# Chapter 1

# The Nature of Mathematical Economics

Mathematical economics is not a distinct branch of economics in the sense that public finance or international trade is. Rather, it is an *approach* to economic analysis, in which the economist makes use of mathematical symbols in the statement of the problem and also draws upon known mathematical theorems to aid in reasoning. As far as the specific subject matter of analysis goes, it can be micro- or macroeconomic theory, public finance, urban economics, or what not.

Using the term *mathematical economics* in the broadest possible sense, one may very well say that every elementary textbook of economics today exemplifies mathematical economics insofar as geometrical methods are frequently utilized to derive theoretical results. More commonly, however, mathematical economics is reserved to describe cases employing mathematical techniques beyond simple geometry, such as matrix algebra, differential and integral calculus, differential equations, difference equations, etc. *It is the purpose of this book to introduce the reader to the most fundamental aspects of these mathematical methods*—those encountered daily in the current economic literature.

## 1.1 Mathematical versus Nonmathematical Economics

Since mathematical economics is merely an approach to economic analysis, it should not and does not fundamentally differ from the *non*mathematical approach to economic analysis. The purpose of any theoretical analysis, regardless of the approach, is always to derive a set of conclusions or theorems from a given set of assumptions or postulates via a process of reasoning. The major difference between "mathematical economics" and "literary economics" is twofold: First, in the former, the assumptions and conclusions are stated in mathematical symbols rather than words and in equations rather than sentences. Second, in place of literary logic, use is made of mathematical theorems—of which there exists an abundance to draw upon—in the reasoning process. Inasmuch as symbols and words are really equivalents (witness the fact that symbols are usually defined in words), it matters little which is chosen over the other. But it is perhaps beyond dispute that symbols are more convenient to use in deductive reasoning, and certainly are more conducive to conciseness and preciseness of statement.

The choice between literary logic and mathematical logic, again, is a matter of little import, but mathematics has the advantage of forcing analysts to make their assumptions explicit at every stage of reasoning. This is because mathematical theorems are usually stated in the "if-then" form, so that in order to tap the "then" (result) part of the theorem for their use, they must first make sure that the "if" (condition) part does conform to the explicit assumptions adopted.

Granting these points, though, one may still ask why it is necessary to go beyond geometric methods. The answer is that while geometric analysis has the important advantage of being visual, it also suffers from a serious dimensional limitation. In the usual graphical discussion of indifference curves, for instance, the standard assumption is that only *two* commodities are available to the consumer. Such a simplifying assumption is not willingly adopted but is forced upon us because the task of drawing a three-dimensional graph is exceedingly difficult, and the construction of a four- (or higher) dimensional graph is actually a physical impossibility. To deal with the more general case of 3, 4, or *n* goods, we must instead resort to the more flexible tool of equations. This reason alone should provide sufficient motivation for the study of mathematical methods beyond geometry.

In short, we see that the mathematical approach has claim to the following advantages: (1) The "language" used is more concise and precise; (2) there exists a wealth of mathematical theorems at our service; (3) in forcing us to state explicitly all our assumptions as a prerequisite to the use of the mathematical theorems, it keeps us from the pitfall of an unintentional adoption of unwanted implicit assumptions; and (4) it allows us to treat the general *n*-variable case.

Against these advantages, one sometimes hears the criticism that a mathematically derived theory is inevitably *unrealistic*. However, this criticism is not valid. In fact, the epithet "unrealistic" cannot even be used in criticizing economic theory in general, whether or not the approach is mathematical. Theory is by its very nature an abstraction from the real world. It is a device for singling out only the most essential factors and relationships so that we can study the crux of the problem at hand, free from the many complications that do exist in the actual world. Thus the statement "theory lacks realism" is merely a truism that cannot be accepted as a valid criticism of theory. By the same token, it is quite meaningless to pick out any one approach to theory as "unrealistic." For example, the theory of firm under pure competition is unrealistic, as is the theory of firm under imperfect competition, but whether these theories are derived mathematically or not is irrelevant and immaterial.

To take advantage of the wealth of mathematical tools, one must of course first acquire those tools. Unfortunately, the tools that are of interest to economists are widely scattered among many mathematics courses—too many to fit comfortably into the plan of study of a typical economics student. The service the present volume performs is to gather in one place the mathematical methods most relevant to the economics literature, organize them into a logical order of progression, fully explain each method, and then immediately illustrate how the method is applied in economic analysis. By tying together the methods and their applications, the relevance of mathematics to economics is made more transparent than is possible in the regular mathematics courses where the illustrated applications are predominantly tied to physics and engineering. Familiarity with the contents of this book (and, if possible, also its sequel volume: Alpha C. Chiang, *Elements of Dynamic Optimization*, McGraw-Hill, 1992, now published by Waveland Press, Inc.) should therefore enable you to comprehend most of the professional articles you will come across in

such periodicals as the *American Economic Review, Quarterly Journal of Economics, Journal of Political Economy, Review of Economics and Statistics,* and *Economic Journal.* Those of you who, through this exposure, develop a serious interest in mathematical economics can then proceed to a more rigorous and advanced study of mathematics.

## 1.2   Mathematical Economics versus Econometrics

The term *mathematical economics* is sometimes confused with a related term, *econometrics.* As the "metric" part of the latter term implies, econometrics is concerned mainly with the measurement of economic data. Hence it deals with the study of *empirical* observations using statistical methods of estimation and hypothesis testing. Mathematical economics, on the other hand, refers to the application of mathematics to the purely *theoretical* aspects of economic analysis, with little or no concern about such statistical problems as the errors of measurement of the variables under study.

In the present volume, we shall confine ourselves to mathematical economics. That is, we shall concentrate on the application of mathematics to deductive reasoning rather than inductive study, and as a result we shall be dealing primarily with theoretical rather than empirical material. This is, of course, solely a matter of choice of the scope of discussion, and it is by no means implied that econometrics is less important.

Indeed, empirical studies and theoretical analyses are often complementary and mutually reinforcing. On the one hand, theories must be tested against empirical data for validity before they can be applied with confidence. On the other, statistical work needs economic theory as a guide, in order to determine the most relevant and fruitful direction of research.

In one sense, however, mathematical economics may be considered as the more basic of the two: for, to have a meaningful statistical and econometric study, a good theoretical framework—preferably in a mathematical formulation—is indispensable. Hence the subject matter of the present volume should be useful not only for those interested in theoretical economics, but also for those seeking a foundation for the pursuit of econometric studies.