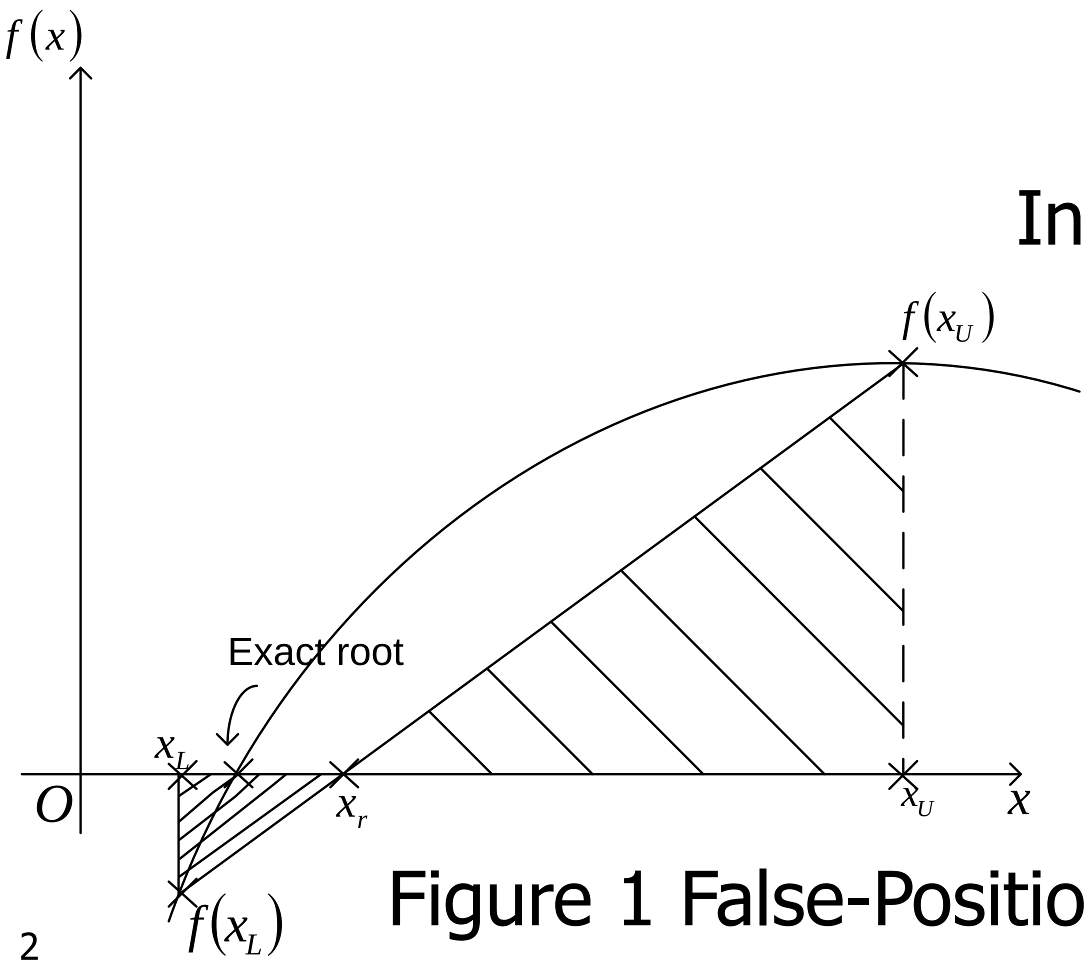
Lecture # 1

# Chapter 03.06: False-Position Method of Solving a Nonlinear Equation

# Introduction




$$f(x) = 0 \qquad (1)$$

In the Bisection method

$$f(x_L) * f(x_U) < 0 \qquad (2)$$

$$x_r = \frac{x_L + x_U}{2} \qquad (3)$$

Figure 1 False-Position Method

# False-Position Method

Based on two similar triangles, shown in Figure 1, one gets:

$$\frac{f(x_L)}{x_r - x_L} = \frac{f(x_U)}{x_r - x_U} \tag{4}$$

The signs for both sides of Eq. (4) is consistent, since:

$$f(x_L) < 0; \; x_r - x_L > 0$$
$$f(x_U) > 0; \; x_r - x_U < 0$$

From Eq. (4), one obtains

$$\left(x_r - x_L\right) f\left(x_U\right) = \left(x_r - x_U\right) f\left(x_L\right)$$

$$x_U f\left(x_L\right) - x_L f\left(x_U\right) = x_r \left\{ f\left(x_L\right) - f\left(x_U\right) \right\}$$

The above equation can be solved to obtain the next predicted root $x_r$, as

$$x_r = \frac{x_U f\left(x_L\right) - x_L f\left(x_U\right)}{f\left(x_L\right) - f\left(x_U\right)} \qquad (5)$$

The above equation,

$$x_r = x_U - \frac{f(x_U)\{x_L - x_U\}}{f(x_L) - f(x_U)} \quad (6)$$

or

$$x_r = x_L - \frac{f(x_L)}{\left\{\dfrac{f(x_U) - f(x_L)}{x_U - x_L}\right\}} \quad (7)$$

# Step-By-Step False-Position Algorithms

1. Choose $x_L$ and $x_U$ as two guesses for the root such that

$$f(x_L)f(x_U)<0$$

2. Estimate the root, $x_m = \dfrac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$

3. Now check the following

(a) If $f(x_L)f(x_m)<0$ , then the root lies between $x_L$ and $x_m$; then $x_L = x_L$ and $x_U = x_m$

(b) If $f(x_L)f(x_m)>0$ , then the root lies between $x_m$ and $x_U$; then $x_L = x_m$ and $x_U = x_U$

(c) If $f(x_L)f(x_m)=0$ , then the root is $x_m$.

Stop the algorithm if this is true.

4. Find the new estimate of the root

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

Find the absolute relative approximate error as

$$|\in_a| = \left| \frac{x_m^{new} - x_m^{old}}{x_m^{new}} \right| \times 100$$

where

$x_m^{new}$ = estimated root from present iteration

$x_m^{old}$ = estimated root from previous iteration

5. $say \in_s = 10^{-3} = 0.001.$ If $|\in_a| > \in_s$, then go to step 3, else stop the algorithm.

**Notes:** The False-Position and Bisection algorithms are quite similar. The only difference is the formula used to calculate the new estimate of the root $x_m$ ,shown in steps #2 and 4!

# Example 1

The floating ball has a specific gravity of 0.6 and has a radius of 5.5cm.

You are asked to find the depth to which the ball is submerged when floating in water.

The equation that gives the depth $x$ to which the ball is submerged under water is given by

$$x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$$

Use the false-position method of finding roots of equations to find the depth $x$ to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration, and the number of significant digits at least correct at the converged iteration.

# Solution

From the physics of the problem

$$0 \leq x \leq 2R$$

$$0 \leq x \leq 2(0.055)$$

$$0 \leq x \leq 0.11$$

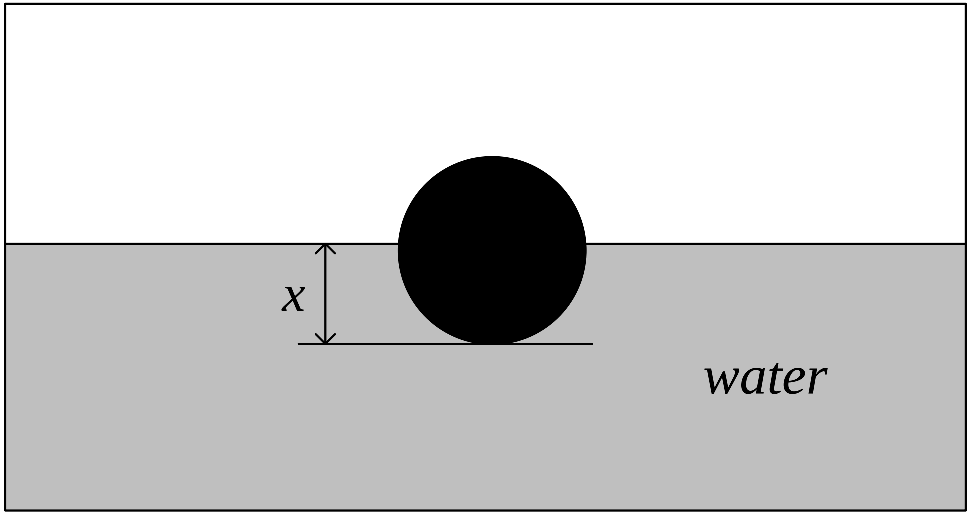



Figure 2 : Floating ball problem

Let us assume

$$x_L = 0,\ x_U = 0.11$$

$$f(x_L) = f(0) = (0)^3 - 0.165(0)^2 + 3.993\times10^{-4} = 3.993\times10^{-4}$$
$$f(x_U) = f(0.11) = (0.11)^3 - 0.165(0.11)^2 + 3.993\times10^{-4} = -2.662\times10^{-4}$$

Hence,

$$f(x_L)f(x_U) = f(0)f(0.11) = \left(3.993\times10^{-4}\right)\left(-2.662\times10^{-4}\right) < 0$$

# Iteration 1

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

$$= \frac{0.11 \times 3.993 \times 10^{-4} - 0 \times \left(-2.662 \times 10^{-4}\right)}{3.993 \times 10^{-4} - \left(-2.662 \times 10^{-4}\right)}$$

$$= 0.0660$$

$$f(x_m) = f(0.0660) = (0.0660)^3 - 0.165(0.0660)^2 + \left(3.993 \times 10^{-4}\right)$$

$$= -3.1944 \times 10^{-5}$$

$$f(x_L) f(x_m) = f(0) f(0.0660) = (+)(-) < 0$$

$$x_L = 0,\ x_U = 0.0660$$

# Iteration 2

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

$$= \frac{0.0660 \times 3.993 \times 10^{-4} - 0 \times (-3.1944 \times 10^{-5})}{3.993 \times 10^{-4} - (-3.1944 \times 10^{-5})}$$

$$= 0.0611$$

$$f(x_m) = f(0.0611) = (0.0611)^3 - 0.165(0.0611)^2 + (3.993 \times 10^{-4})$$

$$= 1.1320 \times 10^{-5}$$

$$f(x_L) f(x_m) = f(0) f(0.0611) = (+)(+) > 0$$

Hence, $x_L = 0.0611, x_U = 0.0660$

$$\in_a = \left| \frac{0.0611 - 0.0660}{0.0611} \right| \times 100 \cong 8\%$$

**<u>Iteration 3</u>**

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

$$= \frac{0.0660 \times 1.132 \times 10^{-5} - 0.0611 \times \left(-3.1944 \times 10^{-5}\right)}{1.132 \times 10^{-5} - \left(-3.1944 \times 10^{-5}\right)}$$

$$= 0.0624$$

$$f\left(x_m\right) = -1.1313 \times 10^{-7}$$

$$f\left(x_L\right) f\left(x_m\right) = f\left(0.0611\right) f\left(0.0624\right) = (+)(-) < 0$$

Hence,

$$x_L = 0.0611,\ x_U = 0.0624$$

$$\in_a = \left| \frac{0.0624 - 0.0611}{0.0624} \right| \times 100 \cong 2.05\%$$

# Table 1: Root of $f(x) = x^3 - 0.165x^2 + 3.993\times 10^{-4} = 0$ for False-Position Method.

| Iteration | $x_L$ | $x_U$ | $x_m$ | $\lvert\in_a\rvert\%$ | $f(x_m)$ |
|---|---|---|---|---|---|
| 1 | 0.0000 | 0.1100 | 0.0660 | N/A | $-3.1944\text{x}10^{-5}$ |
| 2 | 0.0000 | 0.0660 | 0.0611 | 8.00 | $1.1320\text{x}10^{-5}$ |
| 3 | 0.0611 | 0.0660 | 0.0624 | 2.05 | $-1.1313\text{x}10^{-7}$ |
| 4 | 0.0611 | 0.0624 | 0.0632377619 | 0.02 | $-3.3471\text{x}10^{-10}$ |

$$|\in_a| \le 0.5 \times 10^{2-m}$$

$$0.02 \le 0.5 \times 10^{2-m}$$

$$0.04 \le 10^{2-m}$$

$$\log(0.04) \le 2 - m$$

$$m \le 2 - \log(0.04)$$

$$m \le 2 - (-1.3979)$$

$$m \le 3.3979$$

$$So,\ m = 3$$

The number of significant digits at least correct in the estimated root of 0.062377619 at the end of 4th iteration is 3.

# References

1. S.C. Chapra, R.P. Canale, Numerical Methods for Engineers, Fourth Edition, Mc-Graw Hill.

# THE END

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