SOLUTION OF ECONOMIC EQUATIONS

1. DEMAND'S FUNCTIONAL EQUATION

The law of demand states that the quantity demanded depends upon price, other things remaining the same. This is shown as: $Qd = f(P, Y, Ps, T)$

where Qd = quantity demanded, P = price, Y = income of consumers, T = tastes of consumers. Such all, i.e., Y, Ps and T are accorded as constant or parameters of the law of demand. Accordingly, they are held constant while describing the law of demand. Thus the general demand function in the presence of parameters will be as:

$$Qd = f(P, \overline{Y}, \overline{Ps}, \overline{T})$$

If we exclude these parameters, the demand function in its general form will be as:

$$Qd = f(P)$$

As we know there exists an inverse relationship between P and Q, i.e., when price rises demand contracts ; when price falls demand expands.

Thus in the presence of all the parameters and inverse behaviour between P and Q, we present standard demand function as : $Q = a - bP$

where a and b represent parameters (the variables which have been held constant), the negative sign shows the inverse relationship between P and Q.

DERIVATION OF DEMAND'S FUNCTION

Normally the demand curve is straight line and the slope of a straight line is same in between all the points. On the basis of such fact we derive demand equation

As there are two points on a demand curve (P_1, Q_1), (P_2, Q_2). Then slope of demand function will be : $\frac{Q_2 - Q_1}{P_2 - P_1}$. Again, if we take a general point on the demand curve like (P, Q), then with this point and (P_1, Q_1) we can get the slope of the same demand curve as $\frac{Q - Q_1}{P - P_1}$. As the slope at all points of the demand curve is the same, we write

$$\frac{Q - Q_1}{P - P_1} = \frac{Q_2 - Q_1}{P_2 - P_1}$$

Cross multiplying and solving for $Q - Q_1$, we get

$$(Q - Q_1)(P_2 - P_1) = (P - P_1)(Q_2 - Q_1) \Rightarrow Q - Q_1 = \frac{Q_2 - Q_1}{P_2 - P_1}(P - P_1)$$

Thus in the form of last equation we get the formula of derivation of demand functional equation.

EXAMPLE – 1. If $P_1 = 80$, $P_2 = 60$, $Q_1 = 10$, $Q_2 = 20$, then we derive a specific demand functional equation by putting these values in the formula. (GCUF:2013/I)

$$Q - Q_1 = \frac{Q_2 - Q_1}{P_2 - P_1}(P - P_1)$$

$$Q - 10 = \frac{20 - 10}{60 - 80}(P - 80)$$

$$Q - 10 = \frac{10}{-20}(P - 80)$$

$$Q - 10 = -\frac{1}{2}(P - 80)$$

$$Q - 10 = -\frac{1}{2}P + 40$$

$$Q = -\frac{1}{2}P + 40 + 10$$

$$Q = -\frac{1}{2}P + 50$$

$$Q = 50 - \frac{1}{2}P$$

Thus this is the required specific demand equation.

EXAMPLE – 2. A consumer purchases 20 pens at the price of Rs.5/- per pen while the demand for pens decreased to 15 when price is Rs.10 per pen. We derive the demand equation and draw its curve.

Here $P_1 = 5$, $Q_1 = 20$, $P_2 = 10$, $Q_2 = 15$, then we derive a specific demand functional equation by putting these values in the formula

$$Q - Q_1 = \frac{Q_2 - Q_1}{P_2 - P_1}(P - P_1)$$

$$Q - 20 = \frac{15 - 20}{10 - 5}(P - 5)$$

$$Q - 20 = \frac{-5}{5}(P - 5)$$

$$Q - 20 = -1(P - 5)$$

$$Q - 20 = -P + 5$$

$$Q = -P + 5 + 20$$

$$Q = -P + 25$$

$$Q = 25 - P$$

Thus this is the required specific demand equation.

To draw the diagram of this demand equation, we need two points on it.

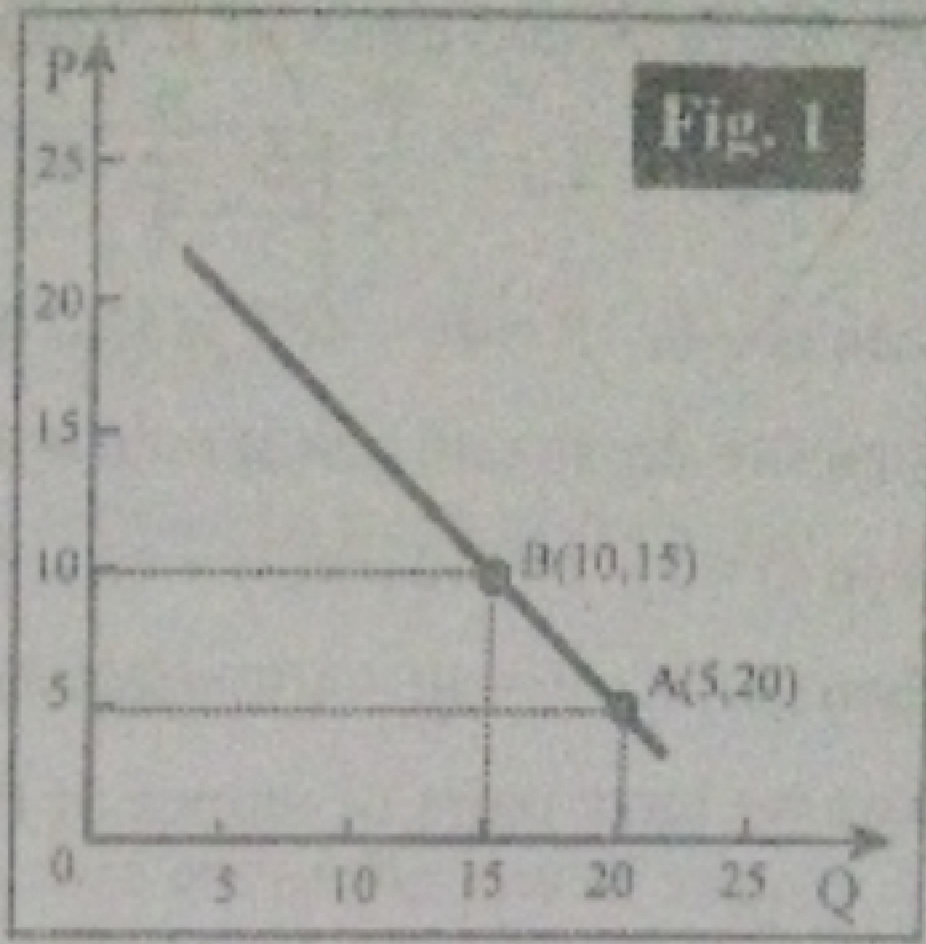
Taking P as 5, 10When $P = 5\ (= P_1)$, then

$$Q = 25 - 5 = 20\ (= Q_1)$$

When $P = 10\ (= P_2)$,

$$Q = 25 - 10 = 15\ (= Q_2)$$

Thus we get two pairs of values of P and Q, i.e. A (5, 20), B(10, 15). By joining these points we get the demand curve which falls negatively.



SOLUTION OF DEMAND EQUATIONS

EXAMPLE–1. If $3Qd + 4P = 240$, find values of Q if the values of P are as $P = 12, 9, 0$.

Solution. The restriction over P and Qd is as : P, Qd ≥ 0.

Now $3Qd + 4P = 240$

$$3Qd = 240 - 4P$$

Putting the values of P, we get

When $P = 12$, $3Qd = 240 - 4(12)$

$$3Qd = 240 - 48 = 192$$

$$Qd = \frac{192}{3} = 64$$

When $P = 9$, $3Qd = 240 - 4(9)$

$$3Qd = 240 - 36 = 204$$

$$Qd = \frac{204}{3} = 68$$

When $P = 0$, $3Qd = 240 - 4(0)$

$$3Qd = 240 - 0 = 240$$

$$Qd = \frac{240}{3} = 80$$

Thus we have pairs of values of P and Q as

P	12	9	0
Q	64	68	80

With these pairs of values of P and Q we can construct the demand curve.

EXAMPLE–2. If $Qd = 10 - \frac{P}{4}$, find values of Q if the values of P are as $P = 0, 4, 16, 25$.

Solution. Now Qd $Qd = 10 - \frac{P}{4}$

When $P = 0$, $Qd = 10 - \frac{0}{4}$

$$Qd = 10 - 0 = 10$$

When $P = 4$, $Qd = 10 - \frac{4}{4}$

$$Qd = 10 - 1 = 9$$

When $P = 16$, $Qd = 10 - \frac{16}{4}$

$$Qd = 10 - 4 = 6$$

When $P = 25$, $Qd = 10 - \frac{25}{4}$

$$Qd = 10 - 6.25 = 3.75$$

Thus we have pairs of values of P and Q as

P	0	4	16	25
Q	10	9	6	3.75

Now we find the values of P while the values of Q are as Q = 0,2,7, 9.

$$Qd = 10 - \frac{P}{4} \Rightarrow \frac{P}{4} = Q - 10 \Rightarrow 4\left(\frac{P}{4}\right) = 4(Q-10) \Rightarrow P = 40 - 4Q$$

When Q = 0, P = 40 − 4(0) = 40 − 0 = 40

When Q = 2, P = 40 − 4(2) = 40 − 8 = 32

When Q = 7, P = 40 − 4(7) = 40 − 28 = 12

When Q = 9, P = 40 − 4(9) = 40 − 36 = 4

Thus we have pairs of values of P and Q as

> Q. From the following demand function Qd=400-8P (i) Calculate Qd when price is Rs 10/: (ii) Calculate Qd when P is increased by 30% (iii) Calculate Qd when price falls 20%. (iv) find the maximum price that can be paid for the product (v) find the demand for product when it is free.(UOH:2009). Its solution is in next box below

Thus we have pairs of values of P and Q as:

Q	0	2	7	9
P	40	32	12	4

EXAMPLE–3. If $Q + \frac{1}{2}P - 50 = 0$, find values of Q while the values of P are as P = 10, 20, 30, 40. Writing given equation in standard form: $Q = 50 - \frac{1}{2}P$

Putting the values of P, we get

When P = 10, $Q = 50 - \frac{1}{2}(10) = 50 - 5 = 45$

When P = 20, $Q = 50 - \frac{1}{2}(20) = 50 - 10 = 40$

When P = 30, $Q = 50 - \frac{1}{2}(30) = 50 - 15 = 35$

When P = 40, $Q = 50 - \frac{1}{2}(40) = 50 - 20 = 30$

Thus we have pairs of values of P and Q as

P	10	20	30	40
Q	45	40	35	30

EXAMPLE–4. If PQ = 1200, find values of Q while the values of P are as" P = 100, 200, 300, 400.

Solution. Writing given equation in standard form: $Q = \frac{1200}{P}$

Putting the values of P, we get

When P = 100, $Q = \frac{1200}{100} = 12$, When P = 200, $Q = \frac{1200}{200} = 6$

When P = 300, $Q = \frac{1200}{300} = 4$, When P = 400, $Q = \frac{1200}{400} = 3$

> (i) Qd=400-8(10)=320
> (ii)Qd when **P ↑ 30%**
> $P = \frac{30}{100} \times 10 + 10$ =3+10=13. Qd = 400−8 (13) =296. It shows when P rises to 13, Qd falls to 296. (iii) Qd when $P = -20/100 \times 10 =$ −2+10=8 Qd=400-8(8) = 336. It shows when P falls to 8, Qd rises to 336.
> (iv) Qd = 400-8P. keeping Qd=0 as at maximum price on demand curve the Qd = 0. 0=400-8P ⇒ 8P=400 ⇒ P=50 the maximum price paid for the product is 50
> (v) If P = 0 ⇒ Qd=400−8(0) = 400, thus when P = 0, Qd = 400

Thus we have pairs of values of P and Q as

P	100	200	300	400
Q	12	6	4	3

2. SUPPLY'S FUNCTIONAL EQUATION (GCUF:2013/I)

According to the law of supply " Other things remaining the same, the quantity supplied varies directly with change in price ". It is written in general form as :

$$Qs = f(P, W, C, Tech.)$$

where Qs = quantity supplied, P = price W = Weather,

C = costs , Tech.= technology.

The W, C and Tech. are known as constants and parameters of the law of supply. Such all change but they are held constant. Thus in the presence of such parameters, the supply function is as : $Qs = f\left(P, \overline{W}, \overline{C}, \overline{Tech}\right)$

If we exclude these parameters, the supply function in its general form will be as :

$$Qs = f(P)$$

As we know there exists a direct relationship between P and Qs, i.e., when price rises supply also rises and vice versa. Thus in the presence of all the parameters and positive relation between P and Q we present a standard supply function as:

$$Q = a + bP$$

where a and b represent parameters (the variables which are held constant), Q and P are variables and the positive sign shows positive relationship between P and Q.

DERIVATION OF SUPPLY'S FUNCTION

The formal derivation of supply's functional equation will be same as that of demand's functional equation —as we did earlier. Thus the formula to derive supply's functional equation will be as :

$$\boxed{Q - Q_1 = \frac{Q_2 - Q_1}{P_2 - P_1}(P - P_1)}$$

EXAMPLE – 1. If $P_1 = 60$. $P_2 = 80$, $Q_1 = 10$, $Q_2 = 20$, then we derive a specific supply functional equation by putting these values in the formula

$$Q - Q_1 = \frac{Q_2 - Q_1}{P_2 - P_1}(P - P_1)$$

$$Q - 10 = \frac{20 - 10}{80 - 60}(P - 60)$$

$$Q - 10 = \frac{10}{20}(P - 60)$$

$$Q - 10 = \frac{1}{2}(P - 60)$$

$$Q - 10 = \frac{1}{2}P - 30$$

$$Q = \frac{1}{2}P - 30 + 10$$

$$Q = \frac{1}{2}P - 20$$

$$Q = -20 + \frac{1}{2}P$$

Thus this is the required specific supply equation.

EXAMPLE – 2. A firm sells 5000 pens at the price of Rs.3.50 per pen while it sells 2000 pens at the price of Rs.5/- per pen. Derive the supply's functional equation and graph it.

Here $P_1 = 3.50$, $Q_1 = 5000$, $P_2 = 5$, $Q_2 = 2000$. Putting these values in the formula $Q - Q_1 = \frac{Q_2 - Q_1}{P_2 - P_1}(P - P_1)$

$$Q - 5000 = \frac{2000 - 5000}{3.50 - 5}(P - 5)$$

$$Q - 5000 = \frac{-3000}{-1.50}(P - 5)$$

$$Q - 5000 = 2000(P - 5)$$

$$Q - 5000 = 2000P - 10000$$

$$Q = 2000P - 10000$$

$$Q = 2000P - 10000 + 5000$$

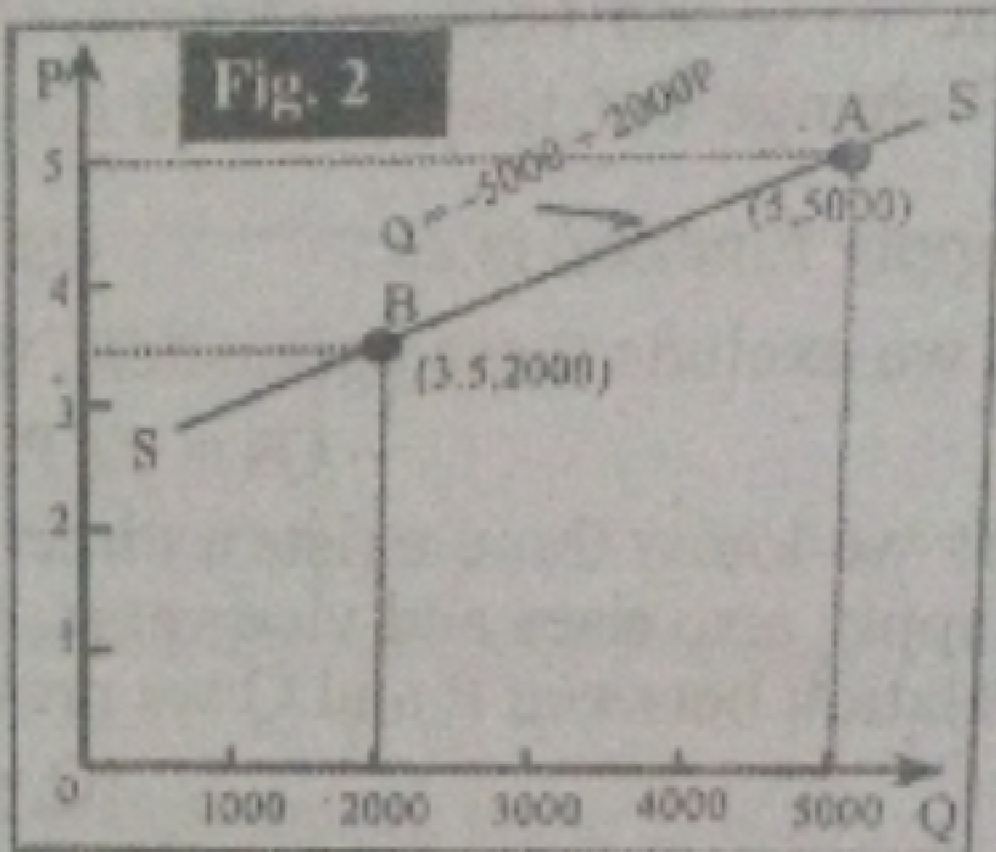
$$Q = 2000P - 5000$$

$$Q = -5000 + 2000P$$

Thus this is the required specific supply equation.

To draw the graph of this functional equation we have already gotten two pairs of values of P and Q, i.e. A(5, 5000), B(3.50, 2000) Joining these points we get the required graph which is a positively sloped curve as shown in Fig. 2.

Fig. 2



SOLUTION OF SUPPLY EQUATIONS

EXAMPLE – 1. If $Q = 4 + 2P$, $P = 2, 4, 6$, we find quantities of supply (Q) for these values of P.

When $P = 2$, $Q = 4 + 2(2) = 4 + 4 = 8$

When $P = 4$, $Q = 4 + 2(4) = 4 + 8 = 12$

When $P = 6$, $Q = 4 + 2(6) = 4 + 12 = 16$

P	2	4	6
Q	8	12	16

With these pairs of values of P and Q we can construct the supply curve which will slope upward.

EXAMPLE – 2. If $Q + 20 - 4P = 0$. Find the values of Q given $P = 5, 10, 15$.

Converting in standard form: $Q = 4P - 20$. Putting the values of P, we get

When $P = 5$, $Q = 4(5) - 20 = 20 - 20 = 0$

When $P = 10$, $Q = 4(10) - 20 = 40 - 20 = 20$

When $P = 15$, $Q = 4(15) - 20 = 60 - 20 = 40$

P	5	10	15
Q	0	20	40

With these pairs of values of P and Q we can construct the supply curve which slopes upward.

EXAMPLE. A firm has to face Rs.350/- as costs while producing 10 tables daily, while it has to face Rs.600/- as costs while producing 20 tables, construct the linear cost function and graph it.

Solution. The general formula to derive a linear function is: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

where y is the dependent and x is the independent variable. In a cost function, costs (C) are dependent variable while output (Q) is the independent variable. Accordingly, the formula to derive the required linear cost function will be as :

$$C - C_1 = \frac{C_2 - C_1}{Q_2 - Q_1}(Q - Q_1)$$

Q	10	20
C	350	600

$Q_1 = 10, Q_2 = 20, C_1 = 350, C_2 = 600,$

$$C - 350 = \frac{600 - 350}{20 - 10}(Q - 10)$$

$$C - 350 = \frac{250}{10}(Q - 10)$$

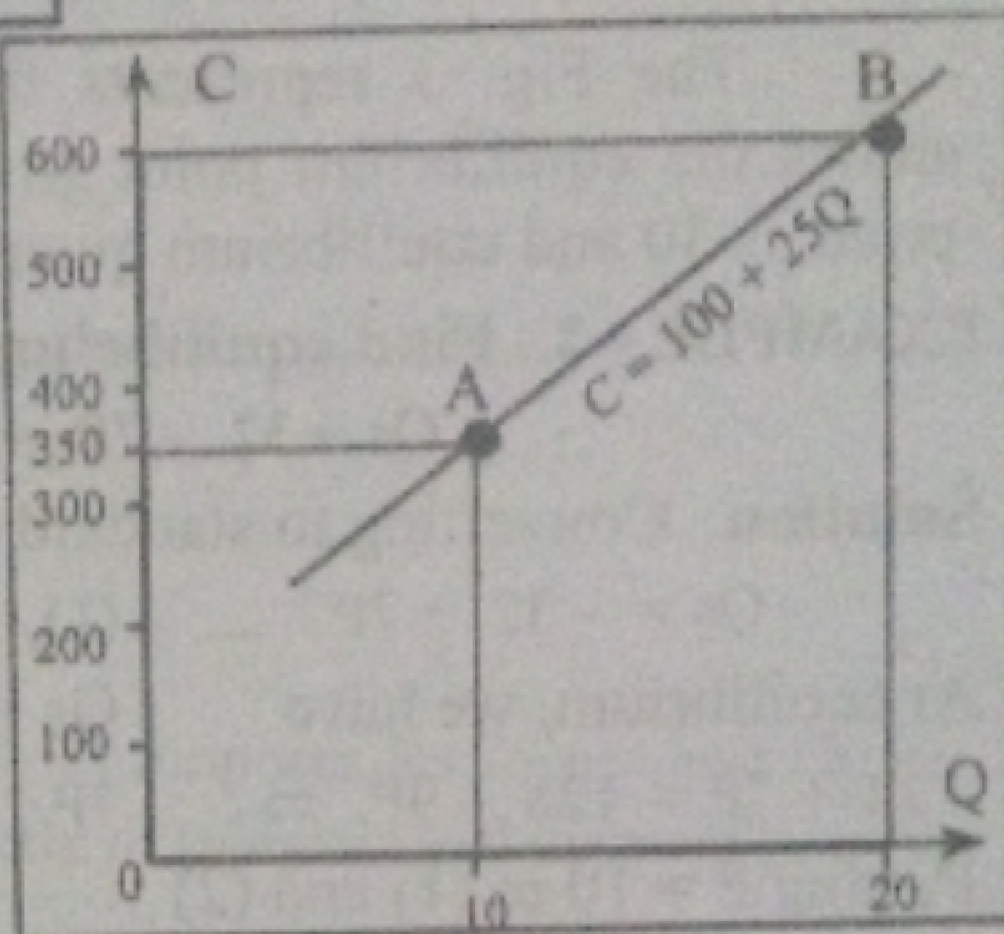
$$C - 350 = 25(Q - 10)$$

$$C - 350 = 25Q - 250$$

$$C = 25Q - 250 + 350$$

$$C = 100 + 25Q$$

is the required cost function.



Putting the values of Q_1 and Q_2 to check, we get

$$C = 100 + 25Q = 100 + 25(10) = 350$$

$$C = 100 + 25Q = 100 + 25(20) = 600$$

Thus graph is also given as shown in the diagram.

DEMAND SUPPLY EQUILIBRIUM (Linear Equations)

—— PARTIAL EQUILIBRIUM ANALYSIS

The price where quantity demanded is equal to quantity supplied is called **Equilibrium Price** while the quantity where demand is equal to supply is called **Equilibrium Quantity**. The point where demand is equal to to supply is called **Equilibrium Point**. This is Partial Equilibrium Analysis where we assumed that prices of other goods remain the same.

Now with the help of demand and supply equations, which are linear, we find equilibrium price and equilibrium quantity.

EXAMPLE – 1. Find equilibrium price and quantity and plot them, given:

$$Qs = -20 + 3P, \quad Qd = 220 - 5P.$$

Solution.

$$Qs = -20 + 3P \quad ... \quad (1)$$

$$Qd = 220 - 5P \quad ... \quad (2)$$

Removing s and d

$$Q = -20 + 3P \quad ... \quad (1)$$

$$\pm Q = \pm 220 \mp 5P \quad ... \quad (2)$$

Subtracting: $0 = -240 + 8P \Rightarrow -8P = -240 \Rightarrow \quad P = \frac{240}{8} = 30$

Putting P = 30 in (1) and (2)

$$Qs = -20 + 3(30) = -20 + 90 = 70$$
$$Qd = 220 - 5(30) = 220 - 150 = 70$$

Thus equilibrium price $= \overline{P} = 30$,

equilibrium quantity $= \overline{Q} = 70$.

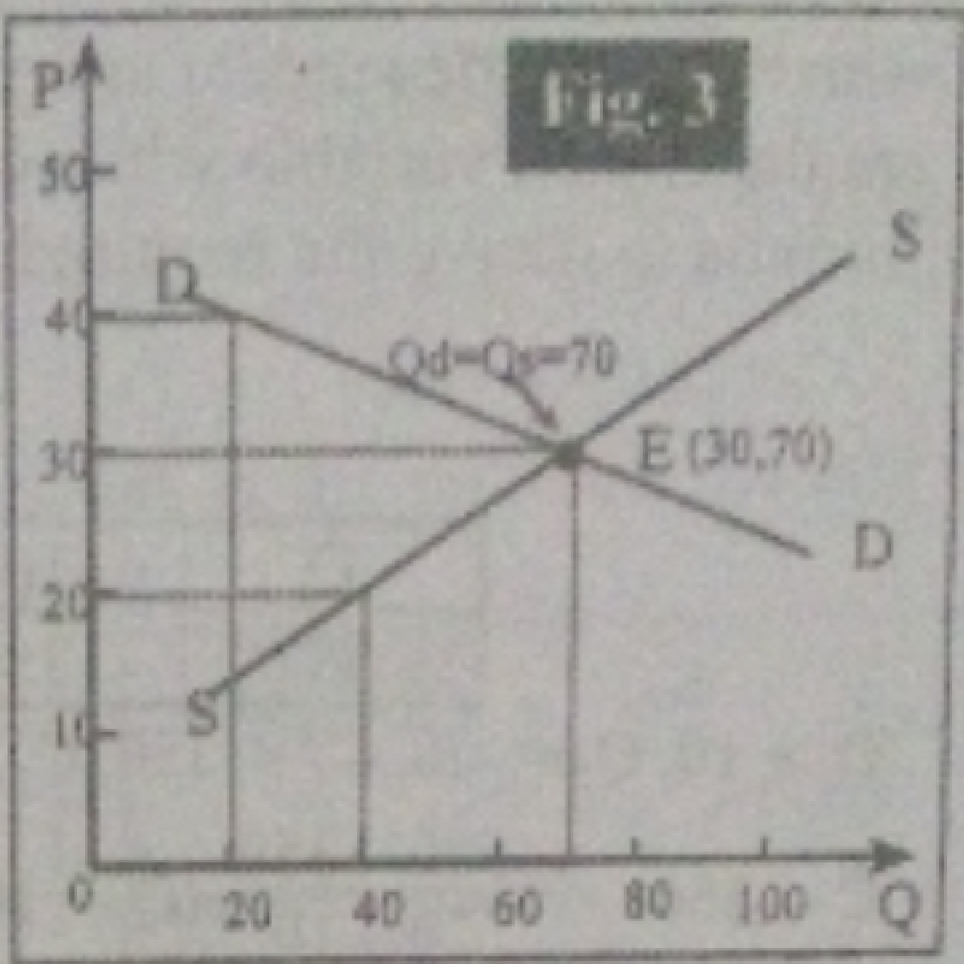


Fig. 3

The Fig. 3 represents that the point E shows the equilibrium point where equilibrium price is 30 and equilibrium quantity is 70.

EXAMPLE – 2. Find equilibrium price and quantity, given :

$$Qs + 32 - 7P = 0, \quad Qd - 128 + 9P = 0.$$

Solution. Converting to standard forms the equations are

$Qs = -32 + 7P \quad ... \quad (1), \qquad Qd = 128 - 9P \quad ... \quad (2)$

At equilibrium, we have $\quad Qs = Qd$

$-32 + 7P = 128 - 9P \Rightarrow \quad 7P + 9P = 128 + 32 \quad \Rightarrow \quad 16P = 160 \Rightarrow P = 10$

Putting P = 10 in (1) and (2)

$$Qs = -32 + 7(10) = -32 + 70 = 38$$
$$Qd = 128 - 9(10) = 128 - 90 = 38$$

Thus equilibrium price $= \overline{P} = 10$ equilibrium quantity $= \overline{Q} = 38$.

EXAMPLE – 3. Find equilibrium price and quantity, given :

$$Qd = 51 - 3P, \quad Qs = -10 + 6P.$$

Solution. Converting to standard forms the equations are

$Qd = 51 - 3P \quad ... \quad (1), \qquad Qs = -10 + 6P \quad ... \quad (2)$

At equilibrium, we have $\quad Qd = Qs$

$51 - 3P = -10 + 6P \quad \Rightarrow -3P - 6P = -10 - 51 \quad \Rightarrow \quad -9P = -61 \Rightarrow P = \frac{61}{9}$

Putting $P = \frac{61}{9}$ in (1) and (2)

$$Qs = 51 - 3\left(\frac{61}{9}\right) = 51 - \frac{61}{3} = \frac{92}{3} = 30.7$$
$$Qd = -10 + 6\left(\frac{61}{9}\right) = -10 + \frac{122}{3} = \frac{92}{3} = 30.7$$

Thus equilibrium price $= \overline{P} = 6.8$, equilibrium quantity $= \overline{Q} = 30.7$.

EXAMPLE – 4. Find equilibrium price and quantity, given :

$$Qd = 50 - \left(\frac{8}{7}\right)P, \quad Qs = 10 + \left(\frac{2}{3}\right)P.$$

Solution. $Qd = 50 - (\frac{8}{7})P$ ... (1), $Qs = 10 + (\frac{2}{3})P$ ... (2)

Removing s and d $Q = 50 - (\frac{8}{7})P$ ... (1)

Substracting: $\underline{\pm Q = \pm 10 \pm (\frac{2}{3})P}$ ... (2)

> Find equilibrium price and quantity $Qd = 30 - P$
> $Qs = 6+5P$ (UOPR:2013)

Subtracting: $0 = 40 - \frac{38}{21}P \Rightarrow \frac{38}{21}P = 40 \Rightarrow P = \frac{40 \times 21}{38} = \frac{840}{38} = 22.10$

Putting $P = 22.10$ in (1) and (2)

$$Qs = 50 - \frac{8}{7}(22.10) = 50 - 25.28 = 24.72$$

$$Qd = 10 + \frac{2}{3}(22.10) = 10 + 14.72 = 24.72$$

Thus equilibrium price $= \overline{P} = 22.10$, equilibrium quantity $= \overline{Q} = 24.72$.

Example 5: Given the following Demand and Supply functions (UOP:2014)

$Qd = 196 - 5P$, $Qs = 20 + 3P$

(a) Find equilibrium Price and equilibrium QTY? (5)

(b) Find new equilibrium Price and new QTY if govt. imposes Rs.8/- per unit Excise duty on the commodity. Also prove with the help of schedule and diagram? (15)

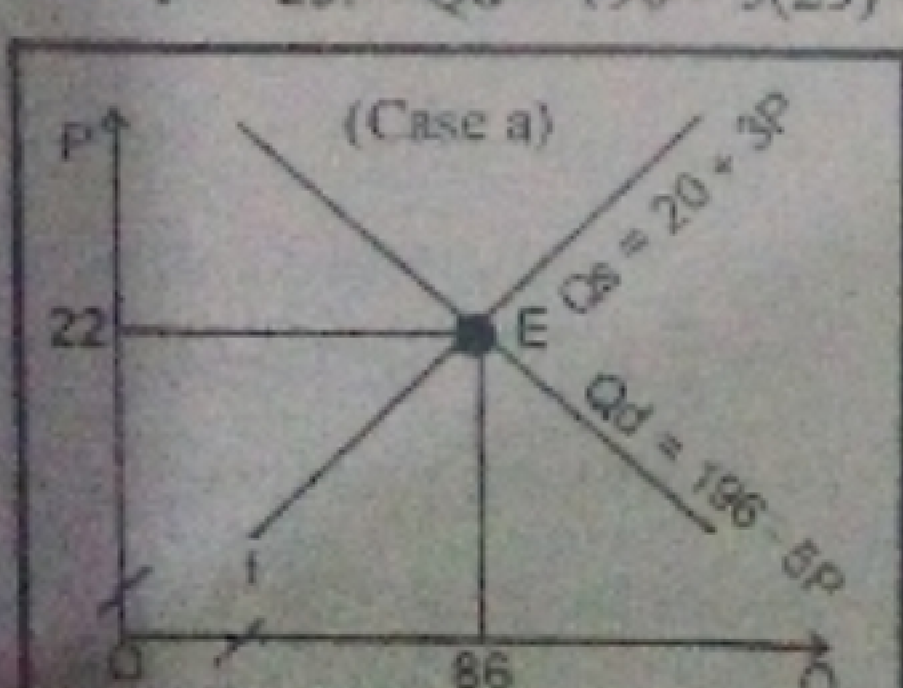
$Qd = Qs \Rightarrow 196 - 5P = 20 + 3P$

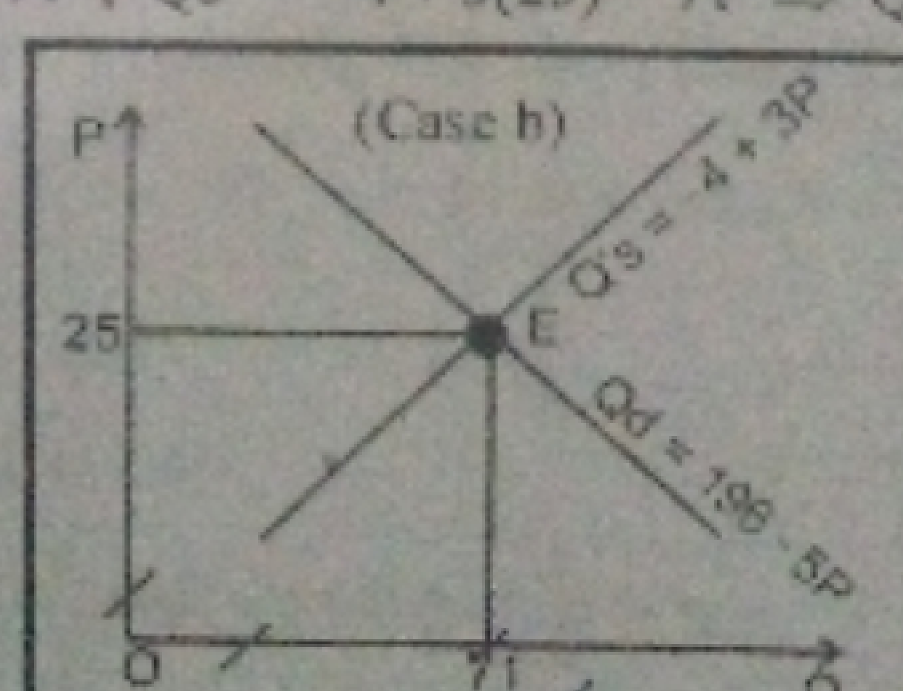
(a): $\Rightarrow -5P - 3P = 20 - 196 \Rightarrow -8P = -176 \Rightarrow P = 22 \Rightarrow \overline{P} = 22$

$Qd = 196 - 5(22) = 86$, $Qs = 20 + 3(22) = 86 \Rightarrow \bar{Q} = 86$

(b): $Qs' = 20 + 3(P - 8) \Rightarrow Qs' = 20 + 3P - 24 \Rightarrow Qs' = -4 + 3P$

$Qd = Qs' \Rightarrow 196 - 5P = -4 + 3P \Rightarrow -5P - 3P = -4 - 196 \Rightarrow -8P = -200 \Rightarrow P = 25 \Rightarrow$
$\overline{P} = 25$. $Qd = 196 - 5(25) = 71$, $Qs' = -4 + 3(25) = 71 \Rightarrow \bar{Q} = 71$





EXAMPLE – 6. Find equilibrium price and quantity, given:

$P = 50 - (\frac{7}{8})Q$, $P = 15 + (\frac{3}{4})Q$.

$$P = 50 - \left(\frac{7}{8}\right)Q \quad \ldots \quad (1)$$

$$\underline{\pm P = \pm 15 \pm \left(\frac{3}{4}\right)Q} \quad \ldots \quad (2)$$

$$0 = 35 - \frac{13}{8}Q \Rightarrow \frac{13}{8}Q = 35 \Rightarrow Q = \frac{35 \times 8}{13} = \frac{280}{13} = 21.54$$

Putting $Q = \frac{280}{13}$ in (1) and (2)

$$P = 50 - \frac{7}{8}\left(\frac{280}{13}\right) = 50 - \frac{245}{13} = \frac{405}{13} = 31.15$$

$$P = 15 + \frac{3}{4}\left(\frac{280}{13}\right) = 15 + \frac{210}{13} = \frac{195 + 210}{13} = \frac{405}{13} = 31.15$$

Thus equilibrium price $= \overline{P} = 31.15$, equilibrium quantity $= \overline{Q} = 21.54$.

EXAMPLE – 7. Find equilibrium price and quantity, given:

$$P = 5 - \left(\frac{1}{2}\right)X, \; 6P = 6 + X.$$

$$P = 5 - \frac{1}{2}X \quad \ldots \quad (1)$$

$$6P = 6 + X \quad \ldots \quad (2)$$

Multiplying (1) by 6

$$6P = 30 - 3X \quad \ldots \quad (1)$$

$$\underline{\pm 6P = \pm 6 \pm X} \quad \ldots \quad (2)$$

Subtracting : $0 = 24 - 4X \Rightarrow 4X = 24 \Rightarrow X = \frac{24}{4} = 6$

Putting $X = 6$ in (1) and (2) $P = 5 - \frac{6}{2} = 5 - 3 = 2 \Rightarrow 6P = 6 + 6 = 12 \Rightarrow P = 2$

Thus equilibrium price $= \overline{P} = 2$, equilibrium quantity $= \overline{X} = 6$.

EXAMPLE – 8. Find equilibrium price and quantity, given:

$$Dx = 40 - P, \; Sx = -10 + \frac{3}{2}P$$

Solution. $D_X = 40 - P \quad \ldots \quad (1), \quad S_X = -10 + \frac{3}{2}P \quad \ldots \quad (2)$

At equilibrium : $D_X = S_X$

$$40 - P = -10 + \frac{3}{2}P \Rightarrow -P - \frac{3}{2}P = -10 - 40 \Rightarrow -2.5P = -50 \Rightarrow P = 20$$

Putting $P = 20$ in (1) and (2) $D_X = 40 - 20 = 20$,

$$S_X = -10 + \frac{3}{2} \times 20 = 20$$

Thus equilibrium price $= \overline{P} = 20$, equilibrium quantity $= \overline{X} = 20$

DEMAND SUPPLY EQUILIBRIUM (Quadratic Equations)
— PARTIAL EQUILIBRIUM ANALYSIS

There are also no-linear models of demand and supply equilibrium. In such models, equilibrium price is also settled where demand is equal to supply, while equilibrium quantity is also determined where demand is equal to supply. When we construct the graph of non-linear model the equation which is quadratic, its graph will be quadratic and the equation which is linear, its graph will be a linear. If both equations are quadratic then the graphs of both demand and supply equations will be quadratic. In such quadratic graphs there may be negative quantities as well as negative prices. But in economics, the prices as well as quantities are not negative. Accordingly, we represent just positive quantities and positive prices in the diagrams.

Now with the help of demand and supply equations, we find equilibrium price and equilibrium quantity.

EXAMPLE – 1. Find equilibrium price and quantity, given:

$Qd = 3 - P^2, \; Qs = 6P - 4.$ (UOS:2006,2009/II, 2012)

Solution. $Qd = 3 - P^2$ ... (1), $Qs = P - 4$ (2)

At equilibrium: $Qd = Qs$

$$3 - P^2 = 6P - 4 \Rightarrow -P^2 - 6P + 3 + 4 = 0$$

or $$-P^2 - 6P + 7 = 0 \Rightarrow P^2 + 6P - 7 = 0$$

Using Quadratic Formula, we get

$$P = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 + 28}}{2} = \frac{-6 \pm \sqrt{64}}{2} = \frac{-6 \pm 8}{2}$$

$$P = \frac{-6 + 8}{2} = \frac{2}{2} = 1, \quad P = \frac{-6 - 8}{2} = \frac{-14}{2} = -7$$

Dropping the negative values of P, we have $P = 1$

Putting $P = 1$ in (1) and (2)

$$Qd = 3 - (1)^2 = 3 - 1 = 2,$$
$$Qs = 6(1) - 4 = 6 - 4 = 2$$

Thus equilibrium price = 1,

equilibrium quantity = 2.

They are represented with figure 4.

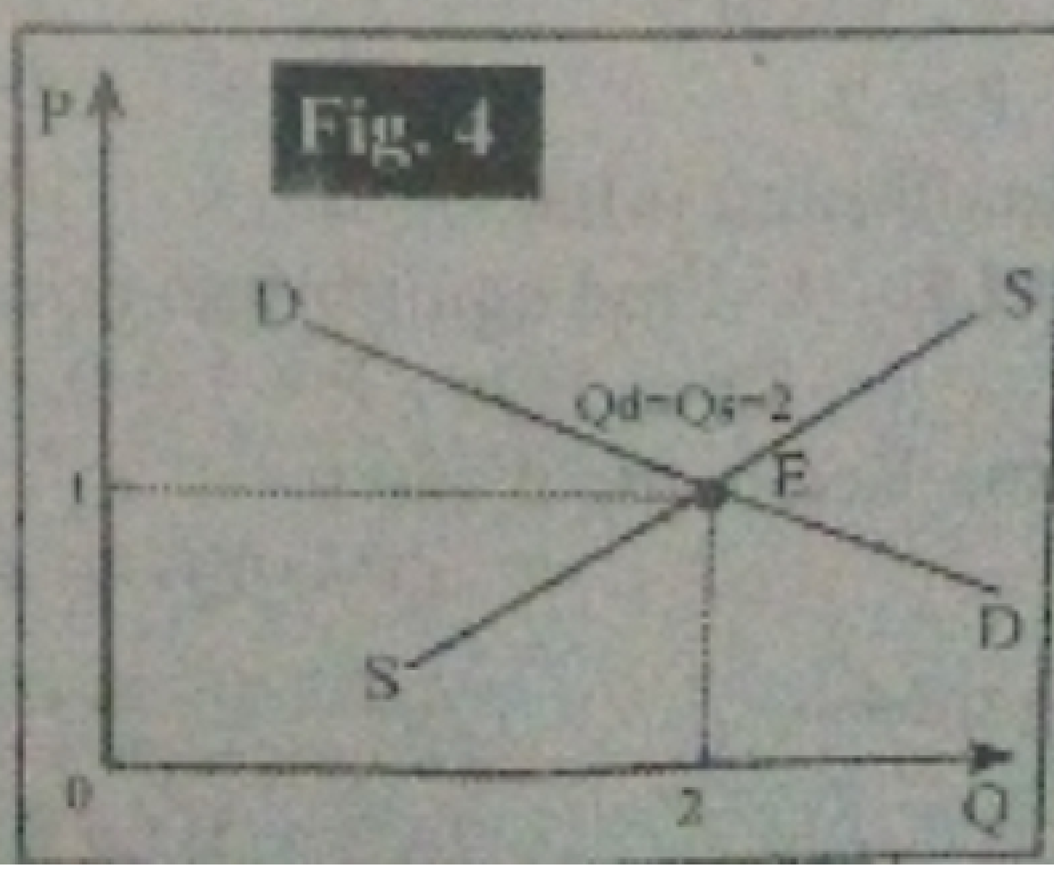


Fig. 4

EFFECTS OF TAXES ON EQUILIBRIUM PRICE AND EQUILIBRIUM QUANTITY

(I) When Taxes are Specific

It is reminded that the **specific tax** is one which is imposed on the per unit of a good sold in a specific amount irrespective of its sale price.

Example (1). $Q_d = 100 - 2P$ Demand function (UOH:2006)

$Q_s = 3P - 50$ Supply function

(i) Equilibrium price and equilibrium quantity is found.

(ii) What will be the effect on equilibrium price and quantity if govt. imposes tax Rs.5/- per unit on the firm ?

(iii) What will be the effect on equilibrium price and quantity if govt. imposes tax Rs.5/- per unit on the consumers ?

(iv) What will be the effect on equilibrium price and quantity if govt. imposes tax Rs.5/- per unit both on consumers and producers ?

(i) *Equilibrium price and equilibrium quantity is found.*

$$Qd = Qs \Rightarrow 100 - 2P = 3P - 50$$

$$\Rightarrow -2P - 3P = -50 - 100 \Rightarrow -5P = -150 \Rightarrow P = 30$$

$$Qd = 100 - 2P = 100 - 2(30) = 40$$

$$Qs = 3P - 50 = 3(30) - 50 = 40$$

Thus, equilibrium price = 30 and equilibrium quantity = 40

(ii) *Effect of tax of Rs. 5/- per unit on firm:*

As the firm has to pay the tax on the price which it is charging. Therefore, Rs.5/- will be subtracted from its price (P – 5). Then its supply function will be as:

$$Q's = 3(P - 5) - 50$$

$$Q's = 3P - 15 - 50$$

$Q's = 3P - 65$ New supply function

$Qd = 100 - 2P$ Previous demand function

$$Qd = Q's$$

$$100 - 2P = 3P - 65$$

$$-2P - 3P = -65 - 100$$

$$-5P = -165$$

$$P = 33$$

New equilibrium quantity is found. $Qd = 100 - 2P = 100 - 2(33) = 34$

$Q's = 3P - 65 = 3(33) - 65 = 34$

Thus, new equilibrium price = 33 and new equilibrium quantity is 34.

This shows that because of such tax on firm, the equilibrium price has gone up to 33 and the equilibrium quantity has decreased to 34.

(iii) *Effect of tax of Rs.5/- per unit on consumers:*

It means that the consumer will have to pay Rs.5/- more in addition to the price which he used to pay earlier. Accordingly, Rs.5/- will be added in the previous price. It is as: (P + 5). Now the new demand function will be as :

$Q'd = 100 - 2(P + 5)$

$Q'd = 100 - 2P - 10$

$Q'd = 90 - 2P$ (New demand function)

$Qs = 3P - 50$ (Previous supply function)

Equilibrium price is found

$Qs = Q'd$

$3P - 50 = 90 - 2P$

$3P + 2P = 90 + 50$

$5P = 140 \Rightarrow P = 28$

New equilibrium quantity is found

$Q'd = 90 - 2P = 90 - 2(28) \quad = 90 - 56 = 34$

$Qs = 3P - 50 = 3(28) - 50 \quad = 84 - 50 = 34$

It is obvious that new equilibrium quantity is 34 and new equilibrium price is 28 as consumer has to pay tax on price. Hence he will have to pay Rs. 33 (= 28 + 5). When tax is imposed on firms, the equilibrium price is 33, as the firm has to pay Rs.5/- to govt. as tax, then it will get 28 (= 33 – 5).

(iv) *Effect of Rs.5/- tax on firm and consumer both.*

$Q'd = 90 - 2P$ New demand function

$Q's = -65 + 3P$ New supply function

Equilibrium price is found thus: $Q'd = Q's$

$90 - 2P = -65 + 3P \Rightarrow -2P - 3P = -65 - 90 \Rightarrow -5P = -155 \Rightarrow P = 31$

$Q'd = 90 - 2P = 90 - 2(31) = 90 - 62 = 28$

$Q's = -65 + 3P = -65 + 3(31) = -65 + 93 = 28$

Now the equilibrium price = 31 and equilibrium quantity is = 28.