chapter 1

Forces

Maintaining Equilibrium or Changing Motion

objectives

When you finish this chapter, you should be able to do the following:

- Define force
- Classify forces
- Define friction force
- Define weight
- Determine the resultant of two or more forces
- Resolve a force into component forces acting at right angles to each other
- Determine whether an object is in static equilibrium, if the forces acting on the object are known
- Determine an unknown force acting on an object, if all the other forces acting on the object are known and the object is in static equilibrium

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A gymnast mainains a precarious position on one foot during a balance beam routine. A rock climber clings by his fingertips to the face of a cliff. A cyclist is motionless on her bicycle at the start of a race. A diver is supported only by his toes on the edge of the diving board before executing a back dive. What are the forces that act on each of these athletes? How do the athletes manipulate these forces in order to maintain balance? The information presented in this chapter provides you with the knowledge you need to answer these questions.

At every instant throughout our lives, our bodies are subjected to forces. Forces are important for motion because they enable us to start moving, stop moving, and change directions. Forces are also important even if we aren't moving. We manipulate the forces acting on us to maintain our balance in stationary positions. To complete a biomechanical analysis of a human movement, we need a basic understanding of forces: how to add them to produce a resultant force, how to resolve forces into component forces, and how forces must act to maintain equilibrium.

What Are Forces?

Simply defined, a **force** is a push or a pull. Forces are exerted by objects on other objects. Forces come in pairs: The force exerted by one object on another is matched by an equal but oppositely directed force exerted by the second object on the first—action and reaction. A force is something that accelerates or deforms an object. In rigid-body mechanics, we ignore deformations and assume that the objects we analyze do not change shape. So, in rigid-body mechanics, forces do not deform objects, but they do accelerate objects if the force is unopposed. Mechanically speaking, something accelerates when it starts, stops, speeds up, slows down, or changes direction. So a force is something that can cause an object to start, stop, speed up, slow down, or change direction.

Simply defined, a force is a push or a pull.

Our most familiar unit of measurement for force is the pound, but the SI unit of measurement for force is the newton, named in honor of the English scientist and mathematician Isaac Newton (we'll learn more about him in chapter 3). The newton is abbreviated as N. One newton of force is defined as the force required to accelerate a 1 kg mass 1 m/s^2, or algebraically as follows:

$$1.0\ \text{N} = (1.0\ \text{kg})(1.0\ \text{m/s}^2) \tag{1.1}$$

One newton of force is equal to 0.225 lb of force, or 1 lb equals 4.448 N. You may remember the story of Isaac Newton's discovery of gravity when an apple fell on his head. This story is probably not true, but it provides a good way to remember the size of a newton. A typical ripe apple weighs about 1 N.

Think about how to describe a force. For instance, suppose you want to describe the force a shot-putter exerted on a shot at the instant shown in figure 1.1. Would describing the size of the force provide enough information about it to predict its effect? What else would we want to know about the force? Some other important characteristics of a force are its point of application, its direction (line of action), and its sense (whether it pushes or pulls along this line). A force is what is known as a vector quantity. A **vector** is a mathematical representation of anything that is defined by its size or magnitude (a number) and its direction (its orientation). To fully describe a force, you must describe its size and direction.

If we want to represent a force (or any other vector) graphically, an arrow makes a good representation. The length of the arrow indicates the size of the force, the shaft of the arrow indicates its line of application, the arrowhead indicates its sense or direction along that line of application, and one of the arrow's ends indicates the point of application of the force. This is a good time to emphasize that the point of application of the force also defines which object the force is acting on (and thus defines which of the pair of forces—action or reaction—we are examining).

In this chapter and the next three chapters, we'll simplify rigid body mechanics even further by assuming that the rigid bodies we analyze are point masses or particles. The objects we'll examine are not really point masses or particles—they have a size and occupy space—but in analyzing the objects as particles, we'll assume that all the forces acting on these objects have the same point of application. Given these assumptions, the dimensions and shape of the objects do not change the effect of the forces acting on the object.

Classifying Forces

Now let's consider the different types of forces and how they are classified. Forces can be classified as internal or external.

Internal Forces

Internal forces are forces that act within the object or system whose motion is being investigated. Remember, forces come in pairs—action and reaction. With internal forces, the action and reaction forces act on different parts of the system (or body). Each of these forces may affect the part of the body it acts on, but the two forces do not affect the motion of the whole body because the forces act in opposition.

> Internal forces are forces that act within the object or system whose motion is being investigated.

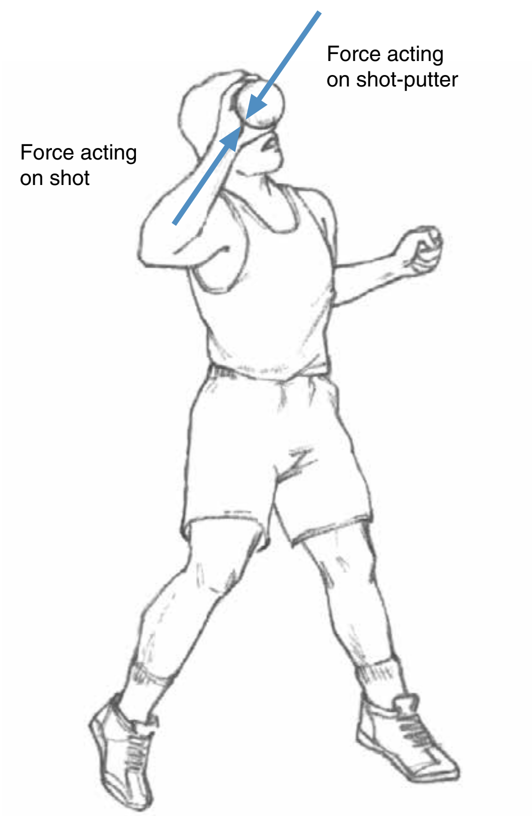


Figure 1.1 The forces acting on a shot-putter and a shot at the instant before release.

In sport biomechanics, the objects whose motion we are curious about are the athlete's body and the implements manipulated by the athlete. The human body is a system of structures—organs, bones, muscles, tendons, ligaments, cartilage, and other tissues. These structures exert forces on one another. Muscles pull on tendons, which pull on bones. At joints, bones push on cartilage, which pushes on other cartilage and bones. If pulling forces act on the ends of an internal structure, the internal pulling forces are referred to as **tensile forces**, and the structure is under **tension**. If pushing forces act on the ends of an internal structure, the internal pushing forces are referred to as **compressive forces**, and the structure is under compression. Internal forces hold things together when the structure is under tension or compression. Sometimes the tensile or compressive forces acting on a structure are greater than the internal forces the structure can withstand. When this happens, the structure fails and breaks. Structural failure in the body occurs when muscles pull, tendons rupture, ligaments tear, and bones break.

We think of muscles as the structures that produce the forces that cause us to change our motion. Actually, because muscles can produce only internal forces, they are incapable of producing changes in the motion of the body's center of mass. It is true that muscle forces can produce motions of the body's limbs, but these motions will not produce any change in motion of the body's center of mass unless external forces are acting on the system. The body is able to change its motion only if it can push or pull against some external object. Imagine a defensive player in basketball jumping up to block a shot (see figure 1.2). If she has been fooled by the shooter and jumps too early, she can't stop herself in midair to wait for the shooter to shoot. The only external force acting on her in this case is gravity. She needs to touch something to create another external force to counteract the force of gravity. So she has to get her feet back on the ground. Then she can push against the ground and create an external reaction force that causes her to jump up again. The ground provides the external force that causes the change in motion of the basketball player.

Internal forces may be important in the study of exercise and sport biomechanics if we are concerned about the nature and causes of injury, but they cannot produce any changes in the motion of the body's center of mass. External forces are solely responsible for that.

External Forces

External forces are those forces that act on an object as a result of its interaction with the environment surrounding it. We can classify external forces as contact forces or noncontact forces. Most of the forces we think about are contact forces. These occur when objects are

Figure 1.2 A basketball player cannot change her motion once she has jumped into the air.

touching each other. Noncontact forces are forces that occur even if the objects are not touching each other. The gravitational attraction of the earth is a noncontact force. Other noncontact forces include magnetic forces and electrical forces.

External forces are those forces that act on an object as a result of its interaction with the environment surrounding it.

In sports and exercise, the only noncontact force we will concern ourselves with is the force of gravity. The force of gravity acting on an object is defined as the weight of the object. Remember that we defined 1 N of force as the force that would accelerate a 1 kg mass 1 m/s^2. If the only force acting on an object is the force of the earth's gravity, then the force of gravity will accelerate the object. This is the case when we drop something (if the force of air resistance can be ignored). Scientists have precisely measured this acceleration for various masses at various locations around the earth. It turns out to be about 9.81 m/s^2 (or 32.2 ft/s^2) downward no matter how large or small the object is. This acceleration is called **gravitational acceleration** or the **acceleration due to gravity** and is abbreviated as g.

Weight is the force of gravity acting on an object.

Now let's see if we can figure out the weight of something if we know its mass. If a 1 N force accelerates a 1 kg mass 1 m/s^2, then how large is the force that would accelerate a 1 kg mass 9.81 m/s^2? Another way of asking this question is, How much does 1 kg weigh?

? N = (1 kg)(9.81 m/s^2) = Weight of 1 kg = Force of gravity acting on 1 kg

If we solve this equation, we find that 1 kg weighs 9.81 N. On the earth, mass (measured in kilograms) and weight (measured in newtons) are proportional to each other by a factor of 9.81. The weight of an object (in newtons) is its mass (in kilograms) times the acceleration due to gravity (9.81 m/s^2), or,

$$W = mg \tag{1.2}$$

where

W = weight (measured in newtons),
m = mass (measured in kilograms), and
g = acceleration due to gravity = 9.81 m/s^2.

To estimate the weight of something, multiplying its mass by 9.81 m/s^2 may be difficult to do in your head. For quick approximations, let's round 9.81 m/s^2 to 10 m/s^2 and use that as our estimate of the acceleration due to gravity. This will make things easier, and our approximation won't be too far off because our estimate of g is only 2% in error. If more accuracy is required, the more precise value of 9.81 m/s^2 should be used for g.

Contact forces are forces that occur between objects in contact with each other. The objects in contact can be solid or fluid. Air resistance and water resistance are examples of fluid contact forces, which are further discussed in chapter 8. The most important contact forces in sport occur between solid objects, such as the athlete and some other object. For a shot-putter to put the shot, the athlete must apply a force to it, and the only way the athlete can apply a force to the shot is to touch it. To jump up in the air, you must be in contact with the ground and push down on it. The reaction force from the ground pushes up on you and accelerates you up into the air. To accelerate yourself forward and upward as you take a running step, you must be in contact with the ground and push backward and downward against it. The reaction force from the ground pushes forward and upward against you and accelerates you forward and upward.

Contact forces can be resolved into parts or components—the component of force that acts perpendicular to the surfaces of the objects in contact and the component of force that acts parallel to the surfaces in contact. We call the first component of contact force a normal contact force (or normal reaction force), where normal refers to the fact that the line of action of this force is perpendicular to the surfaces in contact. During a running step, when the runner pushes down and backward on the ground, the normal contact force is the component of force that acts upward on the runner and downward on the ground. The second component of the contact force is called **friction**. The line of action of friction is parallel to the two surfaces in contact and opposes motion or sliding between the surfaces. So when the runner pushes down and backward on the ground during a running step, the frictional force is the component of force that acts forward on the runner and backward on the ground (see figure 1.3). The frictional force is the component of the contact force responsible for changes in the runner's horizontal motion. Frictional forces are primarily responsible for human locomotion, so an understanding of friction is important.

Friction

The frictional force just described is dry friction, which is also referred to as Coulomb friction. Another type of friction is fluid friction, which develops between two layers of fluid and occurs when dry surfaces are lubricated. The behavior of fluid friction is complicated; and because fluid friction occurs less frequently in sport, we will limit our discussion to dry friction. Dry friction acts between the nonlubricated surfaces of solid objects or rigid bodies in contact and acts parallel to the contact surfaces. Friction arises as a result of interactions between the molecules of the surfaces in contact. When dry friction acts between two surfaces that are not moving relative to each other, it is referred to as **static friction**. Static friction is also referred to as **limiting friction** when we describe the maximum amount of friction that develops just before two surfaces begin to slide. When dry friction acts between two surfaces that are moving relative to each other, it is referred to as **dynamic friction**. Other terms for dynamic friction are *sliding friction* and *kinetic friction.*

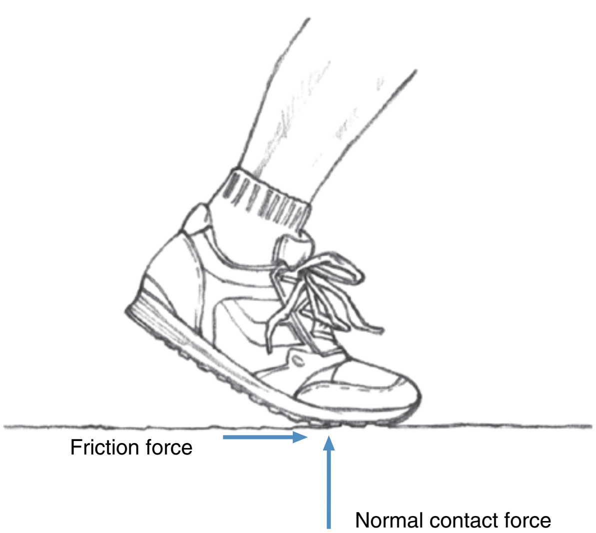


Figure 1.3 Normal contact force and friction force acting on a runner's foot during push-off.

Friction and Normal Contact Force

Try self-experiment 1.1 to see how friction is affected by normal contact force.

Self-Experiment 1.1

Let's do some experimentation to learn more about friction. Place a book on a flat horizontal surface such as a desk or tabletop. Now push sideways against the book and feel how much force you can exert before the book begins to move. What force resists the force that you exert on the book and prevents the book from sliding? The resisting force is static friction, which is exerted on the book by the table or desk. If the book doesn't slide, then the static friction force acting on the book is the same size as the force you exert on the book, but in the opposite direction. So, the effects of these forces are canceled, and the net force acting on the book is zero. Put another book on top of the original book and push again (see figure 1.4). Can you push with a greater force before the books begin to move? Add another book and push again. Can you push with an even greater force now? As you add books to the pile, the magnitude (size) of the force you exert before the books slide becomes bigger, and so does the static friction force.

How did adding books to the pile cause the static friction force to increase? We increased the inertia of the pile by increasing its mass. This shouldn't affect the static friction force, though, because there is no apparent way an increase in mass could affect the interactions of the molecules of the contacting surfaces. It is these interactions that are responsible for friction. We also increased the weight of the pile as we added books to it. Could this affect the static friction force? Well, increasing the weight would increase the normal contact force acting between the two surfaces. This would increase the interactions of the molecules of the contacting surfaces, because they would be pushed together harder. So it is not the weight of the books that caused the increased static friction force, but the increase in the normal contact force. If

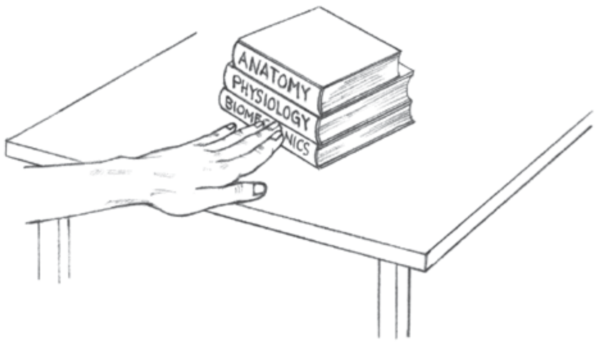


Figure 1.4 Adding books to the stack increases static friction between the bottom book and the table.

we measured this normal contact force and the friction force, we would find that the friction force is proportional to the normal contact force. As one increases, the other increases proportionally. This is true for both static and dynamic friction.

In self-experiment 1.1, the friction force was horizontal and the normal contact force was a vertical force influenced by the weight of the books. Is friction force only a horizontal force? Is the normal contact force always vertical and related to the weight of the object that friction acts on? Try self-experiment 1.2 to answer these questions.

Self-Experiment 1.2

Now try holding the book against a vertical surface, such as a wall (see figure 1.5). Can you do this if you push against the book only with a horizontal force? How hard must you push against the book to keep it from sliding down the wall? What force opposes the weight of the book and prevents the book from falling? The force of your hand pressing against the book is acting horizontally, so it can't oppose the vertical force of gravity pulling down on the book. The force acting upward on the book is friction between the book and the wall (and possibly between the book and your hand). The force you exert against the book affects friction since the book will slide and fall if you don't push hard enough. Again we see that friction is affected by the normal contact force—the contact force acting perpendicular to the friction force and the contact surfaces.

➲ **Friction force is proportional to the normal contact force and acts perpendicular to it.**

Friction and Surface Area

What else affects friction? What about surface area? Let's try another experiment (self-experiment 1.3) to see if increasing or decreasing surface area in contact affects friction force.

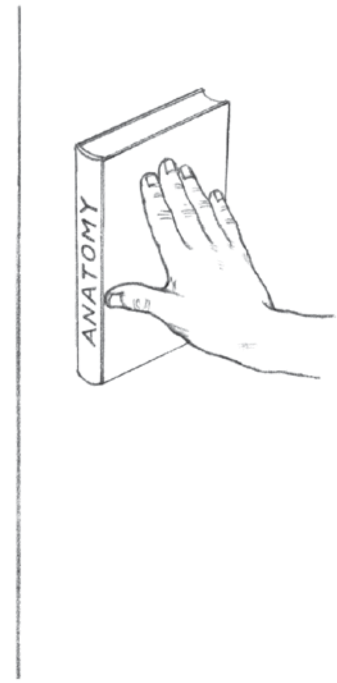


Figure 1.5 Friction force between the book and the wall and between the book and your hand is enough to hold the book up.

Self-Experiment 1.3

Does surface area of contact affect friction? Take a hardback book and lay it on a table or desk. (It's important that you use a hardback book.) Push the book back and forth across the table and get a feeling for how large the dynamic and static friction forces are. Try to exert only horizontal forces on the book. Now, try the same thing with the book standing on its end (as in figure 1.6). Use a rubber band to hold the book closed, but don't let the rubber band touch the table as you're sliding the book. Are there any noticeable differences between the frictional forces you feel with the book in its different orientations? Try it with another hardback book.

With the different orientations in self-experiment 1.3, the surface areas in contact between the book and table varied dramatically, but friction did not change noticeably. In fact, dry friction, both static and dynamic, is not affected by the size of the surface area in contact. This statement is probably not in agreement with your intuitions about friction, but you have just demonstrated it to yourself. If that isn't enough to convince you that dry friction is unaffected by surface area, let's try to explain it.

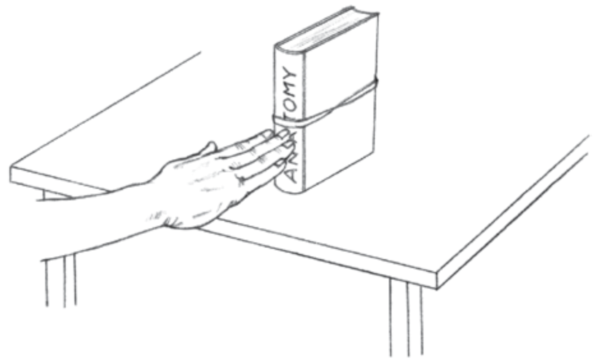


Figure 1.6 A book on its end has a smaller area of contact with the table. Does this reduced area of contact affect the frictional force between the book and the table?

Dry friction arises due to the interaction of the molecules at the surface areas in contact. We have seen that if we press these surfaces together with greater force, the interactions of the molecules will be greater and friction will increase. It makes sense to say that if we increase the area of the surfaces in contact, we also increase the number of molecules that can interact with each other, and thus we create more friction. But if the force pushing the surfaces together remains the same, with the greater surface area in contact, this force is spread over a greater area, and the pressure between the surfaces will be less (pressure is force divided by area). So the individual forces pushing each of the molecules together at the contact surfaces will be smaller, thus decreasing the interactions between the molecules and decreasing the friction. This looks like a trade-off. The increase in surface area increases the number of molecular interactions, but the decrease in pressure decreases the magnitude of these interactions. So the net effect of increasing surface area is zero, and friction is unchanged.

Dry friction is not affected by the size of the surface area in contact.

Friction and Contacting Materials

Friction is affected by the size of the normal contact force, but it is unaffected by the area in contact. What about the nature of the materials that are in contact? Is the friction force on rubber-soled shoes different than the friction force on leather-soled shoes? Let's try one more experiment (self-experiment 1.4) to investigate how the nature of the materials in contact affects the friction force between them.

Self-Experiment 1.4

Let's observe the difference between the frictions of a book on the table and a shoe on the table. Place the book on the table and put an athletic shoe on top of it. Push the book back and forth across the table and get a feeling for how large the dynamic and static friction forces are. Now, put the shoe on the table, sole down, and place the book on top of it. Push the shoe back and forth across the table and get a feeling for how large the dynamic and static friction forces are. Which produced larger frictional forces with the table, the book or the shoe? What changed between the two conditions? In the two conditions, the weight and mass of the objects being moved (the shoe and book) stayed the same. The surface area of contact changed, but we have determined that friction is unaffected by that. The variable that must be responsible for the changes in the observed frictional force is the difference in the type of material that was in contact with the table. Greater friction existed between the table and the softer and rougher sole of the shoe than between the table and the smoother and harder book cover.

One more observation about friction must be made. When you moved the book back and forth across the table in the self-experiments, was it easier to get the book started or to keep the book moving? In other words, was static friction larger or smaller than dynamic friction? It was easier to keep the book moving than to get it started moving, so static friction is larger than dynamic friction.

Let's summarize what we now know about dry friction. Friction is a contact force that acts between and parallel to the two surfaces in contact. Friction opposes relative motion (or impending relative motion) between the surfaces in contact. Friction is proportional to the normal contact force pushing the two surfaces together. This means that as the normal contact force increases, the frictional force increases as well. If the normal contact force doubles in size, the frictional force will double in size also. Friction is affected by the characteristics of the surfaces in contact. Greater friction can be developed between softer and rougher surfaces than between harder and smoother surfaces. Finally, static friction is greater than dynamic friction. Mathematically, we can express static and dynamic friction as

$$F_s = \mu_s R \qquad (1.3)$$

$$F_d = \mu_d R \qquad (1.4)$$

where

F_s = static friction force,
F_d = dynamic friction force,
μ_s = coefficient of static friction,
μ_d = coefficient of dynamic friction, and
R = normal contact force.

The coefficient of friction is a number that accounts for the different effects that materials have on friction. Mathematically, the coefficient of friction, abbreviated with the Greek letter mu, is just the ratio of friction force to normal contact force.

Mathematically, the coefficient of friction is the ratio of friction force over normal contact force.

Friction in Sport and Human Movement

Friction is an important force in every sport and human movement. Locomotion requires frictional forces, so the shoes we wear are designed to provide proper frictional forces between our feet and the supporting surface. In most athletic shoes, we want large frictional forces, so the materials used for the soles have large coefficients of friction. In some activities, such as dancing or bowling, sliding is desirable, so the soles of the shoes used for these activities have smaller coefficients of friction. In snow skiing, we also want small frictional forces, so we wax the bottoms of our skis to decrease the coefficient of friction. In racket sports and other sports involving implements, large frictional forces are desirable so that we don't lose hold of the implement. The grips are made of material such as leather or rubber, which have large coefficients of friction. We may even alter the grips to increase their coefficients of friction by wrapping athletic tape on them, spraying them with tacky substances, or using chalk on our hands. Think about the variety of sports you have been involved in and how friction affects performance in them. In everyday activities, the friction between footwear and floors is important in preventing slips and falls.

We now know about several of the various external forces that can act on us in sport activities; gravity, friction, and contact forces are the major ones. In most sport and exercise situations, more than one of these external forces will act on the individual. How do we add up these forces to determine their effect on the person? What is a net force or a resultant force?

Addition of Forces: Force Composition

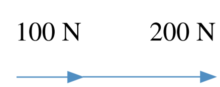
The net force acting on an object is the sum of all the external forces acting on it. This sum is not an algebraic sum; that is, we can't just add up the numbers that represent the sizes of the forces. The net force is the vector sum of all the external forces. Remember that we define a force as a push or pull, and that forces are vector quantities. This means that the full description of a force includes its magnitude (how large is it?) and its direction (which way does it act?). Visually, we can think of forces as arrows, with the length of the shaft representing the magnitude of the force, the orientation of the arrow representing its line of application, and the arrowhead indicating its direction of action along that line. When we add vectors such as forces, we can't just add up the numbers representing their sizes. We must also consider the directions of the forces. Forces are added using the process of vector addition. The result of vector addition of two or more forces is called a **resultant force**. The vector addition of all the external forces acting on an object is the **net force**. It is also referred to as the resultant force, because it results from the addition of all the external forces. Now we will learn how to carry out vector addition of forces.

The vector addition of all the external forces acting on an object is the net force.

Colinear Forces

To begin our discussion of vector addition, let's start with a simple case that involves colinear forces. If you look closely at the word *colinear,* you may notice that the word *line* appears in it. **Colinear forces** are forces that have the same line of action. The forces may act in the same direction or in opposite directions along that line. Now here's the situation. You are on a tug-of-war team with two others. You pull on the rope with a force of 100 N, and your teammates pull with forces of 200 N and 400 N. You are all pulling along the same line—the line of the rope. To find out the resultant of these three forces, we begin by graphically representing each force as an arrow, with the length of each arrow scaled to the size of the force. First, draw the 100 N force that you exerted on the rope. If you were pulling to the right, the force you exerted on the rope might be represented like this:

Now draw an arrow representing the 200 N force. Put the tail of this force at the arrowhead of the 100 N force. If it is scaled correctly, this arrow should be twice as long as the arrow representing the 100 N force.



Now draw the arrow representing the 400 N force. Put the tail of this force at the arrowhead of the 200 N force. This arrow should be four times as long as the arrow representing the 100 N force and twice as long as the arrow representing the 200 N force. Your drawing should look something like this:

An arrow drawn from the tail of the 100 N force to the tip of the 400 N force represents the resultant force, or the vector sum of the 100 N, 200 N, and 400 N forces, if we put the arrowhead on the end where it meets the tip of the 400 N force.

If we measure the length of this arrow, it turns out to be seven times as long as the 100 N force. The resultant force must be a 700 N force acting to the right. But this is what we would have found if we added the magnitudes of the three forces algebraically:

100 N + 200 N + 400 N = 700 N

Does that mean that vector addition and algebraic addition are the same? No! This is true only when the forces all act along the same line and in the same direction.

When forces act along the same line and in the same direction, they can be added using regular algebraic addition.

Now let's consider the forces the opposing team exerts on the rope. That team also consists of three members. The forces they exert on the rope are to the left and are 200 N, 200 N, and 200 N, respectively. What is the resultant of these three forces?

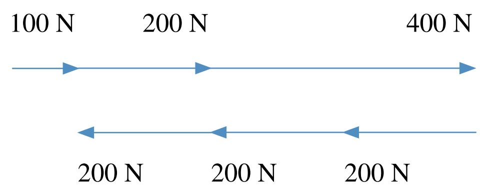
We can determine the resultant by graphically representing the three forces as arrows and connecting the tail of the first arrow with the tip of the last arrow, as we did previously. We could also have added the magnitudes of the forces algebraically, because all three forces acted along the same line in the same direction.

200 N + 200 N + 200 N = 600 N

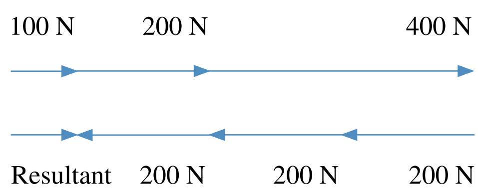
Now what is the resultant force acting on the rope as a result of your team pulling to the right and the opposing team pulling to the left? In this case, we have the three forces from your team pulling to the right

and the three forces from the opposing team pulling to the left:

These forces are all still colinear because they act along the same line, the line of the rope in this case. If we follow the procedure we used before, we add the forces graphically by lining the vectors up tip to tail. We have done this for the three forces from your team. The tail of the 200 N force is lined up with the tip of the 100 N force, and the tail of the 400 N force is lined up with the tip of the 200 N force. We have done this for the opposing team's forces as well. Now, to add up all of these forces, we line up the tail of the 200 N force of the opposing team with the tip of the 400 N force of your team (we also could have lined up the tail of your 100 N force with the tip of the 200 N force from the opposing team):

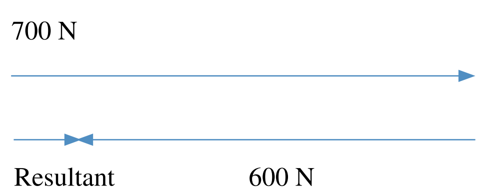


We find the resultant force by drawing an arrow from the tail of the 100 N force to the tip of the last 200 N force, with the tip of the arrow at the end lined up with the tip of the 200 N force and the tail at the end lined up with the tail of the 100 N force:



If we measure the length of this resultant vector, we see that it is the same length as the 100 N force. The resultant force is 100 N to the right.

We could have arrived at the same resultant if we replaced the forces exerted by your team with its 700 N resultant force to the right and the forces by the opposing team with its 600 N resultant force to the left.



Because all of the forces are acting along the same line, the resultant force could also be found through algebraic means. Now, rather than just adding up the forces as we did for each team, we must also consider the direction in which the forces act. Let's arbitrarily say that forces acting to the right are positive. Then forces acting to the left must be considered negative. So the resultant force acting on the rope as a result of your team pulling on it to the right and the opposing team pulling on it to the left can now be determined algebraically by adding up the positive forces from your team and the negative forces from the opposing team.

$$100\text{ N} + 200\text{ N} + 400\text{ N} + (-200\text{ N}) + (-200\text{ N}) + (-200\text{ N}) = +100\text{ N}$$

Adding a negative number is just like subtracting it, so we could also write this as

$$100\text{ N} + 200\text{ N} + 400\text{ N} - 200\text{ N} - 200\text{ N} - 200\text{ N} = +100\text{ N}.$$

The positive sign associated with our answer of 100 N indicates that the resultant force acts in the positive direction. We set up our positive direction to the right, so the resultant force is a force of 100 N to the right.

If forces are colinear, we may add them using vector addition by graphically representing each force as an arrow and arranging the force arrows tip to tail. We determine the resultant force by drawing an arrow from the tail of the first force to the tip of the last force. This arrow has its tip at the tip of the last force, and it represents the resultant force. We may also add colinear forces algebraically if we take into account the senses of the forces on the line along which they act by assigning positive or negative signs to the magnitudes of the forces. Positive forces act in one direction along the line, and negative forces act in the opposite direction along the line.

Concurrent Forces

If forces do not act along the same line but do act through the same point, the forces are **concurrent forces**. As long as we model objects as point masses, the forces acting on these objects will be considered colinear forces if they act along the same line, and concurrent forces if they do not act along the same line. It is not until chapter 5, when we begin modeling objects as true rigid bodies and not point masses, that the forces we consider can be nonconcurrent forces.

Now let's consider a situation in which the external forces are not colinear but are concurrent. A gymnast is about to begin his routine on the high bar. He jumps up and grasps the bar, and his coach stops his swinging by exerting forces on the front and back of the gymnast's torso. The external forces acting on the gymnast are the force of gravity acting on the mass of the gymnast, a horizontal force of 20 N exerted by the coach pushing on the front of the gymnast, a horizontal force of 30 N exerted by the coach pushing on the back of the gymnast, and an upward vertical reaction force of 550 N exerted by the bar on the gymnast's hands. The gymnast's mass is 50 kg. What is the net external force acting on the gymnast?

First, how large is the force of gravity that acts on the gymnast? If you remember, earlier in this chapter we said that the force of gravity acting on an object is the object's weight. What is the gymnast's weight? Weight is defined with equation 1.2

$$W = mg$$

where W represents weight in newtons, m represents mass in kilograms, and g represents the acceleration due to gravity, or 9.81 m/s^2. For a good approximation, we can round 9.81 m/s^2 to 10 m/s^2 and make our computations easier. If we want more accuracy, we should use 9.81 m/s^2 rather than 10 m/s^2. So, using the rougher approximation for g, the gymnast weighs

$$W = mg = (50\text{ kg})(10\text{ m/s}^2) = 500\text{ kg m/s}^2 = 500\text{ N}.$$

This weight is a downward force of 500 N. We now have all the external forces that act on the gymnast. A drawing of the gymnast and all the external forces that act on him is shown in figure 1.7.

SAMPLE PROBLEM 1.1

A spotter assists a weightlifter who is attempting to lift a 1000 N barbell. The spotter exerts an 80 N upward force on the barbell, while the weightlifter exerts a 980 N upward force on the barbell. What is the net vertical force exerted on the barbell?

Solution

Assume that upward is the positive direction. The 80 N force and the 980 N force are positive, and the 1000 N weight of the barbell is negative. Adding these up gives us the following:

$$\Sigma F = (+80\text{ N}) + (+980\text{ N}) + (-1000\text{ N}) = 80\text{ N} + 980\text{ N} - 1000\text{ N} = +60\text{ N}$$

The symbol, Σ, that appears before the F in the above equation is the Greek letter sigma. In mathematics it is the summation symbol. It means to sum or add up all items indicated by the variable following the Σ. In this case, ΣF means sum all of the forces or add up all of the forces.

The net vertical force acting on the barbell is a 60 N force acting upward.

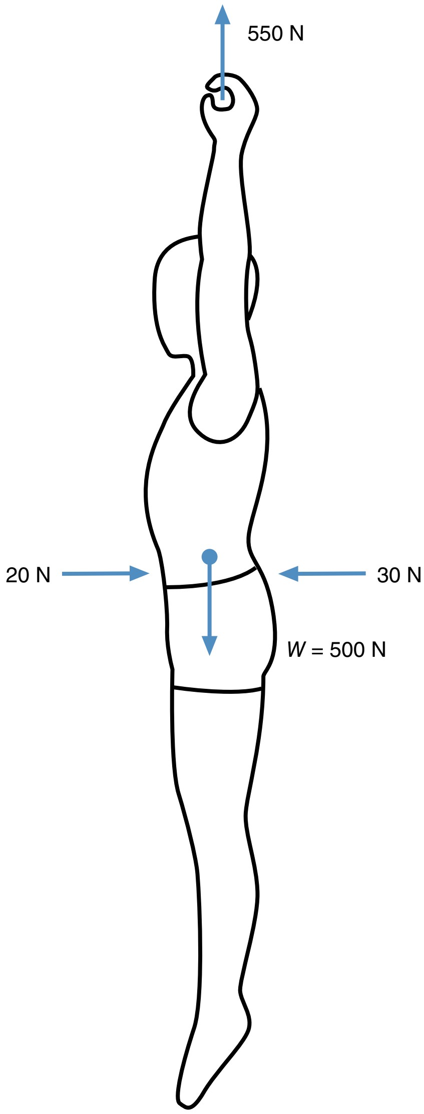


Figure 1.7 Free-body diagram showing the external forces acting on a gymnast hanging from the horizontal bar.

Now we can begin the process of determining the resultant of these forces. Just as we did with the colinear forces, we can represent each force graphically with an arrow, scaling the length of the arrow to represent the magnitude of the force, orienting the arrow to show its line of application, and using an arrowhead to show its sense or direction. As with the colinear forces, if we line up the forces tip to tail, we can find the resultant. Let's do that. First, draw the 20 N horizontal force acting to the right. Now draw the 550 N upward force so that its tail begins at the head of the 20 N force. Draw the 30 N horizontal force to the left so that the tail of this force begins at the head of the 550 N force. Draw the 500 N downward force of gravity so that the tail of this force begins at the head of the 30 N force. You should now have a drawing that looks something like figure 1.8.

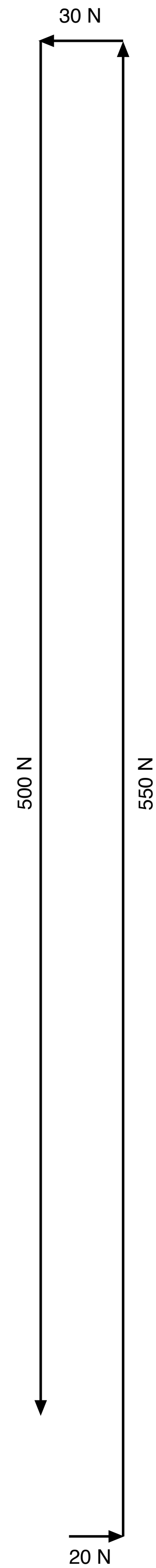


Figure 1.8 Graphic representation of all forces acting on the gymnast.

The head of the 500 N downward force and the tail of the 20 N horizontal force do not connect. The resultant of the four forces can be represented by an arrow connecting the tail of the 20 N horizontal force (the first force in our drawing) with the head of the 500 N downward force (the last force in our drawing). Figure 1.9 shows the construction of the resultant force. This resultant force is directed from the tail of the 20 N horizontal force to the head of the 500 N downward force. The resultant is thus directed upward and slightly to the left. The size of the resultant force is indicated by the length of its arrow. Using the same scale that was used to construct the other forces in figure 1.9, we can estimate that the magnitude of the resultant force is about 51 N.

If we describe the direction of the resultant force as "upward and slightly to the left," we haven't provided a very precise description. Can the direction of the force be described with more precision than that? We could describe the angle that the force makes with a vertical line or a horizontal line. Measuring clockwise from a vertical line, this force is about 11° from vertical. This angular description is much more precise than the description of the force as "upward and slightly to the left."

If vertical and horizontal forces act on a body, we can add forces graphically, as we did to determine the resultant force. Is there any way we can determine the resultant force without using graphical means? Is there a mathematical technique we can use? Let's again consider the four forces acting on the gymnast. Horizontally, there are two forces acting: a 20 N force to the right and a 30 N force to the left. Vertically, there are also two forces acting: a 500 N force downward and a 550 N force upward. Can we just add up all of these forces algebraically? If we did, we would have

$$20\text{ N} + 30\text{ N} + 500\text{ N} + 550\text{ N} = 1100\text{ N}.$$

This is much different than what we determined graphically. Maybe we need to consider the downward forces as negative and the forces to the left as negative. Using this method, we have

$$20\text{ N} + (-30\text{ N}) + (-500\text{ N}) + 550\text{ N} =$$
$$20\text{ N} - 30\text{ N} - 500\text{ N} + 550\text{ N} = 40\text{ N}.$$

This is much closer to the graphical result, but it still is not correct. We also don't know in which direction the resultant acts. Let's try one more method. Consider the horizontal and vertical forces separately and determine what the horizontal resultant force is and what the vertical resultant force is. Now the problem is similar to the colinear force problems we solved earlier.

Horizontally, we have a 20 N force acting to the right and a 30 N force acting to the left. Previously, we arbitrarily decided that forces to the right were positive, and we assigned a negative value to forces that acted to the left, so the resultant horizontal force is

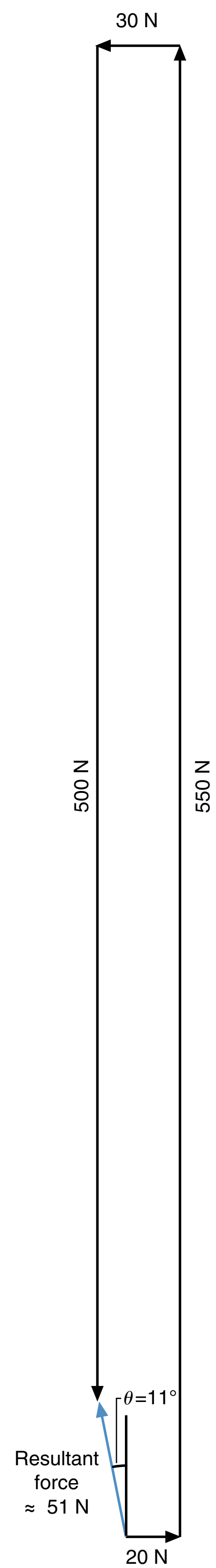


Figure 1.9 Graphic determination of resultant force acting on the gymnast.

$$20\text{ N} + (-30\text{ N}) = 20\text{ N} - 30\text{ N} = -10\text{ N}.$$

The negative sign associated with this force indicates that it acts to the left. The resultant horizontal force is 10 N acting to the left.

Vertically, we have a 500 N force acting downward and a 550 N force acting upward. Let's call upward our positive direction and assign a negative value to the downward force. The resultant vertical force is

$$(-500\text{ N}) + 550\text{ N} = +50\text{ N}.$$

The positive sign associated with this force indicates that it acts in an upward direction. The resultant vertical force is 50 N acting upward.

Using this method, the resultant force can be expressed as a 10 N horizontal force acting to the left and a 50 N vertical force acting upward. Is this equivalent to the 51 N resultant force that acts upward and slightly to the left at 11° from vertical? How can a 51 N force be equivalent to a 50 N force and a 10 N force? Add the horizontal resultant force of 10 N and the vertical force of 50 N graphically to determine their resultant force. Draw the forces tip to tail, as shown in figure 1.10.

Now draw the resultant by connecting the tail of the 10 N horizontal force with the tip of the 50 N vertical force. How does this force compare to the resultant shown in figure 1.9? They look identical. Measure the resultant in figure 1.10, and measure the angle it makes with the vertical. The resultant force is about 51 N and makes an angle of 11° with vertical. It is identical to the resultant force shown in figure 1.9. Apparently, a 50 N force and a 10 N force can be equivalent to a 51 N force.

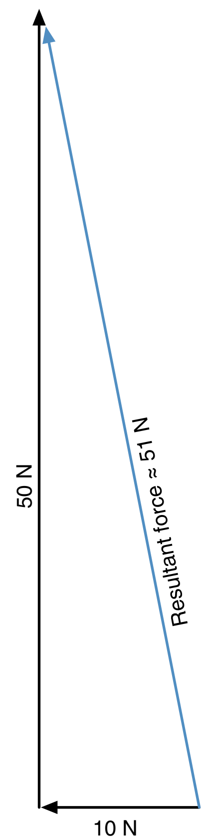


Figure 1.10 Vector sum of the net horizontal force and net vertical force acting on the gymnast.

Trigonometric Technique

Take a closer look at the shape created by the three forces in figure 1.10. It's a triangle. In fact, it's a right triangle—one of the angles in the triangle is a 90° angle. The 90° angle is formed between the sides of the triangle representing the horizontal resultant force and the vertical resultant force. There are special relationships among the sides of a right triangle. One of these relates the lengths of the two sides that make the right angle to the length of the side opposite the right angle. If A and B represent the two sides that make up the right angle and C represents the hypotenuse (the side opposite the right angle), then

$$A^2 + B^2 = C^2. \tag{1.5}$$

This relationship is called the Pythagorean theorem. For our force triangle, then, we can substitute 10 N for A and 50 N for B and then solve for C, which represents the resultant force.

$$(10\text{ N})^2 + (50\text{ N})^2 = C^2$$

$$100\text{ N}^2 + 2500\text{ N}^2 = C^2$$

$$2600\text{ N}^2 = C^2$$

$$C = 51\text{ N}$$

This gives us an answer identical to what we got when we actually measured the graphical representation of the force.

Let's take another look at the right triangle we ended up with in figure 1.10. Besides the Pythagorean theorem, there are other relationships between the sides and the angles of a right triangle. If we know the lengths of any two sides of a right triangle, we can determine the length of the other side and the size of the angle between the sides as well. Conversely, if we know the length of one side of a right triangle and the measurement of one of the angles other than the right angle, we can determine the lengths of the other sides and the measurement of the other angle using trigonometry. Trigonometry was not a prerequisite for using this book, and the intent of this book is not to teach you trigonometry, but a knowledge of some of the tools of trigonometry will assist you in the study of biomechanics.

Basically, what trigonometry tells us is that a ratio exists among the lengths of the sides of right triangles that have similar angles. Look at the right triangles in figure 1.11. They are all different sizes, but the angles are all the same, and the sides all change proportionally. If you lengthened one side of any of these triangles, you would have to lengthen the other sides as well to keep the angles of the triangle unchanged. So relationships exist between the lengths of the sides of a right triangle and the angles in a right triangle.

These relationships can be expressed as ratios of one side to another for each size of angle that may exist between two sides of a right triangle. Here are the relationships that may be helpful:

$$\sin\ \theta = \frac{\text{opposite side}}{\text{hypotenuse}} \tag{1.6}$$

$$\cos\ \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \tag{1.7}$$

$$\tan\ \theta = \frac{\text{opposite side}}{\text{adjacent side}} \tag{1.8}$$

In these equations, θ, which is pronounced "theta," represents the angle; opposite refers to the length of the side of the triangle opposite the angle theta; adjacent refers to the length of the side of the triangle adjacent to the angle theta; and hypotenuse refers to the length of the side of the triangle opposite the right angle. The term *sin* refers to the word *sine; cos* refers to the word *cosine;* and *tan* refers to the word *tangent.* Any modern scientific calculator includes the functions for sine, cosine and tangent. The right triangle in figure 1.12 has these three sides labeled for you.

An easy technique for remembering these trigonometric relationships is the following sentence:

Some **O**f **H**is $\qquad \mathbf{s}\text{in}\,\theta = \dfrac{\mathbf{o}\text{pposite side}}{\mathbf{h}\text{ypotenuse}}$

Children **A**re **H**aving $\qquad \mathbf{c}\text{os}\,\theta = \dfrac{\mathbf{a}\text{djacent side}}{\mathbf{h}\text{ypotenuse}}$

Trouble **O**ver **A**lgebra. $\qquad \mathbf{t}\text{an}\,\theta = \dfrac{\mathbf{o}\text{pposite side}}{\mathbf{a}\text{djacent side}}$

The first letter of each of these words matches the first letter in each of the trigonometric variables listed in the equations. You may know of other mnemonic devices for memorizing these relationships.

Equations 1.6, 1.7, and 1.8 can be used to determine the length of an unknown side of a right triangle if the length of another side is known and one of the two angles other than the 90° angle is known. If the angle and the hypotenuse are known, the opposite side could be determined using equation 1.6, and the adjacent side could be determined using equation 1.7.

If the sides of the right triangle are known, then the inverse of the trigonometric function is used to compute the angle:

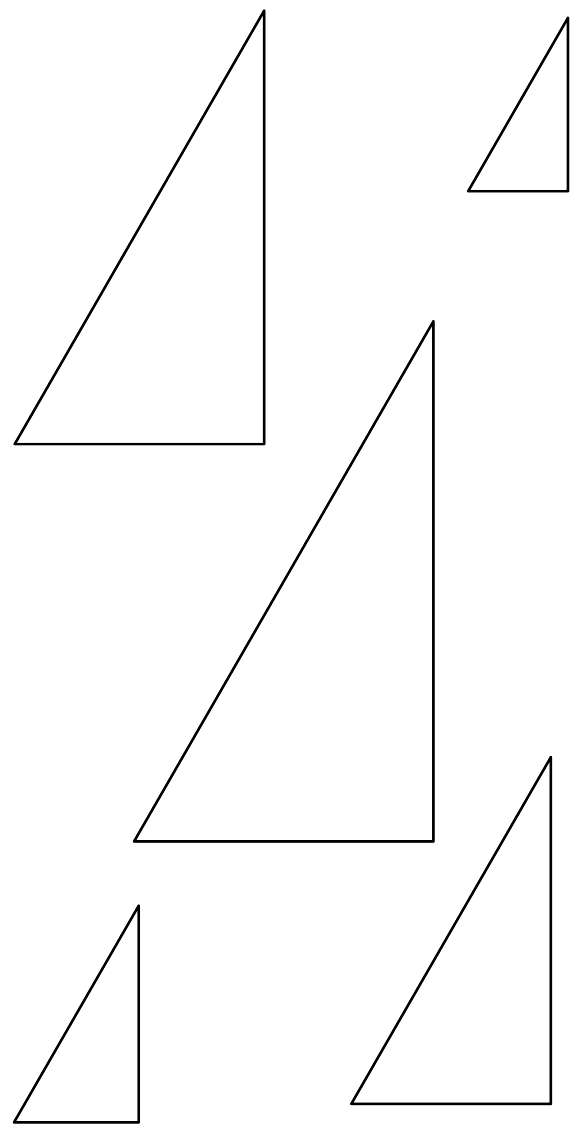


Figure 1.11 Similar right triangles. The triangles are different sizes, but the corresponding angles of each triangle are the same.

$$\theta = \arcsin\left(\frac{\text{opposite side}}{\text{hypotenuse}}\right) \tag{1.9}$$

$$\theta = \arccos\left(\frac{\text{adjacent side}}{\text{hypotenuse}}\right) \tag{1.10}$$

$$\theta = \arctan\left(\frac{\text{opposite side}}{\text{adjacent side}}\right) \tag{1.11}$$

The arcsine, arccosine, and arctangent functions are used to compute one of the angles in a right triangle if the lengths of any two sides are known.

Now let's go back to the resultant forces acting on the gymnast in figure 1.10. We used the Pythagorean theorem

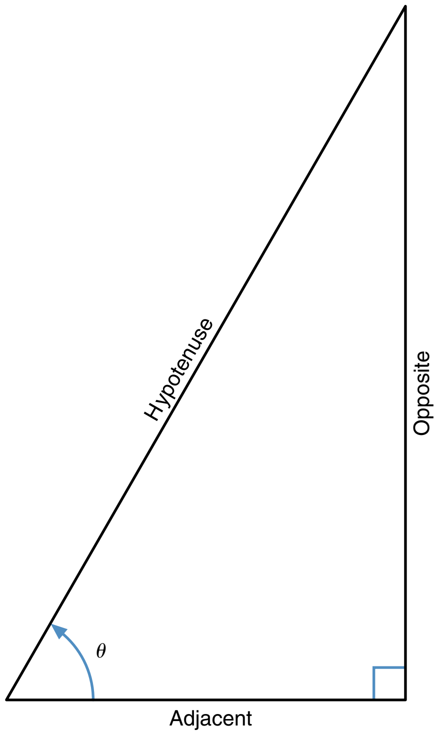


Figure 1.12 Parts of a right triangle.

to compute the size of the resultant force, 51 N. But in what direction is it acting? Let's determine the angle between the 51 N resultant force (the hypotenuse of the triangle) and the 10 N horizontal force (the adjacent side). The 50 N vertical force is the side opposite the angle. Using equation 1.11 gives the following:

$$\theta = \arctan\left(\frac{\text{opposite side}}{\text{adjacent side}}\right)$$

$$\theta = \arctan\left(\frac{50\text{ N}}{10\text{ N}}\right) = \arctan(5)$$

To determine the angle θ, we use inverse of the tangent function or the arctangent. On most scientific calculators, the arctangent function is the second function for the tangent key and is usually abbreviated as $\tan^{-1}$ or atan. Using a calculator (make sure its angle measure is programmed for degrees rather than radians), we find that

$$\theta = \arctan(5) = 78.7°.$$

The angles in a triangle add up to 180°. In a right triangle, one angle is 90°, so the sum of the other two angles is 90°. The other angle in this case is thus 11.3° (i.e., 90° − 78.7°). This is pretty close to the value we arrived at earlier using the graphical method when we measured the angle directly with a protractor.

If forces are concurrent but not colinear, we can add the forces to determine their resultant by graphically representing the forces as arrows and arranging them tip to tail. The resultant force will be represented by an arrow drawn from the tail of the first force to the tip of the last force represented. Alternatively, if the forces are directed only horizontally or vertically, we can algebraically add up all the horizontal forces to determine the resultant horizontal force, then add up all the vertical forces and determine the resultant vertical force. The size of the resultant of these two forces can be determined using the Pythagorean theorem, and its direction can be determined using trigonometry.

Resolution of Forces

What if the external forces acting on the object are not colinear and do not act in a vertical or horizontal direction? Look back at figure 1.1 and consider the forces acting on a shot during the putting action. Imagine that at the instant shown, the athlete exerts a 100 N force on the shot at an angle of 60° above horizontal. The mass of the shot is 4 kg. What is the net force acting on the shot? First, we need to determine the weight of the shot. Using the rough approximation for g, the shot weighs

$$W = mg = (4\text{ kg})(10\text{ m/s}^2) = 40\text{ N}.$$

Now we can determine the net external force by graphically adding the 40 N weight of the shot to the 100 N force exerted by the athlete. Try doing this. Your graphic solution should be similar to figure 1.13. If we measure the resultant force, it appears to be about 68 N. It acts upward and to the right at an angle a little less than 45°.

Is there another method we could use to determine this resultant, as we did with the gymnast problem earlier? Recall that the external forces acting on the gymnast were all horizontal or vertical forces. In that problem, we could just sum the horizontal forces and the vertical forces algebraically to find the resultant horizontal and vertical forces. In the shot-putting problem, we have one vertical force, the shot's weight, but the force from the athlete is acting both horizontally and vertically. It is pushing upward and forward on the shot. Because this 100 N force acts to push the shot both horizontally and vertically, perhaps it can be represented by two different forces: a horizontal force and a vertical force.

Graphical Technique

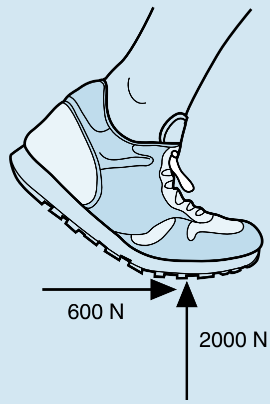
Let's start by looking at the problem graphically. We want to represent the 100 N force that acts forward and upward at 60° above horizontal as a pair of forces. The pair of forces we are trying to find are called the horizontal and vertical components of this 100 N force. You are probably familiar with the word *component.* Components are the parts that make up a system. The horizontal and vertical force components of the 100 N force are the parts that

SAMPLE PROBLEM 1.2

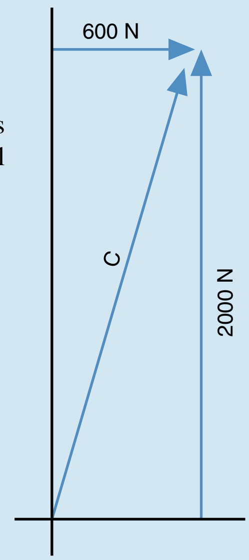
The vertical ground reaction force (normal contact force) acting under a runner's foot is 2000 N, while the frictional force is 600 N acting forward. What is the resultant of these two forces?

Solution:

Step 1: Draw the forces.



Step 2: Draw the resultant force. Let the two known forces represent two sides of a box. Draw the other two sides of the box. The resultant force is the diagonal of this box, with one end at the point of application of the other two forces.



Step 3: Use the Pythagorean theorem (equation 1.5) to compute the size of the resultant force:

$$A^2 + B^2 = C^2$$

$$(2000\ \text{N})^2 + (600\ \text{N})^2 = C^2$$

$$4{,}000{,}000\ \text{N}^2 + 360{,}000\ \text{N}^2 = C^2$$

$$4{,}360{,}000\ \text{N}^2 = C^2$$

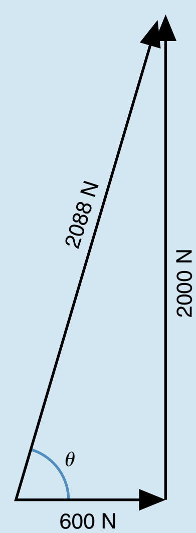
$$2088\ \text{N} = C$$

Step 4: Use the arctangent function (equation 1.11) to determine the angle of the resultant force with horizontal:

$$\theta = \arctan\left(\frac{\text{opposite side}}{\text{adjacent side}}\right)$$

$$\theta = \arctan\left(\frac{2000\ \text{N}}{600\ \text{N}}\right) = \arctan$$

$$\theta = 73.3°$$



make up or have the same effect as the 100 N force. We can think of the 100 N force as the resultant of adding the horizontal and vertical components of this force. Let's draw the 100 N force as a vector, as shown in figure 1.14*a*.

Think about how we graphically determined the resultant of two forces—we lined up the arrows representing these forces end to end and then drew an arrow from the tail of the first force arrow to the tip of the last force arrow in the sequence. This last force arrow we drew was the resultant. Now we want to work that process in reverse. We know what the resultant force is, but we want to know what horizontal and vertical forces can be added together to produce this resultant.

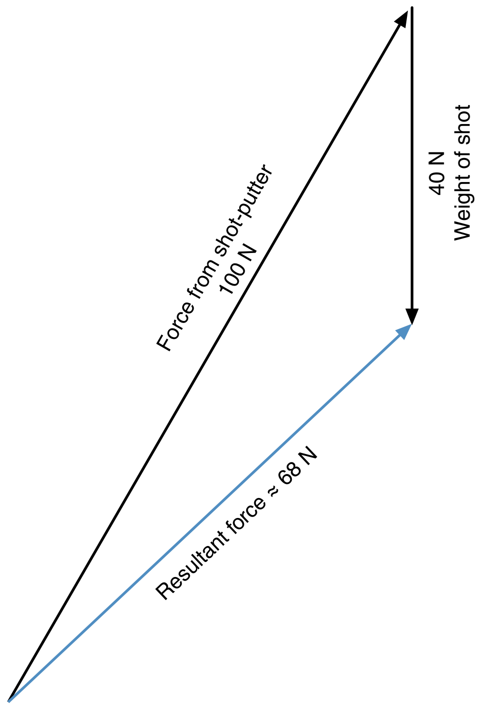


Figure 1.13 Graphic determination of resultant force acting on the shot.

Draw a box around the 100 N force so that the sides of the box align vertically or horizontally and so that the 100 N force runs diagonally through the box from corner to corner (see figure 1.14*b*). Notice that the box is actually two triangles with the 100 N force as the common side. In each triangle, the other two sides represent the horizontal and vertical components of the 100 N force. In the upper triangle, the 100 N resultant force is the outcome when we start with a vertical force and add a horizontal force to it. The tail of the vertical force is the point of application of forces, and we add the horizontal force to it by aligning the tail of the horizontal force to the tip of the vertical force. In the lower triangle, the 100 N force is the outcome when we start with a horizontal force and add a vertical force to it. The tail of the horizontal force is the point of application of the forces, and we add the vertical force to it by aligning the tail of the vertical force to the tip of the horizontal force. The triangles are identical, so we can use either one. Let's choose the lower triangle. Put arrowheads on the horizontal and vertical force components in this triangle (see figure 1.14*c*). Now measure the lengths of these force vectors. The horizontal force component is about 50 N, and the vertical force component is about 87 N.

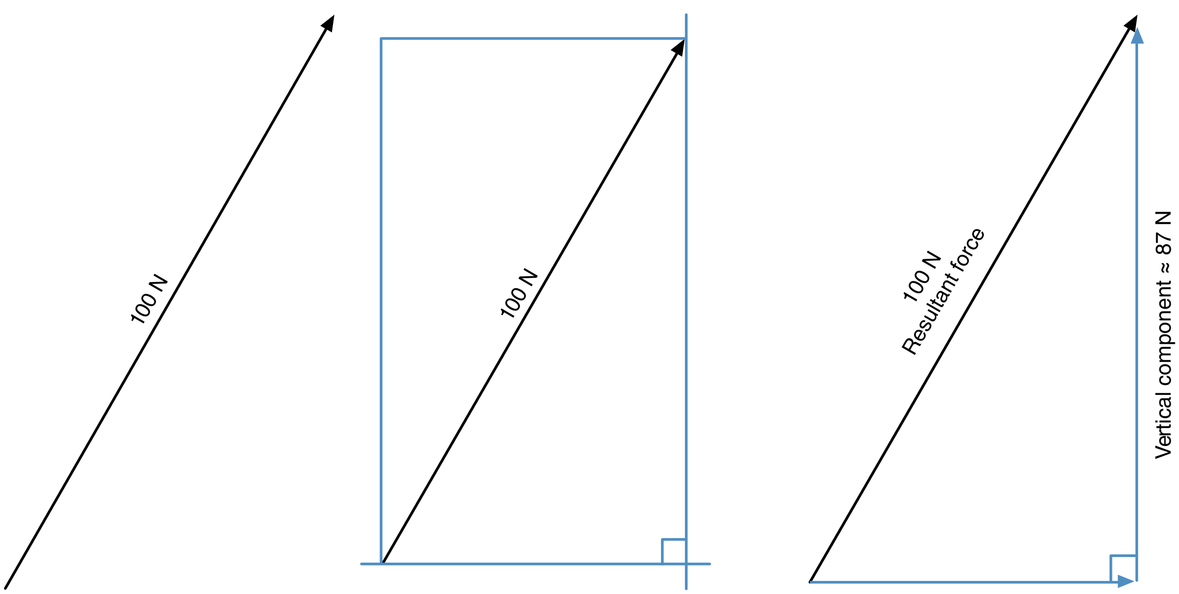


Figure 1.14 Resolution of force exerted by shot-putter. *(a)* Resultant force. *(b)* Construction of force triangle. *(c)* Resolution into component forces.

Since we are working with a right triangle, the Pythagorean theorem (equation 1.5) must apply.

$$A^2 + B^2 = C^2$$

For our force triangle, then, we can substitute 50 N for *A,* 87 N for *B,* and 100 N for *C.* Let's check to see if it works.

$$(50\text{ N})^2 + (87\text{ N})^2 = (100\text{ N})^2$$

$$2500\text{ N}^2 + 7569\text{ N}^2 = 10{,}000\text{ N}^2$$

$$10{,}069\text{ N}^2 \approx 10{,}000\text{ N}^2$$

Although 10,069 doesn't exactly equal 10,000, the difference is less than 1%. That's pretty close, especially considering our accuracy in measuring the length of the force arrows. If the measurement accuracy were increased, the difference between the two numbers would become closer to zero.

To complete the original problem, we would include the 40 N weight of the shot as a downward force. This would be subtracted algebraically from the 87 N upward component of the force exerted by the athlete. The resulting vertical force acting on the shot would be

$$(-40\text{ N}) + 87\text{ N} = +47\text{ N}.$$

A 47 N force acts upward on the shot. We still have the 50 N horizontal component of the force exerted by the athlete. If we add this to the 47 N vertical force, using the Pythagorean theorem (equation 1.5), we get

$$A^2 + B^2 = C^2$$

$$(50\text{ N})^2 + (47\text{ N})^2 = C^2$$

$$2500\text{ N}^2 + 2209\text{ N}^2 = C^2$$

$$4709\text{ N}^2 = C^2$$

$$C = 68.6\text{ N}$$

This is close to the answer we got when we used the graphical technique to solve for the resultant force acting on the shot in figure 1.13. In this problem, we actually resolved a force into components, added these components to other forces along the same lines, and then added the resultant component forces back together to find the net resultant force.

The process of determining what two force components add together to make a resultant force is called force resolution. We resolved a force into its components. The word *resolve* sounds like *re-solve,* which is what we did. We had the resultant force, and we solved the problem backward—we *re-solved* it—to determine the forces that added together to yield this resultant. But it was all done graphically. We want a nongraphical technique for doing this.

Trigonometric Technique

The force triangle we ended up with in figure 1.14*c* is a right triangle. Besides the Pythagorean theorem, there are other relationships between the sides and the angles of a right triangle. Some of these relationships can be described by the sine, cosine, and tangent functions, which were defined by equations 1.6, 1.7, and 1.8. Let's see if we can use any of these relationships to resolve the 100 N force that the shot-putter exerts on the shot into horizontal and vertical components.

First, draw the 100 N force as an arrow acting upward and to the right 60° above horizontal, as we did in figure 1.14*a.* Now, just as we did in figure 1.14*b,* draw a box around this force so that the sides of the box are horizontal or vertical and the 100 N force runs diagonally through the box from corner to corner. Let's consider the lower of the two triangles formed by the box and the 100 N diagonal of the box (see figure 1.15). The 100 N force is the hypotenuse of this right triangle. The horizontal side of the triangle is the side adjacent to the 60° angle. The length of this side can be found using the cosine function defined by equation 1.7:

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\cos 60^\circ = \frac{\text{adjacent side}}{100\text{ N}}$$

$$\begin{aligned}(100\text{ N})\cos 60^\circ &= \text{adjacent side}\\ &= \text{horizontal force component}\end{aligned}$$

Using a scientific calculator (make sure its angle measure is programmed for degrees rather than radians), we find that the cosine of 60° is 0.500. Substitute this number for cos 60° in the previous equation:

$$\begin{aligned}(100\text{ N})\cos 60^\circ &= (100\text{ N})(0.500)\\ &= \text{adjacent side} = 50\text{ N}\end{aligned}$$

The horizontal component of the 100 N force is 50 N. Now find the vertical component of the 100 N force. The side of the triangle opposite the 60° angle represents the vertical component of the 100 N force. We can find the length of this side by using the sine function defined by equation 1.6:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin 60^\circ \frac{\text{opposite side}}{100\text{ N}}$$

$$\begin{aligned}(100\text{ N})\sin 60^\circ &= \text{opposite side}\\ &= \text{vertical force component}\end{aligned}$$