FACTOR - FACTOR RELATIONSHIP

In the previous case, production decisions were confined to the situation where there was only one variable input factor. Production problems generally involve situations where two or more inputs or variables are employed. Let us consider a situation where only two inputs are used in production and the function can be written as

4. $$Y = f\ (X_1,\ X_2 / X_3, \ldots, X_n),$$

where Y is output, while X_1 and X_2 are quantities of concentrates and green fodder (berseem). It is to be noted that these inputs are fed to livestock along with the other inputs which are kept constant at a fixed level.

Our concern here is that what would happen to output when the quantities of inputs X_1 and X_2 are increased or decreased. Besides the effect on output, what would happen to substitution of one variable factor for another by changing the quantities of these inputs. Thus, we would be focusing now on the substitution between X_1 and X_2 in any livestock production process. The farmer could choose various combinations of the factors of production within the limitations of his investment capacity. The economically feasible level of output from different combinations of given level of inputs hinges on how the variable inputs are combined. Thus, the critical feature of studying factor-factor relationship is to determine the possibilities of mixing and substituting two or more factors in the production of a given level of output that is economically feasible.

Iso-Product Curve

Iso-product or Iso-quant is a curve that represents different efficient combinations of X_1 and X_2 that are capable of producing a given level of output. Farmers are always interested in finding out the least cost or cheapest method to produce a given level of output. Sometimes different input combinations can be used to produce a particular level of output. It is possible that a cheapest way exists to produce a product using only one factor and none of the other, but in other cases it might be the case that combination of two inputs is the cheapest method to produce a product. The shape of the iso-product curve depends on the way by which the variable inputs are combined in production.

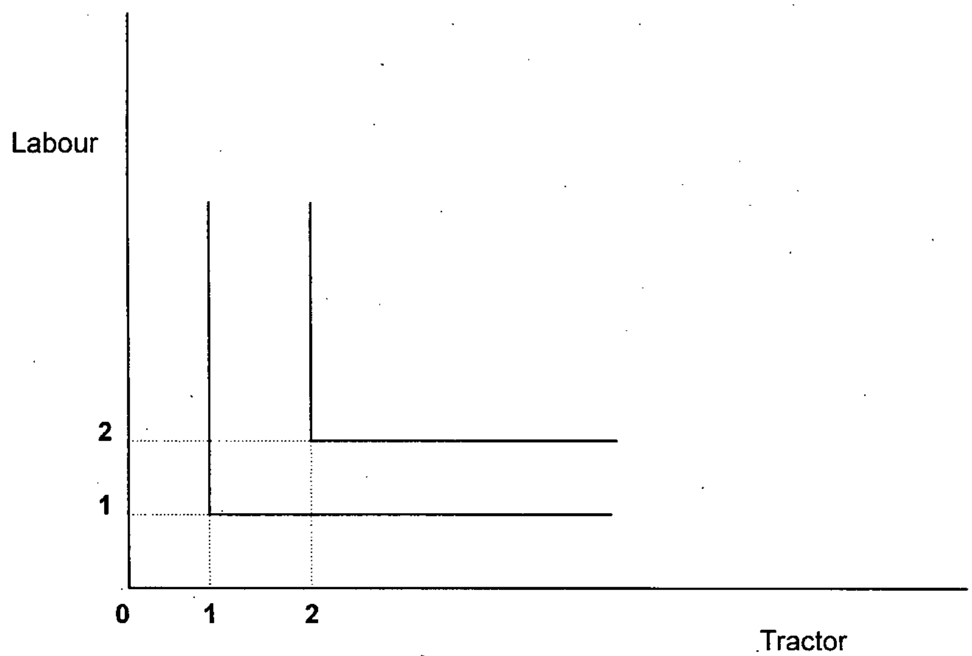
Fixed Proportion

The iso-product curve shown in Figure 3.8 represents that the inputs are combined in fixed proportion in the production of a commodity. For example, neither a tractor nor a pair of bullock (X_1) can be operated without the labour input (X_2).

Constant Rate of Substitution

In this case the iso-product curve is a straight line, which indicate that the marginal rate of technical substitution is constant on any point on the curve; meaning that it is same whatever the relative amounts of factors X_1 and X_2 are being used. <u>The</u>

Figure 3.8: A Graphiclal Representation of fixed Proportion Isoquants



marginal rate of substitution may be defined as the number of units by which the usage of one input (X_2) must be decreased when the usage of the other input (X_1) is increased by one unit along an iso-quant, i.e., when the output is at a particular level. The marginal rate of technical substitution may be written as $\frac{\Delta X_2}{\Delta X_1}$, which is negative of the slope of the iso-quant, and this slope remains constant throughout the line (iso-quant).

Consider a case where a farmer prepares his own feed. He thinks that the major source of energy is either maize or sorghum, and finds that 1.90 kg of maize (X_1) supplies the same energy as 2 kg of sorghum (X_2). Assume that required amount of energy is represented by the product line, and that this energy can be supplied by 200 kgs of sorghum or 190 kg of maize. The same amount of energy can also be obtained by using different combinations of sorghum and maize (see Table 3.3). Figure 3.9 shows that the iso-quant is a straight line, and thus, its slope $\Delta X_2/\Delta X_1$, which is the marginal rate of technical substitution, is constant on all the points of the curve.

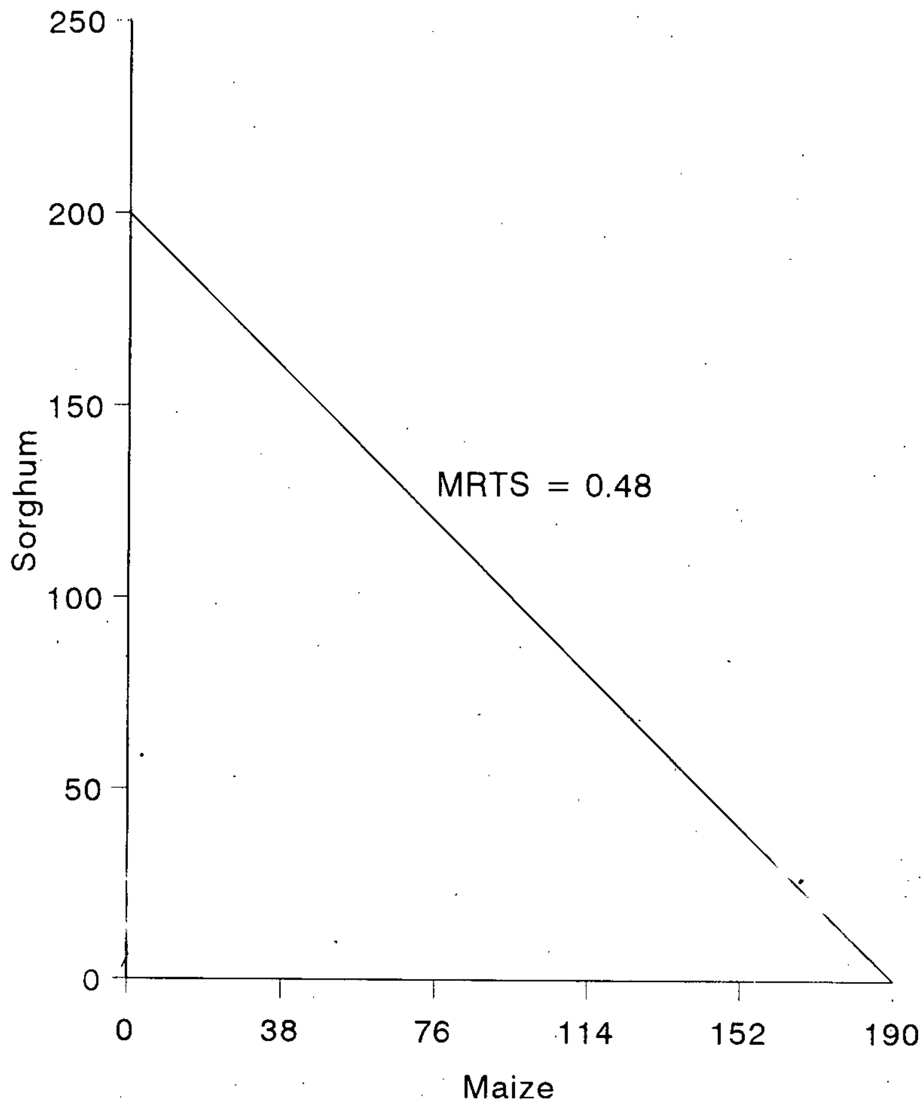
Diminishing Rate of Substitution

In this case, each unit increase in X_1 replaces less and less of X_2 in the production of a given level of output. The slope of the iso-quant becomes smaller as more of X_1 is used relative to X_2 The phenomenon of decreasing rate of substitution can easily be explained using the law of diminishing marginal productivity. As the use of X_1 increases, its marginal product tends to decrease. On the other hand the use of X_2 decreases and, thus, its marginal product increases. Since the marginal rate of substitution equals the marginal product of X_2 (which is

Table 3.3: Constant Rate of Substitution

Sorghum (X_1)	Maize (X_2)	Change in X1 & X2		MRTS
		ΔX_1	ΔX_2	$\Delta X_2/\Delta X_1$
200	0	-	-	0.48
160	38	40	19	0.48
120	76	40	19	0.48
80	114	40	19	0.48
40	152	40	19	0.48
0	190	40	19	0.48

Figure 3.9: An Isoquant Representing Constant Rate of Substitution



declining) divided by Marginal product of X_1 (which is increasing), it must be diminishing as X_1 is substituted for X_2. Declining marginal rate of substitution as X_1 increases and X_2 decreases, implies that the iso-product curve is convex to the origin.

Consider an example of cotton seed and berseem inputs, which substitute each other for the production of milk in such a manner that leads to diminishing marginal rate of substitution. It is assumed here that 10 kgs of milk can be obtained with different combinations of cotton seed (X_2) and berseem (X_1). These combinations are 13.55 kgs of X_2 and 1 kg of X_1 or 5.22 kgs of X_2 and 6 kgs of X_1 or 2.74 kgs of X_2 and 11 kgs of X_1 or 1.77 kgs of X_2 and 16 kgs of X_1 or 1.24 kgs of X_2 and 21 kgs of X_1 or 0.90 kgs of X_2 and 28 kgs of X_1. This particular example indicates that X_1 and X_2 are substitutable and the column 4 of Table 3.4 clearly indicates that the MRTS declines as X_2 is substituted for X_1. This has also been presented in Figure 3.10, where the iso-product is negatively sloped and convex to the origin.

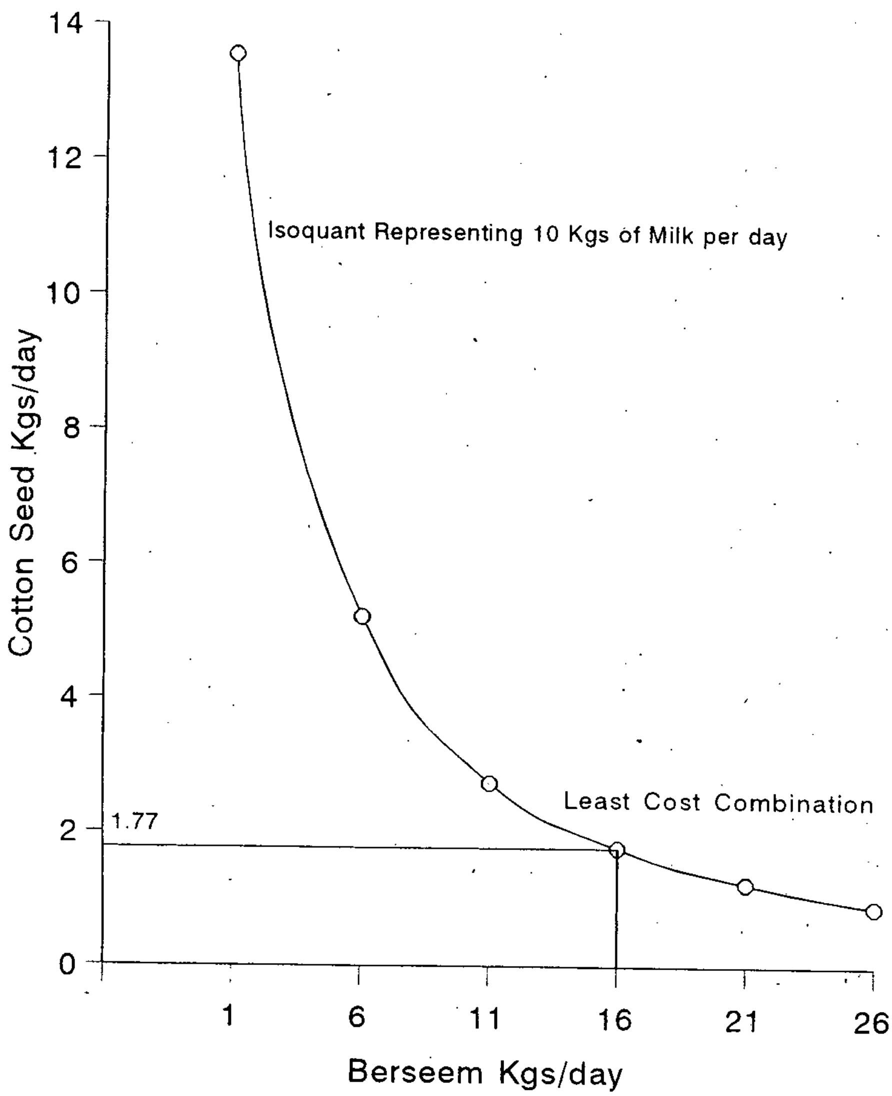
Least Cost Combination of Inputs

To determine the least cost combination of inputs, we need 1) the rate at which the inputs can be exchanged in production, i.e., the marginal rate of technical substitution (MRTS) $\frac{\Delta X_2}{\Delta X_1}$ and 2) the rate at which the inputs can be exchanged in the market, i.e., the ratio of the prices of the two inputs [$\frac{PX_1}{PX_2}$] where PX_1 is the price of X_1 and PX_2 is the price of X_2.

Table 3.4 **Determination of Least Cost Input Combination**

1	2	3		4	5	6	7	8	9	10
(X_1) Berseem (kgs)	(X_2) Cotton seed (kgs)	Change in X_1 and X_2		MRTS $\frac{\Delta X_2}{\Delta X_1}$	$\frac{P_{X1}}{P_{X2}}$	$\Delta X_1\ P_{X1}$	$\Delta X_2.P_{X2}$	$P_{X1}.X_1$	$P_{X2}.X_2$	Total Cost 8+9
		ΔX_1	ΔX_2							
1	13.55	-	-	-	-	-	-	0.80	34.44	34.94
6	5.22	5	8.13	1.63	0.19	2.50	20.98	3.00	13.47	16.47
11	2.74	5	2.48	0.50	0.19	2.50	6.40	5.50	7.07	12.57
16	1.77	5	0.97	0.19	0.19	2.50	2.50	8.00	4.57	12.07
21	1.24	5	0.53	0.11	0.19	2.50	1.37	11.50	3.20	13.70
26	0.90	5	0.34	0.07	0.19	2.50	0.88	13.00	2.32	15.32

Figure 3.10: An Isoquant Representing 10 Kgs. of Milk/day.



The combination of inputs that minimizes the cost of given quantity of output is attained when the marginal rate of technical substitution (MRTS) is equal to the inverse factor price ratio, i.e., $\frac{\Delta X_2}{\Delta X_1} = \frac{P_{X_1}}{P_{X_2}}$. This condition is met when the slope of the iso-quant, $\frac{\Delta X_2}{\Delta X_1}$, is equal to the slope of iso-cost line, $\frac{PX^{Y}}{PX_2}$. $If\ \frac{\Delta X_2}{\Delta X_1} > \frac{PX_{1}}{PX_2}$[40] then cost can be reduced by decreasing the use of X_2 and increasing the use of X_1. If $\frac{\Delta X_2}{\Delta X_1} < \frac{PX_1}{PX_2}$, then cost can be reduced by increasing the use of X_2 and decreasing the use of X_1

The least cost combination can also be given as $\Delta X_1\ P_{X_1} = \Delta X_2\ P_{X_2}$ It states that the cost of adding X_1 is equal to the reduction in cost from using less of X_2. If $\Delta X_1 PX_1 > \Delta X_2 PX_2$, then an increase in the use of X_2 and a decrease in the use of X_1 will reduce cost. If $\Delta X_1 PX_1 < \Delta X_2 PX_2$, then more use of X_1 and less use of X_2 will decrease cost. If Px_1 =Rs 2.58/kg and Px_2 =Rs 0.5/kg, milk is produced with lowest possible cost using 1.77 kgs of X_1 and 16 kgs of X_2 (Figure 3.10 and Table 3.4).

---

40 [1]Iso-cost line represents all possible combinations of two inputs, which can be purchased at a give level of resource income.