$$F^\mu = \frac{dP^\mu}{d\tau} = m a^\mu = m\frac{d^2x^\mu}{d\tau^2} \leftarrow$$ covariant form of Newton's 2nd law

$$F^\mu = q F^{\mu\nu} u_\nu \leftarrow$$ covariant form of Lorentz force law

$$\left.\begin{aligned} \partial_\mu F^{\mu\nu} &= J^\nu/\epsilon_0 \\ \partial_\mu \tilde{F}^{\mu\nu} &= 0 \end{aligned}\right\} \leftarrow$$ covariant Maxwell's eqns

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\partial_\mu J^\mu = 0 \leftarrow$$ covariant eqn. of continuity.

## Newton's gravitation force law

First discovered but remains the least understood force,

$$\vec{F}_G = -\frac{G m_1 m_2}{r_{12}^2}\hat{r}_{12} \; ; \quad G = 6.67 \times 10^{-11}\ N\text{-}m^2/kg^2$$

$$r_{12}^2 = |\vec{r}_1(t) - \vec{r}_2(t)|^2 \rightarrow$$ instantaneous interactions

↙

prohibited in special relativity

↙

"No signal can travel faster than light"

* Gravity is unscreened, since there are no -ve gravitational charges to cancel the +ve ones. Gravity is always attractive.
* Gravity is a long-range interaction.
* Gravit is the weakest of four fundamental forces

$$F_G/F_e \sim 10^{-36}$$ among two protons.

## Gravitation field

$$\vec{f}_e = q\vec{E} \quad \longleftrightarrow \quad \vec{F}_G = m_g \vec{g}\,, \quad \vec{g} = -G\frac{M_g}{|\vec{r}|^2}\hat{r}$$

$$\vec{E} = -\vec{\nabla}\phi \quad \longleftrightarrow \quad \vec{g} = -\vec{\nabla}\Phi_G \longrightarrow \text{gravitational potential}$$

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \quad \longleftrightarrow \quad \vec{\nabla}\cdot\vec{g} = -4\pi G\rho$$

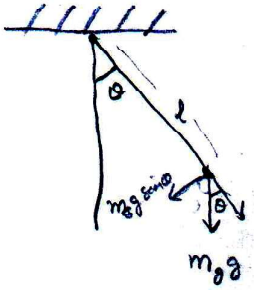
$$\nabla^2\Phi_G = 4\pi G\rho \qquad \text{poisson's eqn.}$$

## The principle of equivalence

$$\vec{f} = m_i\vec{a}\,, \quad \vec{f}_G = m_G\vec{g}\,; \quad m_i \overset{?}{=} m_G$$

($m_i$: inertial mass; $m_G$: gravitational mass)

1680, Newton performed an experiment with a pendulum to decide the question.




$$m_i\frac{d^2(L\theta)}{dt^2} = -m_G g\theta$$

$$\frac{d^2\theta}{dt^2} + \left(\frac{m_G}{m_i}\right)\frac{g}{L}\theta = 0$$

period of oscillation $= T = 2\pi\sqrt{\left(\frac{m_i}{m_G}\right)\frac{L}{g}}$

Newton found his measurements consistent with $T = 2\pi\sqrt{\frac{L}{g}}$

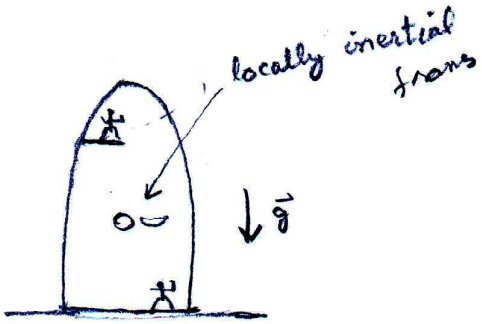
$\Rightarrow$ $m_i = m_G$, but he failed to provide the reason for this equality

Today $\frac{\Delta m}{m} \sim 10^{-12}$ with 12 order of magnitude precesion

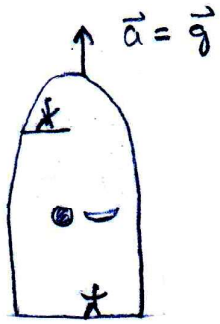
1907, Einstein's thought experiment: "the happiest thought of my life"

"No experiment can distinguish a uniform acceleration from a uniform gravitation field."

$$m_i\vec{a} = m_G\vec{g}\,, \quad \because \vec{a} = \vec{g} \Rightarrow m_i = m_G$$




Earth



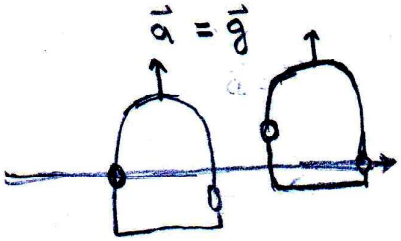

Free space

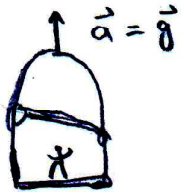
elevators / space-ships

"Freely falling frames are locally inertial frames"

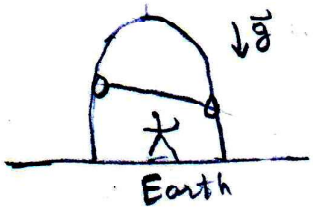
Consequence of EP:

Equivalence principle implies that light falls in a gravitational field, otherwise we donot know how to calculate the effect of gravity on light.




free space



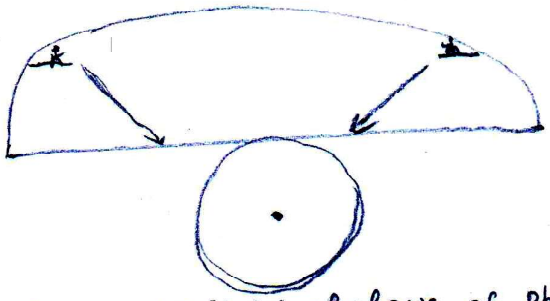

Earth

inertial observer.

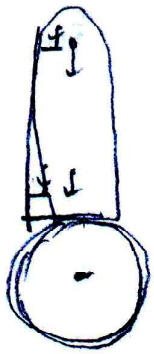
① Gravitational deflection of light.

② Gravitational redshift of light

Earth



non-uniform gravitational effects.



↳ no global inertial frame.

WEP → gravitational laws of Physics.

SEP → all laws of physics, gravitational & others.

## The principle of general covariance

Extension of the priciple of relativity by including the non-inertial frames also.

" The Laws of physics are tensorial in nature, i.e. laws are form invariant w.r.t general co-ordinate transformation "