maximizing the horizontal displacement or range of a projectile is the objective of several projectile sports. Examples of these include many of the field events in track and field, including the shot put, hammer throw, discus throw, javelin throw, and long jump. In the discus throw and javelin throw, the effects of air resistance are large enough that our projectile equations may not be accurate in describing the flight of the discus or javelin. For the shot put, hammer throw, and long jump, air resistance is too small to significantly affect things, so our projectile equations are valid. Our analysis of these situations may require the use of equations. If we want to maximize horizontal displacement, then equation 2.24 may be useful.

$$\Delta x = v\Delta t$$

This equation describes horizontal displacement (Δx) as a function of initial horizontal velocity *(v)* and time (Δt). But time in this case would be total time in the air or the flight time of the projectile. We just saw that the flight time of a projectile is determined by its initial height and its initial vertical velocity. The horizontal displacement of a projectile is thus determined by three things: initial horizontal velocity, initial vertical velocity, and initial height. If the initial height of release is zero (the same as the landing height), then the resultant velocity (the sum of vertical and horizontal velocities) at release determines the horizontal displacement of the projectile. The faster you can throw something, the farther it will go. But what direction should you throw in—more upward (vertical) or more outward (horizontal)?

If the initial velocity of the ball is totally vertical (a projection angle of 90°), the initial horizontal velocity (v in equation 2.24) would be zero, and the horizontal displacement would be zero as well. If the initial velocity of the ball is totally horizontal (a projection angle of zero), the flight time (Δt in equation 2.24) would be zero, and the horizontal displacement would be zero as well. Obviously, a combination of horizontal and vertical initial velocities (and a projection angle somewhere between 0° and 90°) would be better. What combination works best? If the resultant velocity is the same no matter what the angle of projection, then maximum horizontal displacement will occur if the horizontal and vertical components of the initial velocity are equal, or when the projection angle is 45°. If we look at equation 2.24, this makes sense. Horizontal displacement is determined by initial horizontal velocity and time in the air, but time in the air is determined by initial vertical velocity alone (if height of release is zero). It makes sense that these two variables—initial horizontal and vertical velocities—would have equal influence on horizontal displacement.

Let's check to see if this reasoning is confirmed by observations of projection angles in the sport of shot putting. At the 1995 World Track and Field Championships, the average angle of release for the best throw by the six medalists (three men and three women) in the shot put was 35° (Bartonietz and Borgtom 1995). This is much less than the optimal angle of 45°. But wait, does a shot have a height of release? Yes, the shot is released more than 2 m high. Look at figure 2.10, which shows a shot-putter near the instant he releases the shot. The shot is well above the ground. This height is its initial height. The height of release will give the shot more time in the air, so the time in the air does not have to be created by the vertical velocity at release. If the shot-putter doesn't have to give the shot as much vertical velocity at release, he can put more effort into generating horizontal velocity. The optimal projection angle will thus be less than 45°. The higher the height of release, the lower the projection angle.

Figure 2.10 The shot has an initial height at the instant of release.

Is there any other reason why the optimal release angle for shot putting should be less than 45° (other than the fact that shot-putters have a release height of 2 m or more)? Maybe. Our conclusion that 45° was an optimal projection angle for maximizing the horizontal displacement of a projectile relied on two conditions—first, that the release height was zero, and second, that the resultant velocity of the projectile was the same no matter what the projection angle was. For the shot putter, the first assumption was incorrect, so the release angle was less than 45°. What about the second assumption? In shot putting, does the resultant velocity of the shot change if you change the release angle? To answer this question, consider another question: Is it easier to move something faster horizontally or vertically upward? If you have access to a shot, determine whether you can roll it across the floor (move it horizontally) faster than you can throw it straight up. It's more difficult to accelerate objects upward and produce a large upward velocity than it is to accelerate objects horizontally and produce large horizontal velocities. In shot putting (and in most other throwing events), the resultant velocity of the shot increases as the angle of projection decreases below 45°.

If we examine projection angles for the discus throw or the javelin throw, we find that they are even lower than those of the shot put—even though the height of release is lower for the discus and javelin. Why? During the flight of the discus or javelin, the implement is acted on by another force besides gravity—air resistance. If the javelin or discus is thrown correctly, the air resistance force will exert some upward force on the javelin or discus during its flight. This upward force reduces the net downward force acting on the implement and thus causes its downward acceleration to be smaller as well. The result is that the javelin or discus stays airborne longer. Since the lift force gives the javelin or discus more time in the air, the time in the air does not have to be created by the vertical velocity at release. Once again, if the thrower doesn't have to give the javelin or discus as much vertical velocity at release, the athlete can put more effort into generating horizontal velocity. An extreme example of the lift effect of air resistance providing a projectile with more time in the air would be throwing a flying disc or ring such as a Frisbee or an Aerobie for distance. The lift effect of air resistance is so large on these projectiles that the optimal angle of release for maximizing horizontal distance is not much above horizontal.

Let's summarize what we now know about projectiles in sports.

1. If you want to maximize the time of flight or the height reached by a projectile, the vertical component of release velocity should be maximized, and the projection angle should be above 45°.
2. If you want to minimize the time of flight of a projectile, the upward component of release velocity should be minimized (perhaps so much so that the vertical velocity at release is downward). The projection angle should be much lower than 45° and in some situations may even be below horizontal.
3. If you want to maximize the horizontal displacement of a projectile, release velocity should be maximized and a higher release height is better. The horizontal component of release velocity should be slightly faster than the vertical component so that the projection angle is slightly lower than 45°. The higher the release height and the greater the lift effects of air resistance on the projectile, the farther below 45° the projection angle should be.

The equations governing projectile motion dictate the path that a ball or other thrown object will take once it leaves our hands. Once you release a ball, you no longer have control over it. Likewise, if you yourself become a projectile, the path taken by your body in the air is predetermined by your velocity and position at the instant you leave the ground. Once you have left the ground, if the only force acting on you is the force of gravity, you no longer have control over the path your body will take or your velocity.

Summary

Motion may be classified as linear, angular, or a combination of the two (general motion). Most examples of human movement are general motion, but separating the linear and angular components of the motion makes it easier to analyze the motion. Linear displacement is the straight-line distance from starting point to finish, whereas linear distance traveled represents the length of the path followed from start to finish. Velocity is the rate of change of displacement, whereas speed is the rate of change of distance. Acceleration is the rate of change of velocity. Displacement, velocity, and acceleration are vector quantities and are described by size and direction.

The vertical and horizontal motion of a projectile can be described by a set of simple equations if the only force acting on the projectile is the force of gravity. The horizontal velocity of a projectile is constant, and its vertical velocity is constantly changing at the rate of 9.81 m/s^2. The path of a projectile and its velocity are set once the projectile is released or is no longer in contact with the ground.

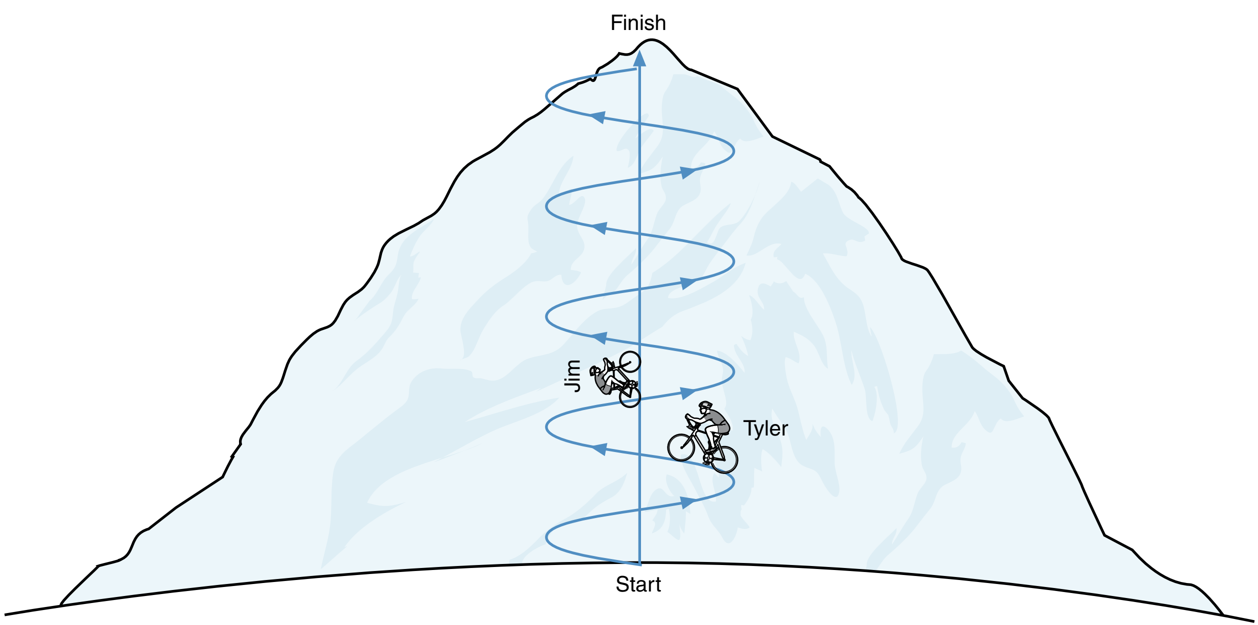
We now have the terms to describe many aspects of the linear motion of an object—distance traveled, displacement, speed, velocity, and acceleration. But what causes linear motion of objects? How do we affect our motion and the motion of things around us? We've gotten some hints in this and the previous chapter. In the next chapter, we will explore the causes of linear motion more thoroughly.

KEY TERMS

acceleration (p. 66)
angular motion (p. 53)
average acceleration (p. 66)
average speed (p. 60)
average velocity (p. 62)
Cartesian coordinate system (p. 54)
curvilinear translation (p. 52)
displacement (p. 57)
distance traveled (p. 57)
general motion (p. 53)
instantaneous acceleration (p. 66)
instantaneous speed (p. 62)
instantaneous velocity (p. 63)
linear motion (p. 52)
position (p. 54)
projectile (p. 68)
rectilinear translation (p. 52)
resultant displacement (p. 57)
speed (p. 60)
uniform acceleration (p. 68)
velocity (p. 60)

REVIEW QUESTIONS

1. Give an example of a human movement involving the whole body that represents curvilinear motion. Do not use the examples given at the beginning of the chapter.
2. Give an example of a human movement involving the whole body that represents rectilinear motion. Do not use the examples given at the beginning of the chapter.
3. Give an example of a human movement involving the whole body that represents angular motion. Do not use the examples given at the beginning of the chapter.
4. Tyler and Jim race each other up a mountain on their bicycles. Tyler rides a road bike on the switchbacks of the twisting and turning mountain road. Jim rides a mountain bike and follows a direct, but steeper, straight-line path up the mountain. They start at the same time and place at the bottom of the mountain and finish at the same time and place at the top of the mountain. From start to finish,
   a. whose distance traveled was longer?
   b. whose displacement was longer?
   c. which rider had the faster average speed?
   d. which rider had the faster average velocity?
   e. who won the race?

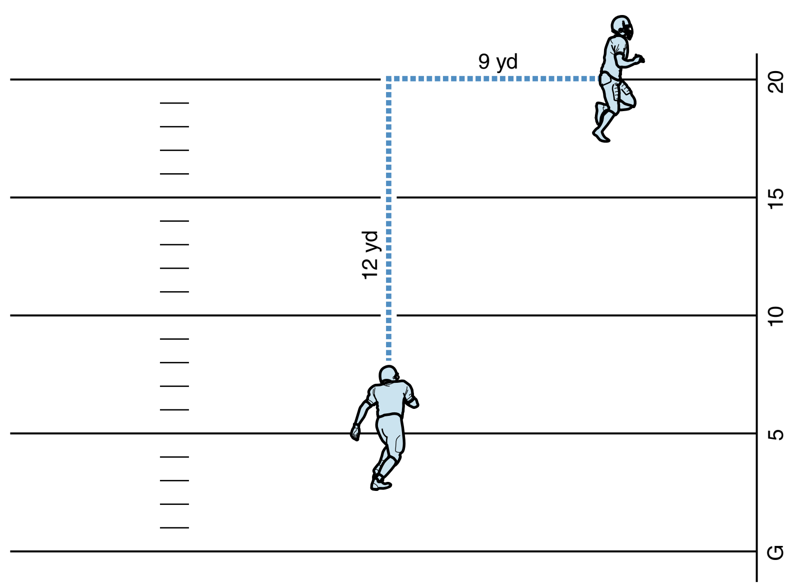


5. Are the sizes of the goals in ice hockey, lacrosse, soccer, field hockey, and team handball related to the speeds of the balls (or puck) used in these games? If so, explain the relationship.
6. Are the sizes of the courts in tennis, volleyball, racquetball, squash, table tennis, and badminton related to the speeds of the balls (or shuttlecock) used in these games? If so, explain the relationship.
7. What factors affect the speeds of the balls and implements listed in table 2.3?
8. When a 100 m sprinter has reached her maximum speed, is her average horizontal velocity faster during the support phase of a step (when her foot is on the ground) or during the flight phase of a step (when she is not in contact with the ground)? Explain.
9. Can a runner moving around a curve at constant speed be accelerating? Explain.

10. If Jim runs around a circle counterclockwise, in which direction (relative to the circle) is his acceleration? Explain.
11. List as many examples as you can of sports or situations in sport in which maximizing a projectile's time in the air is important.
12. List as many examples as you can of sports in which minimizing a projectile's time in the air is important.
13. Elite long jumpers have takeoff angles around 20°. Why do elite long jumpers have takeoff angles so much lower than the theoretically optimal takeoff angle of 45°?

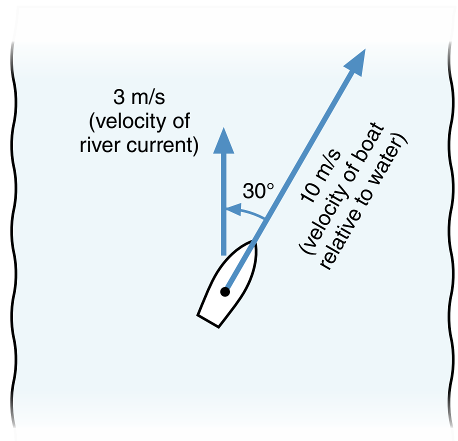
PROBLEMS

1. Sam receives the kicked football on the 3 yd line and runs straight ahead toward the goal line before cutting to the right at the 15 yd line. He then runs 9 yd along the 15 yd line directly toward the right sideline before being tackled.
   a. What was Sam's distance traveled?
   b. What was Sam's resultant displacement?
   c. How many yards did Sam gain in this play (how far was the ball advanced toward the goal line)?



2. During an ice hockey game, Phil had two shots on goal—one shot from 5 m away at 10 m/s and one shot from 10 m away at 40 m/s. Which shot did Brian, the hockey goalie, have a better chance of blocking?
3. The horizontal velocity of Bruce's fastball pitch is 40 m/s at the instant it's released from his hand. If the horizontal distance from Bruce's hand to home plate is 17.5 m at the instant of release, how much time does the batter have to react to the pitch and swing the bat?
4. The world-record times for the men's 50 m, 100 m, 200 m, and 400 m sprint races are 5.47 s, 9.58 s, 19.19 s, and 43.18 s, respectively. Which world-record race was run at the fastest average speed?
5. Matt is sailboarding northeast across the river with a velocity of 10 m/s relative to the water. The river current is moving the water north at a velocity of 3 m/s downstream. If the angle between

the relative velocity of the sailboard and the river current is 30°, what is the resultant or true velocity of the sailboard?



6. Sean is running a 100 m dash. When the starter's pistol fires, he leaves the starting block and continues speeding up until 6 s into the race, when he reaches his top speed of 11 m/s. He holds this speed for 2 s; then his speed has slowed to 10 m/s by the time he crosses the finish line 11 s after he started the race.
   a. What was Sean's average acceleration during the first 6 s of the race?
   b. What was Sean's average acceleration from 6 to 8 s into the race?
   c. What was Sean's average velocity for the whole race?
   d. What was Sean's average acceleration from 8 to 11 s into the race?
7. It is the final seconds of an ice hockey game between the Flyers and the Bruins. The Bruins are down by 1 point. With 20 s left in the game, the Bruins pull the goalie and have him play as a forward in an attempt to tie the game. The Flyers successfully defend their goal for 9 s. With only 1.25 s remaining on the game clock, a Flyer shoots the puck on the ice past the skates and sticks of the other players and toward the Bruins' goal. The puck is 37 m from the goal when it leaves the stick with an initial horizontal velocity of 30 m/s. The shot is perfectly directed toward the empty goal, but the ice slows the puck down at a constant rate of 0.50 m/s^2 as it slides toward the goal. None of the Bruins can stop the puck before it reaches the goal.
   a. Where is the puck when the game clock reaches zero and the horn sounds to end the game?
   b. Do the Flyers win the game by 1 or 2 points?
8. Mike clears a crossbar while pole vaulting. He releases the pole before he achieves his peak height. It takes him 1 s to fall from his peak height to the landing pit. The landing pit is 1 m high. How high above the ground was Mike at the peak height of his vault?
9. Brian is attempting to high jump over a crossbar set at 2.44 m (8 ft). At the instant of takeoff (when he is no longer in contact with the ground) his vertical velocity is 4.0 m/s, and his center of gravity is 1.25 m high.
   a. What is Brian's vertical acceleration at the instant of takeoff?
   b. How much time elapses after takeoff until Brian reaches his peak height?
   c. What peak height does Brian's center of gravity achieve?
10. Oliver punts a football into the air. The football has an initial vertical velocity of 15 m/s and an initial horizontal velocity of 15 m/s when it leaves the Oliver's foot. The ball experiences a constant vertical acceleration of 9.81 m/s^2 downward while it is in the air.
    a. What is the ball's horizontal velocity 2 s after it leaves the kicker's foot?
    b. What is the ball's vertical velocity 2 s after it leaves the kicker's foot?

c. What is the ball's horizontal displacement 2 s after it leaves the kicker's foot?

d. What is the ball's vertical displacement 2 s after it leaves the kicker's foot?

11. Gerri leaves the long jump takeoff board with a vertical velocity of 2.8 m/s and a horizontal velocity of 7.7 m/s.

a. What is Gerri's resultant velocity at takeoff?

b. What is Gerri's takeoff angle—the angle of her resultant takeoff velocity with horizontal?

c. What is Gerri's horizontal velocity just before she lands?

d. If Gerri is in the air for 0.71 s, what is her horizontal displacement during this time in the air?

e. What is Gerri's vertical velocity at the end of her 0.71 s flight?

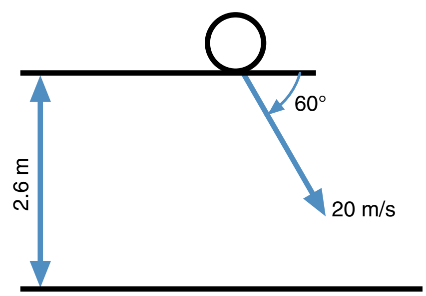
f. If Gerri's center of gravity was 1.0 m high at the instant of takeoff, how high will it be at the peak of her flight?

g. How high is Gerri's center of gravity at the end of her flight, when she first hits the pit?

12. Louise spikes a volleyball. At the instant the ball leaves her hand, its height is 2.6 m and its resultant velocity is 20 m/s downward and forward at an angle of 60° below horizontal.

a. How long will it take for the ball to strike the floor if the opposing team does not block it?

b. How far will the ball travel horizontally before it strikes the floor?



13. Chloe has a vertical velocity of 3 m/s when she leaves the 1 m diving board. At this instant, her center of gravity is 2.5 m above the water.

a. How high will Chloe go?

b. How long will Chloe be in the air before she touches the water? Assume that she first touches the water when her center of gravity is 1 m above the water.

14. Sam fields a baseball hit to him in the left field. He then throws the ball to third to force out the base runner, Mike. Sam releases the ball 1.80 m above the ground with a vertical velocity of 8 m/s and a horizontal velocity of 25 m/s. At the instant Sam releases the ball, he is 41 m from the third baseman, Charlie, and Mike is 13 m from third base and running at 8 m/s toward third. Assume that air resistance does not affect the flight of the ball when answering the following questions.

a. How high in the air does the ball go?

b. How much time does it take for the ball to reach the third baseman?

c. How high is the ball when it reaches the third baseman?

d. If Mike maintains a constant velocity of 8 m/s toward third base, does he reach third base before the ball reaches the third baseman?

15. At the Dallas Cowboys Stadium, the minimum clearance height between the football field and the gigantic video screens that hang over the field is only 90 ft (27.43 m). In the first game played at the Cowboys Stadium on August 21, 2009, a punter's kick hit the video screen.

a. What minimum initial vertical velocity would a football need to have to hit the video screens if it were kicked from a height 1 m above the playing field surface? Assume that air resistance does not affect the flight of the football.

b. If the video screens were not in the way, what would be the hang time for a football kicked 27.43 m high? Again, assume that air resistance does not affect the flight of the football and that the ball is 1 m above the playing field when it is kicked.

Motion Analysis Exercises Using MaxTRAQ

If you haven't done so already, review the instructions for downloading and using the educational version of the MaxTRAQ motion analysis software at the beginning of this book, then download and install the software. Once this is done, you are ready to try the following two-dimensional kinematic analyses using MaxTRAQ.

1. Open MaxTRAQ. Select Tools in the menu bar and then open Options under the Tools menu. In the Options submenu, select Video. In the upper right side of the Video window, under Video Aspect Ratio, make sure that Default-Used Preferred Aspect Ratio is selected. In the lower half of the right side of the Video window, under Deinterlace Options, select BOB, select Use Odd Lines First, and select Stretch Image Vertically. Click OK. Close MaxTRAQ and reopen it to have the deinterlace options take effect. Now open the *Run Slow* video from within MaxTRAQ. The video clips will have been downloaded with the software and saved to your local disk under Program Files\Motion Analysis\MaxTRAQP\VideoFiles. In the Open window, make sure that the drop-down menu at the bottom is set to Video Files.
2. When you open the *Run Slow* video, you should see a running track with three yellow balls in the middle lane. These balls are 2.5 m apart, so the yellow ball on the left is 5 m from the yellow ball on the right. Make sure that the scaling/calibration tool is activated by clicking View on the menu bar, then selecting Tools from the drop-down menu, and then making sure that Show Scale is checked. Open the scaling tool by clicking on Tools on the menu bar and selecting Scale. In the Scaling Tool window that opens, set the gauge length to 500 cm, then click OK. Now place the cursor over the left ball and click the left mouse button once (nothing will appear on screen yet); then place the cursor over the right ball and click the left mouse button a second time. The scale should appear in the video window. Hide the scale by selecting View in the menu, then click Tools, and uncheck Show Scale (this appears in the submenu).
   a. What is the stride length of the runner? Advance the video (using the Step Forward button in the video controls panel at the bottom of the screen) until the runner's right foot touches the track; then continue to advance the video to the last instant when the right foot is in contact with the track. Activate the digitizing function by clicking on the Digitize button on the right side of the screen. Place the cursor over the toe of the runner's right foot and click the left mouse button. A mark should appear on the toe along with the horizontal and vertical coordinates of the toe. If the coordinates do not appear, select Point Markers from the View drop-down menu and make sure that Show Coordinates is checked. Record the horizontal coordinate. Advance the video through one full stride to the next instant of takeoff of the right foot. Back up one frame to the last instant of contact of the right foot. Digitize the toe of the runner again and record the horizontal coordinate. The difference between the horizontal coordinates is the runner's stride length in centimeters.
   b. What is the stride rate of the runner in the *Run Slow* video? Use the frame number/time window at the bottom of the MaxTRAQ window to compute stride rate. Determine the time that elapses from takeoff of the right foot until the next takeoff of the right foot (the two points you digitized in part a). To improve accuracy, compute the elapsed time by multiplying the difference in frame numbers by 1/60 second (or just divide by 60). Divide one by this stride duration to calculate the number of strides per second.
   c. What is the average velocity of the runner in the *Run Slow* video during the one full stride you measured in parts a and b?
3. Open the *Run Medium* video in MaxTRAQ. If you did not set the video aspect ratio and deinterlace options in exercise 1, do so now by following those instructions. As you did in exercise 1,

make sure that the scaling/calibration tool is activated by clicking View, then Tools, and making sure that Show Scale is checked. Open the scaling tool by clicking on Tools on the menu bar and selecting Scale. Set the gauge length to 500 cm, then click once over the left ball and once over the right ball to set the scale. Hide the scale by selecting View, then Tools, and unchecking Show Scale.

a. What is the stride length of the runner in the *Run Medium* video? Measure the stride length from the instant of takeoff of the left foot until the next instant of takeoff of the left foot.

b. What is the stride rate of the runner in the *Run Medium* video?

c. What is the average velocity of the runner over one full stride in the *Run Medium* video?

4. Open the *Run Fast* video in MaxTRAQ. If you have not set the video aspect ratio and deinterlace options, do so now by following the instructions in exercise 1. As you did in exercise 1, make sure that Show Scale is checked under View—Tools, then open the scaling tool, set the gauge length to 500 cm, and click once over the left ball and once over the right ball to set the scale.

   a. What is the stride length of the runner in the *Run Fast* video? Measure the stride length from the instant of takeoff of the right foot until the next instant of takeoff of the right foot.

   b. What is the stride rate of the runner in the *Run Fast* video?

   c. What is the average velocity of the runner over one full stride in the *Run Fast* video?

5. Open the *Run Fastest* video in MaxTRAQ. If you have not set the video aspect ratio and deinterlace options, do so now by following the instructions in exercise 1. As you did in exercise 1, make sure that Show Scale is checked under View—Tools, then open the scaling tool, set the gauge length to 500 cm, and click once over the left ball and once over the right ball to set the scale.

   a. What is the stride length of the runner in the *Run Fastest* video? Measure the stride length from the instant of takeoff of the left foot until the next instant of takeoff of the left foot.

   b. What is the stride rate of the runner in the *Run Fastest* video?

   c. What is the average velocity of the runner over one full stride in the *Run Fastest* video?

6. Open the *Run Spring Stilts* video in MaxTRAQ. If you have not set the video aspect ratio and deinterlace options, do so now by following the instructions in exercise 1. As you did in exercise 1, make sure that Show Scale is checked under View—Tools, then open the scaling tool, set the gauge length to 500 cm, and click once over the left ball and once over the right ball to set the scale.

   a. What is the stride length of the athlete in the *Run Spring Stilts* video? Measure the stride length from the instant of takeoff of the left stilt tip until the next instant of takeoff of the left stilt tip.

   b. What is the stride rate of the athlete in the *Run Spring Stilts* video?

   c. What is the average velocity of the athlete over one full stride in the *Run Spring Stilts* video?

7. Open the *Ball Drop* video in MaxTRAQ. If you have not previously set the video aspect ratio and deinterlace options, do so now by following the instructions in exercise 1. You should see the author standing on a ladder against the wall of a building. The bricks of the wall are about 1 ft square, and horizontal white strips of tape are spaced vertically on the wall 1 m apart to the left of the ladder. Make sure the scaling/calibration tool is activated by clicking View on the menu bar, then selecting Tools from the drop down menu, and then making sure that Show Scale is checked. Open the scaling tool by clicking on Tools on the menu bar and selecting Scale. In the Scaling Tool window that opens, set the gauge length to 500 cm, then click OK. Now place the cursor over the left upper corner of the highest strip of tape and click the left mouse button once (nothing will appear on screen yet); then place the cursor over the left upper corner of the lowest strip of tape and click the left mouse button a second time. The scale should appear in the video window. Hide the scale by selecting View in the menu bar, then click Tools, and uncheck Show Scale. What is the vertical displacement of the ball 1 s (60 frames) after it was dropped? The ball was released in the first frame of video. Theoretically, what should the vertical displacement be?

8. Open the *Ball Toss Up* video in MaxTRAQ. If you have not previously set the video aspect ratio and deinterlace options, do so now by following the instructions in exercise 1. Set up the scaling/calibration tool as you did with the *Ball Drop* video using the 500 cm distance between the uppermost and lowermost strips of tape on the wall.
   a. If the ball reaches peak height between frames 47 and 48, or about 0.79 seconds into the video, what is the vertical displacement of the ball during the 0.75 s (45 frames) before peak height (from frame 2 to frame 47)?
   b. The ball reaches peak height between frames 47 and 48 or about 0.79 seconds into the video. What is the vertical displacement of the ball during the 0.75 s (45 frames) after peak height (from frame 48 to frame 93)?
   c. Is the upward displacement of the ball during the 0.75 s before it reaches peak height similar to the downward displacement of the ball during the 0.75 s after it reaches peak height? Should it be?
9. Open the *Ball Toss* video in MaxTRAQ. If you have not previously set the video aspect ratio and deinterlace options, do so now by following the instructions in exercise 1. Set up the scaling/calibration tool as you did with the *Ball Drop* and *Ball Toss Up* videos using the 500 cm distance between the uppermost and lowermost strips of tape on the wall.
   a. Determine the horizontal displacement of the ball over every 9-frame interval starting at frame 5 and ending at frame 86. Are these nine displacements similar to each other? Should they be?
   b. Determine the vertical displacement of the ball over every 9-frame interval starting at frame 5 and ending at frame 86. Are any of the first four displacements similar to any of the last four displacements? Should there be similarities?
   c. The ball reaches peak height between frames 45 and 46 or about 0.76 seconds into the video. What are the horizontal and vertical displacements of the ball during the 0.67 s (40 frames) before peak height (from frame 5 to frame 45)?
   d. The ball reaches peak height between frames 45 and 46 or about 0.76 seconds into the video. What are the horizontal and vertical displacements of the ball during the 0.67 s (40 frames) after peak height (from frame 46 to frame 86)?
   e. Are the answers to question c similar to the answers to question d? Should they be similar?