CHAPTER 4

SERIES IMPEDANCE OF TRANSMISSION LINES

An electric transmission line has four parameters which affect its ability to fulfill its function as part of a power system: *resistance*, *inductance*, *capacitance*, *and conductance*. In this chapter we discuss the first two of these parameters, and we shall consider capacitance in the next chapter. The fourth parameter, conductance, exists between conductors or between conductors and the ground. Conductance accounts for the leakage current at the insulators of overhead lines and through the insulation of cables. Since leakage at insulators of overhead lines is negligible, the conductance between conductors of an overhead line is usually neglected.

Another reason for neglecting conductance is that since it is quite variable, there is no good way of taking it into account. Leakage at insulators, the principal source of conductance, changes appreciably with atmospheric conditions and with the conducting properties of dirt that collects on the insulators. Corona, which results in leakage between lines, is also quite variable with atmospheric conditions. It is fortunate that the effect of conductance is such a negligible component of shunt admittance.

Some of the properties of an electric circuit can be explained by the electric and magnetic fields which accompany its current flow. Figure 4.1 shows a single-phase line and its associated magnetic and electric fields. The lines of magnetic flux form closed loops linking the circuit, and the lines of electric flux originate on the positive charges on one conductor and terminate on the

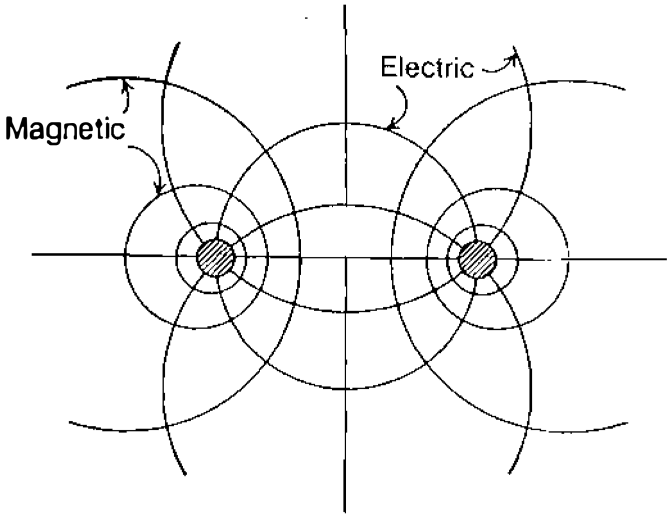

FIGURE 4.1
Magnetic and electric fields associated with a two-wire line.

negative charges on the other conductor. Variation of the current in the conductors causes a change in the number of lines of magnetic flux linking the circuit. Any change in the flux linking a circuit induces a voltage in the circuit which is proportional to the rate of change of flux. The inductance of the circuit relates the voltage induced by changing flux to the rate of change of current.

The capacitance which exists between the conductors is defined as the charge on the conductors per unit of potential difference between them.

The resistance and inductance uniformly distributed along the line form the series impedance. The conductance and capacitance existing between conductors of a single-phase line or from a conductor to neutral of a three-phase line form the shunt admittance. Although the resistance, inductance, and capacitance are distributed, the equivalent circuit of a line is made up of lumped parameters, as we shall see when we discuss them.

4.1 TYPES OF CONDUCTORS

In the early days of the transmission of electric power conductors were usually copper, but aluminum conductors have completely replaced copper for overhead lines because of the much lower cost and lighter weight of an aluminum conductor compared with a copper conductor of the same resistance. The fact that an aluminum conductor has a larger diameter than a copper conductor of the same resistance is also an advantage. With a larger diameter, the lines of electric flux originating on the conductor will be farther apart at the conductor surface for the same voltage. This means there is a lower voltage gradient at the conductor surface and less tendency to ionize the air around the conductor. Ionization produces the undesirable effect called *corona*.

Symbols identifying different types of aluminum conductors are as follows:

AAC	all-aluminum conductors
AAAC	all-aluminum-alloy conductors
ACSR	aluminum conductor, steel-reinforced
ACAR	aluminum conductor, alloy-reinforced

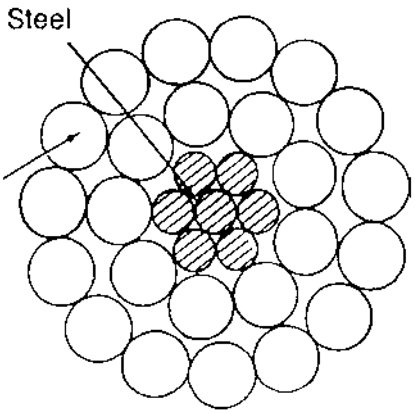

FIGURE 4.2
Cross section of a steel-reinforced conductor, 7 steel strands, and 24 aluminum strands.

Aluminum-alloy conductors have higher tensile strength than the ordinary electrical-conductor grade of aluminum. ACSR consists of a central core of steel strands surrounded by layers of aluminum strands. ACAR has a central core of higher-strength aluminum surrounded by layers of electrical-conductor-grade aluminum.

Alternate layers of wire of a stranded conductor are spiraled in opposite directions to prevent unwinding and to make the outer radius of one layer coincide with the inner radius of the next layer. Stranding provides flexibility for a large cross-sectional area. The number of strands depends on the number of layers and on whether all the strands are of the same diameter. The total number of strands in concentrically stranded cables, where the total annular space is filled with strands of uniform diameter, is 7, 19, 37, 61, 91, or more.

Figure 4.2 shows the cross section of a typical steel-reinforced aluminum cable (ACSR). The conductor shown has 7 steel strands forming a central core, around which there are two layers of aluminum strands. There are 24 aluminum strands in the two outer layers. The conductor stranding is specified as 24 Al/7 St, or simply 24/7. Various tensile strengths, current capacities, and conductor sizes are obtained by using different combinations of steel and aluminum.

Appendix Table A.3 gives some electrical characteristics of ACSR. Code names, uniform throughout the aluminum industry, have been assigned to each conductor for easy reference.

A type of conductor known as *expanded* ACSR has a filler such as paper separating the inner steel strands from the outer aluminum strands. The paper gives a larger diameter (and hence, lower corona) for a given conductivity and tensile strength. Expanded ACSR is used for some extra-high-voltage (EHV) lines.

4.2 RESISTANCE

The resistance of transmission-line conductors is the most important cause of power loss in a transmission line. The term "resistance," unless specifically qualified, means *effective resistance*. The effective resistance of a conductor is

$$R = \frac{\text{power loss in conductor}}{|I|^2}\ \Omega \tag{4.1}$$

where the power is in watts and I is the rms current in the conductor in amperes. The effective resistance is equal to the dc resistance of the conductor only if the distribution of current throughout the conductor is uniform. We shall discuss nonuniformity of current distribution briefly after reviewing some fundamental concepts of dc resistance.

Direct-current resistance is given by the formula

$$R_0 = \frac{\rho l}{A}\,\Omega \tag{4.2}$$

where ρ = resistivity of conductor

l = length

A = cross-sectional area

Any consistent set of units may be used. In power work in the United States l is usually given in feet, A in circular mils (cmil), and ρ in ohm-circular mils per foot, sometimes called ohms per circular mil-foot. In SI units l is in meters, A in square meters and ρ in ohm-meters.[1]

A circular mil is the area of a circle having a diameter of 1 mil. A mil is equal to 10^{-3} in. The cross-sectional area of a solid cylindrical conductor in circular mils is equal to the square of the diameter of the conductor expressed in mils. The number of circular mils multiplied by $\pi/4$ equals the number of square mils. Since manufacturers in the United States identify conductors by their cross-sectional area in circular mils, we must use this unit occasionally. The area in square millimeters equals the area in circular mils multiplied by 5.067×10^{-4}.

The international standard of conductivity is that of annealed copper. Commercial hard-drawn copper wire has 97.3% and aluminum has 61% of the conductivity of standard annealed copper. At 20°C for hard-drawn copper ρ is $1.77 \times 10^{-8}\ \Omega \cdot \text{m}$ ($10.66\ \Omega \cdot \text{cmil/ft}$). For aluminum at 20°C ρ is $2.83 \times 10^{-8}\ \Omega \cdot \text{m}$ ($17.00\ \Omega \cdot \text{cmil/ft}$).

The dc resistance of stranded conductors is greater than the value computed by Eq. (4.2) because spiraling of the strands makes them longer than the conductor itself. For each mile of conductor the current in all strands except the one in the center flows in more than a mile of wire. The increased resistance due to spiraling is estimated as 1% for three-strand conductors and 2% for concentrically stranded conductors.

The variation of resistance of metallic conductors with temperature is practically linear over the normal range of operation. If temperature is plotted on the vertical axis and resistance on the horizontal axis, as in Fig. 4.3, extension

[1]SI is the official designation for the International System of Units.

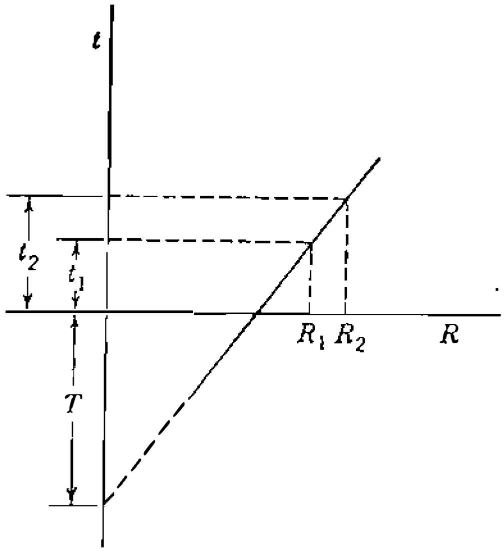

FIGURE 4.3
Resistance of a metallic conductor as a function of temperature.

of the straight-line portion of the graph provides a convenient method of correcting resistance for changes in temperature. The point of intersection of the extended line with the temperature axis at zero resistance is a constant of the material. From the geometry of Fig. 4.3

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1} \tag{4.3}$$

where R_1 and R_2 are the resistances of the conductor at temperatures t_1 and t_2, respectively, in degrees Celsius and T is the constant determined from the graph. Values of the constant T in degrees Celsius are as follows:

$$T = \begin{cases} 234.5 & \text{for annealed copper of 100\% conductivity} \\ 241 & \text{for hard-drawn copper of 97.3\% conductivity} \\ 228 & \text{for hard-drawn aluminum of 61\% conductivity} \end{cases}$$

Uniform distribution of current throughout the cross section of a conductor exists only for direct current. As the frequency of alternating current increases, the nonuniformity of distribution becomes more pronounced. An increase in frequency causes nonuniform current density. This phenomenon is called *skin effect*. In a circular conductor the current density *usually* increases from the interior toward the surface. For conductors of sufficiently large radius, however, a current density oscillatory with respect to radial distance from the center may result.

As we shall see when discussing inductance, some lines of magnetic flux exist inside a conductor. Filaments on the surface of a conductor are not linked by internal flux, and the flux linking a filament near the surface is less than the flux linking a filament in the interior. The alternating flux induces higher voltages acting on the interior filaments than are induced on filaments near the surface of the conductor. By Lenz's law the induced voltage opposes the changes of current producing it, and the higher induced voltages acting on the inner filaments cause the higher current density in filaments nearer the surface, and therefore higher effective resistance results. Even at power system frequencies, skin effect is a significant factor in large conductors.

4.3 TABULATED RESISTANCE VALUES

The dc resistance of various types of conductors is easily found by Eq. (4.2), and the increased resistance due to spiraling can be estimated. Temperature corrections are determined by Eq. (4.3). The increase in resistance caused by skin effect can be calculated for round wires and tubes of solid material, and curves of R/R_0 are available for these simple conductors.[2] This information is not necessary, however, since manufacturers supply tables of electrical characteristics of their conductors. Table A.3 is an example of some of the data available.

Example 4.1. Tables of electrical characteristics of all-aluminum *Marigold* stranded conductor list a dc resistance of 0.01558 Ω per 1000 ft at 20°C and an ac resistance of 0.0956 Ω/mi at 50°C. The conductor has 61 strands and its size is 1,113,000 cmil. Verify the dc resistance and find the ratio of ac to dc resistance.

Solution. At 20°C from Eq. (4.2) with an increase of 2% for spiraling

$$R_0 = \frac{17.0 \times 1000}{1113 \times 10^3} \times 1.02 = 0.01558 \ \Omega \text{ per 1000 ft}$$

At a temperature of 50°C from Eq. (4.3)

$$R_0 = 0.01558 \frac{228 + 50}{228 + 20} = 0.01746 \ \Omega \text{ per 1000 ft}$$

$$\frac{R}{R_0} = \frac{0.0956}{0.01746 \times 5.280} = 1.037$$

Skin effect causes a 3.7% increase in resistance.

4.4 INDUCTANCE OF A CONDUCTOR DUE TO INTERNAL FLUX

The inductance of a transmission line is calculated as flux linkages per ampere. If permeability μ is constant, sinusoidal current produces sinusoidally varying flux in phase with the current. The resulting flux linkages can then be expressed

[2]See The Aluminum Association, *Aluminum Electrical Conductor Handbook*, 2d ed., Washington, DC, 1982.

as a phasor λ, and

$$L = \frac{\lambda}{I} \tag{4.4}$$

If i, the instantaneous value of current, is substituted for the phasor I in Eq. (4.4), then λ should be the value of the instantaneous flux linkages produced by i. Flux linkages are measured in weber-turns, Wbt.

Only flux lines external to the conductors are shown in Fig. 4.1. Some of the magnetic field, however, exists inside the conductors, as we mentioned when considering skin effect. The changing lines of flux inside the conductors also contribute to the induced voltage of the circuit and therefore to the inductance. The correct value of inductance due to internal flux can be computed as the ratio of flux linkages to current by taking into account the fact that each line of internal flux links only a fraction of the total current.

To obtain an accurate value for the inductance of a transmission line, it is necessary to consider the flux inside each conductor as well as the external flux. Let us consider the long cylindrical conductor whose cross section is shown in Fig. 4.4. We assume that the return path for the current in this conductor is so far away that it does not appreciably affect the magnetic field of the conductor shown. Then, the lines of flux are concentric with the conductor.

By Ampere's law the magnetomotive force (mmf) in ampere-turns around any closed path is equal to the net current in amperes enclosed by the path, as discussed in Sec. 2.1. The mmf equals the line integral around the closed path of the component of the magnetic field intensity tangent to the path and is given by Eq. (2.4), now written as Eq. (4.5):

$$\text{mmf} = \oint H \cdot ds = I \text{ At} \tag{4.5}$$

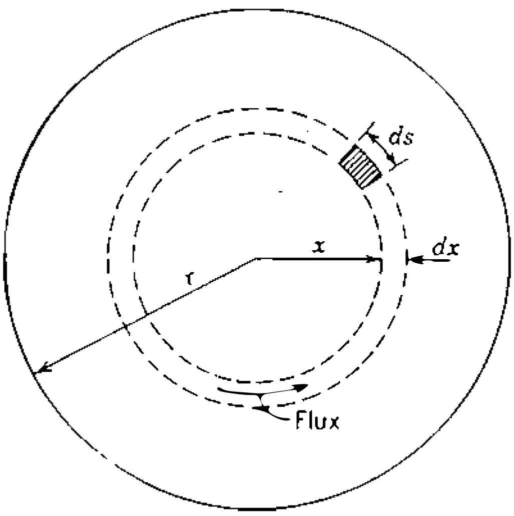

FIGURE 4.4
Cross section of a cylindrical conductor.

where H = magnetic field intensity, At/m

s = distance along path, m

I = current enclosed, A

Note that H and I are shown as phasors to represent sinusoidally alternating quantities since our work here applies equally to alternating and direct current. For simplicity the current I could be interpreted as a direct current and H as a real number. We recall that the dot between H and ds indicates that the value of H is the component of the field intensity tangent to ds.

Let the field intensity at a distance x meters from the center of the conductor be designated H_x. Since the field is symmetrical, H_x is constant at all points equidistant from the center of the conductor. If the integration indicated in Eq. (4.5) is performed around a circular path concentric with the conductor at x meters from the center, H_x is constant over the path and tangent to it. Equation (4.5) becomes

$$\oint H_x \, ds = I_x \tag{4.6}$$

and

$$2\pi x H_x = I_x \tag{4.7}$$

where I_x is the current enclosed. Then, assuming uniform current density,

$$I_x = \frac{\pi x^2}{\pi r^2} I \tag{4.8}$$

where I is the total current in the conductor. Then, substituting Eq. (4.8) in Eq. (4.7) and solving for H_x, we obtain

$$H_x = \frac{x}{2\pi r^2} I \text{ At/m} \tag{4.9}$$

The flux density x meters from the center of the conductor is

$$B_x = \mu H_x = \frac{\mu x I}{2\pi r^2} \text{ Wb/m}^2 \tag{4.10}$$

where μ is the permeability of the conductor.[3]

In the tubular element of thickness dx the flux $d\phi$ is B_x times the cross-sectional area of the element normal to the flux lines, the area being dx

[3] In SI units the permeability of free space is $\mu_0 = 4\pi \times 10^{-7}$ H/m, and the relative permeability is $\mu_r = \mu/\mu_0$.

times the axial length. The flux per meter of length is

$$d\phi = \frac{\mu x I}{2\pi r^2} dx \text{ Wb/m} \tag{4.11}$$

The flux linkages $d\lambda$ per meter of length, which are caused by the flux in the tubular element, are the product of the flux per meter of length and the fraction of the current linked. Thus,

$$d\lambda = \frac{\pi x^2}{\pi r^2} d\phi = \frac{\mu I x^3}{2\pi r^4} dx \text{ Wbt/m} \tag{4.12}$$

Integrating from the center of the conductor to its outside edge to find λ_{int}, the total flux linkages inside the conductor, we obtain

$$\lambda_{int} = \int_0^r \frac{\mu I x^3}{2\pi r^4} dx = \frac{\mu I}{8\pi} \text{ Wbt/m} \tag{4.13}$$

For a relative permeability of 1, $\mu = 4\pi \times 10^{-7}$ H/m, and

$$\lambda_{int} = \frac{I}{2} \times 10^{-7} \text{ Wbt/m} \tag{4.14}$$

$$L_{int} = \frac{1}{2} \times 10^{-7} \text{ H/m} \tag{4.15}$$

We have computed the inductance per unit length (henrys per meter) of a round conductor attributed only to the flux inside the conductor. Hereafter, for convenience, we refer to *inductance per unit length* simply as *inductance*, but we must be careful to use the correct dimensional units.

The validity of computing the internal inductance of a solid round wire by the method of partial flux linkages can be demonstrated by deriving the internal inductance in an entirely different manner. Equating energy stored in the magnetic field within the conductor per unit length at any instant to $L_{int}i^2/2$ and solving for L_{int} will yield Eq. (4.15).

4.5 FLUX LINKAGES BETWEEN TWO POINTS EXTERNAL TO AN ISOLATED CONDUCTOR

As a step in computing inductance due to flux external to a conductor, let us derive an expression for the flux linkages of an isolated conductor due only to that portion of the external flux which lies between two points at D_1 and D_2 meters from the center of the conductor. In Fig. 4.5 P_1 and P_2 are two such points. The conductor carries a current of I A. Since the flux paths are

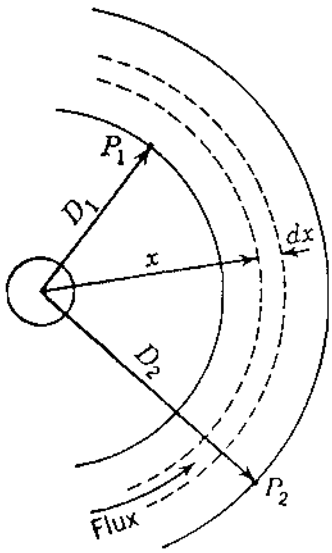

FIGURE 4.5
A conductor and external points P_1 and P_2.

concentric circles around the conductor, all the flux between P_1 and P_2 lies within the concentric cylindrical surfaces (indicated by solid circular lines) which pass through P_1 and P_2. At the tubular element which is x meters from the center of the conductor the field intensity is H_x. The mmf around the element is

$$2\pi x H_x = I \tag{4.16}$$

Solving for H_x and multiplying by μ yield the flux density B_x in the element so that

$$B_x = \frac{\mu I}{2\pi x} \text{ Wb/m}^2 \tag{4.17}$$

The flux $d\phi$ in the tubular element of thickness dx is

$$d\phi = \frac{\mu I}{2\pi x} dx \text{ Wb/m} \tag{4.18}$$

The flux linkages $d\lambda$ per meter are numerically equal to the flux $d\phi$ since flux external to the conductor links all the current in the conductor only once. So, between P_1 and P_2 the flux linkages are

$$\lambda_{12} = \int_{D_1}^{D_2} \frac{\mu I}{2\pi x} dx = \frac{\mu I}{2\pi} \ln \frac{D_2}{D_1} \text{ Wbt/m} \tag{4.19}$$

or for a relative permeability of 1

$$\lambda_{12} = 2 \times 10^{-7} I \ln \frac{D_2}{D_1} \text{ Wbt/m} \tag{4.20}$$

The inductance due only to the flux included between P_1 and P_2 is

$$L_{12} = 2 \times 10^{-7} \ln \frac{D_2}{D_1} \text{ H/m} \tag{4.21}$$

4.6 INDUCTANCE OF A SINGLE-PHASE TWO-WIRE LINE

We can now determine the inductance of a simple two-wire line composed of solid round conductors. Figure 4.6 shows such a line having two conductors of radii r_1 and r_2. One conductor is the return circuit for the other. First, consider only the flux linkages of the circuit caused by the current in conductor 1. A line of flux set up by current in conductor 1 at a distance equal to or greater than $D + r_2$ from the center of conductor 1 does not link the circuit. At a distance less than $D - r_2$ the fraction of the total current linked by a line of flux is 1.0. Therefore, it is logical when D is much greater than r_1 and r_2 to assume that D can be used instead of $D - r_2$ or $D + r_2$. In fact, it can be shown that calculations made with this assumption are correct even when D is small.

We add inductance due to internal flux linkages determined by Eq. (4.15) to inductance due to external flux linkages determined by Eq. (4.21) with r_1 replacing D_1 and D replacing D_2 to obtain

$$L_1 = \left(\frac{1}{2} + 2 \ln \frac{D}{r_1} \right) \times 10^{-7} \text{ H/m} \tag{4.22}$$

which is the inductance of the circuit due to the current in conductor 1 only.

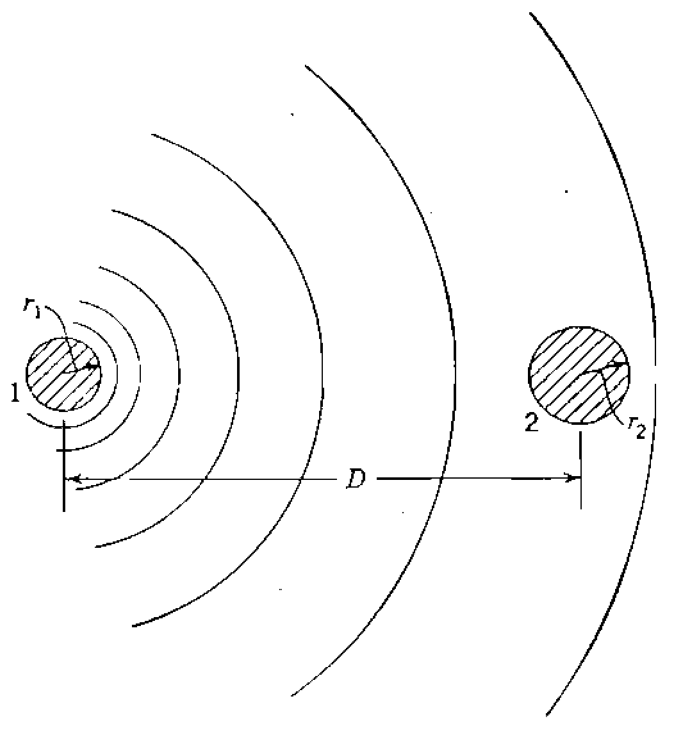

FIGURE 4.6
Conductors of different radii and the magnetic field due to current in conductor 1 only.

The expression for inductance may be put in a more concise form by factoring Eq. (4.22) and by noting that $\ln \varepsilon^{1/4} = 1/4$, whence

$$L_1 = 2 \times 10^{-7}\left(\ln \varepsilon^{1/4} + \ln \frac{D}{r_1}\right) \tag{4.23}$$

Upon combining terms, we obtain

$$L_1 = 2 \times 10^{-7} \ln \frac{D}{r_1 \varepsilon^{-1/4}} \tag{4.24}$$

If we substitute r_1' for $r_1 \varepsilon^{-1/4}$,

$$L_1 = 2 \times 10^{-7} \ln \frac{D}{r_1'} \text{ H/m} \tag{4.25}$$

The radius r_1' is that of a fictitious conductor assumed to have no internal flux but with the same inductance as the actual conductor of radius r_1. The quantity $\varepsilon^{-1/4}$ is equal to 0.7788. Equation (4.25) omits the term accounting for internal flux but compensates for it by using an adjusted value for the radius of the conductor. The multiplying factor of 0.7788, which adjusts the radius in order to account for internal flux, applies only to solid round conductors. We consider other conductors later.

Since the current in conductor 2 flows in the direction opposite to that in conductor 1 (or is 180° out of phase with it), the flux linkages produced by current in conductor 2 considered alone are in the same direction through the circuit as those produced by current in conductor 1. The resulting flux for the two conductors is determined by the sum of the mmfs of both conductors. For constant permeability, however, the flux linkages (and likewise the inductances) of the two conductors considered separately may be added.

By comparison with Eq. (4.25), the inductance due to current in conductor 2 is

$$L_2 = 2 \times 10^{-7} \ln \frac{D}{r_2'} \text{ H/m} \tag{4.26}$$

and for the complete circuit

$$L = L_1 + L_2 = 4 \times 10^{-7} \ln \frac{D}{\sqrt{r_1' r_2'}} \text{ H/m} \tag{4.27}$$

If $r_1' = r_2' = r'$, the total inductance reduces to

$$L = 4 \times 10^{-7} \ln \frac{D}{r'} \text{ H/m} \tag{4.28}$$

This value of inductance is sometimes called the *inductance per loop meter* or *per loop mile* to distinguish it from that component of the inductance of the circuit attributed to the current in one conductor only. The latter, as given by Eq. (4.25), is one-half the total inductance of a single-phase line and is called the *inductance per conductor*.

4.7 FLUX LINKAGES OF ONE CONDUCTOR IN A GROUP

A more general problem than that of the two-wire line is presented by one conductor in a group of conductors where the sum of the currents in all the conductors is zero. Such a group of conductors is shown in Fig. 4.7. Conductors $1, 2, 3, \ldots, n$ carry the phasor currents $I_1, I_2, I_3, \ldots, I_n$. The distances of these conductors from a remote point P are indicated on the figure as $D_{1P}, D_{2P}, D_{3P}, \ldots, D_{nP}$. Let us determine λ_{1P1}, the flux linkages of conductor 1 due to I_1 including internal flux linkages but excluding all the flux beyond the point P. By Eqs. (4.14) and (4.20)

$$\lambda_{1P1} = \left(\frac{I_1}{2} + 2I_1 \ln \frac{D_{1P}}{r_1} \right) 10^{-7} \tag{4.29}$$

$$\lambda_{1P1} = 2 \times 10^{-7} I_1 \ln \frac{D_{1P}}{r_1'} \text{ Wbt/m} \tag{4.30}$$

The flux linkages λ_{1P2} with conductor 1 *due to* I_2 but excluding flux beyond point P is equal to the flux produced by I_2 between the point P and conductor

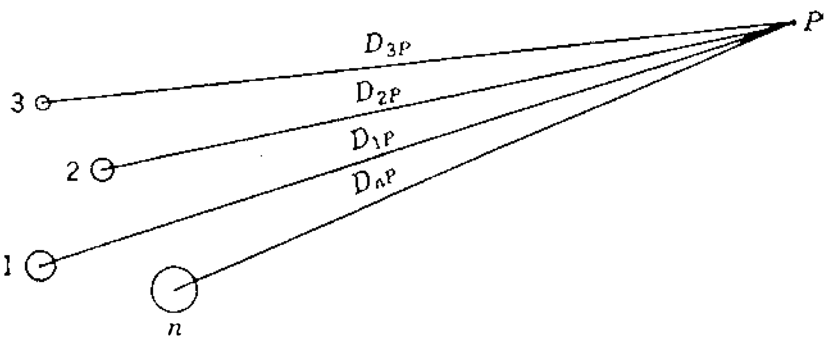

FIGURE 4.7
Cross-sectional view of a group of n conductors carrying currents whose sum is zero. Point P is remote from the conductors.

1 (that is, within the limiting distances D_{2P} and D_{12} from conductor 2), and so

$$\lambda_{1P2} = 2 \times 10^{-7} I_2 \ln \frac{D_{2P}}{D_{12}} \tag{4.31}$$

The flux linkages λ_{1P} with conductor 1 *due to all the conductors* in the group but excluding flux beyond point P is

$$\lambda_{1P} = 2 \times 10^{-7}\left(I_1 \ln \frac{D_{1P}}{r_1'} + I_2 \ln \frac{D_{2P}}{D_{12}} + I_3 \ln \frac{D_{3P}}{D_{13}} + \cdots + I_n \ln \frac{D_{nP}}{D_{1n}}\right) \tag{4.32}$$

which becomes, by expanding the logarithmic terms and regrouping,

$$\lambda_{1P} = 2 \times 10^{-7}\left(I_1 \ln \frac{1}{r_1'} + I_2 \ln \frac{1}{D_{12}} + I_3 \ln \frac{1}{D_{13}} + \cdots + I_n \ln \frac{1}{D_{1n}} + I_1 \ln D_{1P} + I_2 \ln D_{2P} + I_3 \ln D_{3P} + \cdots + I_n \ln D_{nP}\right) \tag{4.33}$$

Since the sum of all the currents in the group is zero,

$$I_1 + I_2 + I_3 + \cdots + I_n = 0$$

and solving for I_n, we obtain

$$I_n = -(I_1 + I_2 + I_3 + \cdots + I_{n-1}) \tag{4.34}$$

Substituting Eq. (4.34) in the second term containing I_n in Eq. (4.33) and recombining some logarithmic terms, we have

$$\lambda_{1P} = 2 \times 10^{-7}\left(I_1 \ln \frac{1}{r_1'} + I_2 \ln \frac{1}{D_{12}} + I_3 \ln \frac{1}{D_{13}} + \cdots + I_n \ln \frac{1}{D_{1n}} + I_1 \ln \frac{D_{1P}}{D_{np}} + I_2 \ln \frac{D_{2P}}{D_{nP}} + I_3 \ln \frac{D_{3P}}{D_{nP}} + \cdots + I_{n-1} \ln \frac{D_{(n-1)P}}{D_{nP}}\right) \tag{4.35}$$

Now letting the point P move infinitely far away so that the set of terms containing logarithms of ratios of distances from P becomes infinitesimal, since the ratios of the distances approach 1, we obtain

$$\lambda_1 = 2 \times 10^{-7}\left(I_1 \ln \frac{1}{r_1'} + I_2 \ln \frac{1}{D_{12}} + I_3 \ln \frac{1}{D_{13}} + \cdots + I_n \ln \frac{1}{D_{1n}} \right) \text{ Wbt/m} \tag{4.36}$$

By letting point P move infinitely far away, we have included all the flux linkages of conductor 1 in our derivation. Therefore, Eq. (4.36) expresses all the flux linkages of conductor 1 in a group of conductors, provided the sum of all the currents is zero. If the currents are alternating, they must be expressed as instantaneous currents to obtain instantaneous flux linkages or as complex rms values to obtain the rms value of flux linkages as a complex number.

4.8 INDUCTANCE OF COMPOSITE-CONDUCTOR LINES

Stranded conductors come under the general classification of *composite* conductors, which means conductors composed of two or more elements or strands electrically in parallel. We limit ourselves to the case where all the strands are identical and share the current equally. The values of internal inductance of specific conductors are generally available from the various manufacturers and can be found in handbooks. The method to be developed indicates the approach to the more complicated problems of nonhomogeneous conductors and unequal division of current between strands. The method is applicable to the determination of inductance of lines consisting of circuits electrically in parallel since two conductors in parallel can be treated as strands of a single composite conductor.

Figure 4.8 shows a single-phase line composed of two conductors. In order to be more general, each conductor forming one side of the line is shown as an arbitrary arrangement of an indefinite number of conductors. The only restrictions are that the parallel filaments are cylindrical and share the current equally. Conductor X is composed of n identical, parallel filaments, each of which carries the current I/n. Conductor Y, which is the return circuit for the current in conductor X, is composed of m identical, parallel filaments, each of which carries the current $-I/m$. Distances between the elements will be designated by the letter D with appropriate subscripts. Applying Eq. (4.36) to

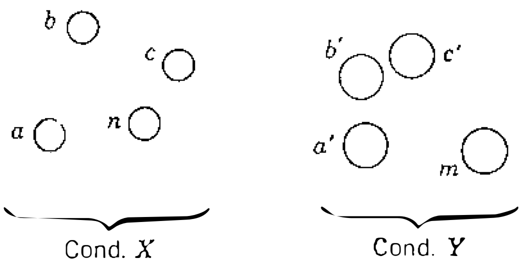

FIGURE 4.8
Single-phase line consisting of two composite conductors.

filament a of conductor X, we obtain for flux linkages of filament a

$$\lambda_a = 2 \times 10^{-7} \frac{I}{n}\left(\ln \frac{1}{r'_a} + \ln \frac{1}{D_{ab}} + \ln \frac{1}{D_{ac}} + \cdots + \ln \frac{1}{D_{an}}\right)$$

$$- 2 \times 10^{-7} \frac{I}{m}\left(\ln \frac{1}{D_{aa'}} + \ln \frac{1}{D_{ab'}} + \ln \frac{1}{D_{ac'}} + \cdots + \ln \frac{1}{D_{am}}\right) \quad (4.37)$$

from which

$$\lambda_a = 2 \times 10^{-7} I \ln \frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'} \cdots D_{am}}}{\sqrt[n]{r'_a D_{ab} D_{ac} \cdots D_{an}}} \text{ Wbt/m} \quad (4.38)$$

Dividing Eq. (4.38) by the current I/n, we find that the inductance of filament a is

$$L_a = \frac{\lambda_a}{I/n} = 2n \times 10^{-7} \ln \frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'} \cdots D_{am}}}{\sqrt[n]{r'_a D_{ab} D_{ac} \cdots D_{an}}} \text{ H/m} \quad (4.39)$$

Similarly, the inductance of filament b is

$$L_b = \frac{\lambda_b}{I/n} = 2n \times 10^{-7} \ln \frac{\sqrt[m]{D_{ba'}D_{bb'}D_{bc'} \cdots D_{bm}}}{\sqrt[n]{D_{ba} r'_b D_{bc} \cdots D_{bn}}} \text{ H/m} \quad (4.40)$$

The average inductance of the filaments of conductor X is

$$L_{av} = \frac{L_a + L_b + L_c + \cdots + L_n}{n} \quad (4.41)$$

Conductor X is composed of n filaments electrically in parallel. If all the filaments had the same inductance, the inductance of the conductor would be $1/n$ times the inductance of one filament. Here all the filaments have different inductances, but the inductance of all of them in parallel is $1/n$ times the average inductance. Thus, the inductance of conductor X is

$$L_X = \frac{L_{av}}{n} = \frac{L_a + L_b + L_c + \cdots + L_n}{n^2} \quad (4.42)$$

Substituting the logarithmic expression for inductance of each filament in Eq.

(4.42) and combining terms, we obtain

$$L_X = 2 \times 10^{-7}$$

$$\times \ln \frac{\sqrt[mn]{(D_{aa'}D_{ab'}D_{ac'} \cdots D_{am})(D_{ba'}D_{bb'}D_{bc'} \cdots D_{bm}) \cdots (D_{na'}D_{nb'}D_{nc'} \cdots D_{nm})}}{\sqrt[n^2]{(D_{aa}D_{ab}D_{ac} \cdots D_{an})(D_{ba}D_{bb}D_{bc} \cdots D_{bn}) \cdots (D_{na}D_{nb}D_{nc} \cdots D_{nn})}} \text{ H/m} \tag{4.43}$$

where r'_a, r'_b, and r'_n have been replaced by D_{aa}, D_{bb}, and D_{nn}, respectively, to make the expression appear more symmetrical.

Note that the numerator of the argument of the logarithm in Eq. (4.43) is the mnth root of mn terms, which are the products of the distances from all the n filaments of conductor X to all the m filaments of conductor Y. For *each* filament in conductor X there are m distances to filaments in conductor Y, and there are n filaments in conductor X. The product of m distances for each of n filaments results in mn terms. The mnth root of the product of the mn distances is called the *geometric mean distance* between conductor X and conductor Y. It is abbreviated D_m or GMD and is also called the *mutual* GMD between the two conductors.

The denominator of the argument of the logarithm in Eq. (4.43) is the n^2 root of n^2 terms. There are n filaments, and for each filament there are n terms consisting of r' for that filament times the distances from that filament to every other filament in conductor X. Thus, we account for n^2 terms. Sometimes r'_a is called the distance from filament a to itself, especially when it is designated as D_{aa}. With this in mind, the terms under the radical in the denominator may be described as the product of the distances from every filament in the conductor to itself and to every other filament. The n^2 root of these terms is called the *self* GMD of conductor X, and the r' of a separate filament is called the self GMD of the filament. Self GMD is also called *geometric mean radius*, or GMR. The correct mathematical expression is self GMD, but common practice has made GMR more prevalent. We use GMR in order to conform to this practice and identify it by D_s.

In terms of D_m and D_s Eq. (4.43) becomes

$$L_X = 2 \times 10^{-7} \ln \frac{D_m}{D_s} \text{ H/m} \tag{4.44}$$

The reader should compare Eqs. (4.44) and (4.25).

The inductance of conductor Y is determined in a similar manner, and the inductance of the line is

$$L = L_X + L_Y$$

Example 4.2. One circuit of a single-phase transmission line is composed of three solid 0.25-cm-radius wires. The return circuit is composed of two 0.5-cm-radius wires. The arrangement of conductors is shown in Fig. 4.9. Find the inductance due to the current in each side of the line and the inductance of the complete line in henrys per meter (and in millihenrys per mile).

Solution. Find the GMD between sides X and Y:

$$D_m = \sqrt[6]{D_{ad} D_{ae} D_{bd} D_{be} D_{cd} D_{ce}}$$

$$D_{ad} = D_{be} = 9 \text{ m}$$

$$D_{ae} = D_{bd} = D_{ce} = \sqrt{6^2 + 9^2} = \sqrt{117}$$

$$D_{cd} = \sqrt{9^2 + 12^2} = 15 \text{ m}$$

$$D_m = \sqrt[6]{9^2 \times 15 \times 117^{3/2}} = 10.743 \text{ m}$$

Then, find the GMR for side X

$$D_s = \sqrt[9]{D_{aa} D_{ab} D_{ac} D_{ba} D_{bb} D_{bc} D_{ca} D_{cb} D_{cc}}$$

$$= \sqrt[9]{(0.25 \times 0.7788 \times 10^{-2})^3 \times 6^4 \times 12^2} = 0.481 \text{ m}$$

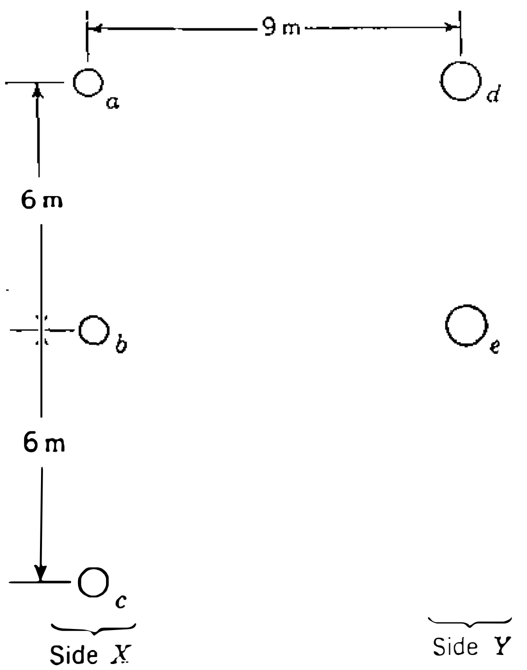

FIGURE 4.9
Arrangement of conductors for Example 4.2.

and for side Y

$$D_s = \sqrt[4]{(0.5 \times 0.7788 \times 10^{-2})^2 \times 6^2} = 0.153 \text{ m}$$

$$L_X = 2 \times 10^{-7} \ln \frac{10.743}{0.481} = 6.212 \times 10^{-7} \text{ H/m}$$

$$L_Y = 2 \times 10^{-7} \ln \frac{10.743}{0.153} = 8.503 \times 10^{-7} \text{ H/m}$$

$$L = L_X + L_Y = 14.715 \times 10^{-7} \text{ H/m}$$

$$(L = 14.715 \times 10^{-7} \times 1609 \times 10^3 = 2.37 \text{ mH/mi})$$

In Example 4.2 the conductors in parallel on one side of the line are separated by 6 m, and the distance between the two sides of the line is 9 m. Here the calculation of mutual GMD is important. For stranded conductors the distance between sides of a line composed of one conductor per side is usually so great that the mutual GMD can be taken as equal to the center-to-center distance with negligible error.

If the effect of the steel core of ACSR is neglected in calculating inductance, a high degree of accuracy results, provided the aluminum strands are in an even number of layers. The effect of the core is more apparent for an odd number of layers of aluminum strands, but the accuracy is good when the calculations are based on the aluminum strands alone.

4.9 THE USE OF TABLES

Tables listing values of GMR are generally available for standard conductors and provide other information for calculating inductive reactance as well as shunt capacitive reactance and resistance. Since industry in the United States continues to use units of inches, feet, and miles, so do these tables. Therefore, some of our examples will use feet and miles, but others will use meters and kilometers.

Inductive reactance rather than inductance is usually desired. The inductive reactance of one conductor of a single-phase two-conductor line is

$$X_L = 2\pi f L = 2\pi f \times 2 \times 10^{-7} \ln \frac{D_m}{D_s}$$

$$= 4\pi f \times 10^{-7} \ln \frac{D_m}{D_s} \ \Omega/\text{m} \tag{4.45}$$

or

$$X_L = 2.022 \times 10^{-3} f \ln \frac{D_m}{D_s} \ \Omega/\text{mi} \tag{4.46}$$

where D_m is the distance between conductors. Both D_m and D_s must be in the same units, usually either meters or feet. The GMR found in tables is an equivalent D_s, which accounts for skin effect where it is appreciable enough to affect inductance. Of course, skin effect is greater at higher frequencies for a conductor of a given diameter. Values of D_s listed in Table A.3 of the Appendix are for a frequency of 60 Hz.

Some tables give values of inductive reactance in addition to GMR. One method is to expand the logarithmic term of Eq. (4.46), as follows:

$$X_L = \underbrace{2.022 \times 10^{-3} f \ln \frac{1}{D_s}}_{X_a} + \underbrace{2.022 \times 10^{-3} f \ln D_m}_{X_d} \ \Omega/\text{mi} \tag{4.47}$$

If both D_s and D_m are in feet, the first term in Eq. (4.47) is the inductive reactance of one conductor of a two-conductor line having a distance of 1 ft between conductors, as may be seen by comparing Eq. (4.47) with Eq. (4.46). Therefore, the first term of Eq. (4.47) is called the *inductive reactance at 1-ft spacing* X_a. It depends on the GMR of the conductor and the frequency. The second term of Eq. (4.47) is called the *inductive reactance spacing factor* X_d. This second term is independent of the type of conductor and depends on frequency and spacing only. Table A.3 includes values of inductive reactance at 1-ft spacing, and Table A.4 lists values of the inductive reactance spacing factor.

Example 4.3. Find the inductive reactance per mile of a single-phase line operating at 60 Hz. The conductor is *Partridge*, and spacing is 20 ft between centers.

Solution. For this conductor Table A.3 lists $D_s = 0.0217$ ft. From Eq. (4.46) for one conductor

$$X_L = 2.022 \times 10^{-3} \times 60 \ln \frac{20}{0.0217}$$

$$= 0.828 \ \Omega/\text{mi}$$

The above calculation is used only if D_s is known. Table A.3, however, lists inductive reactance at 1-ft spacing $X_a = 0.465 \ \Omega/\text{mi}$. From Table A.4 the inductive reactance spacing factor is $X_d = 0.3635 \ \Omega/\text{mi}$, and so the inductive reactance of one conductor is

$$0.465 + 0.3635 = 0.8285 \ \Omega/\text{mi}$$

Since the conductors composing the two sides of the line are identical, the inductive reactance of the line is

$$2X_L = 2 \times 0.8285 = 1.657 \ \Omega/\text{mi}$$

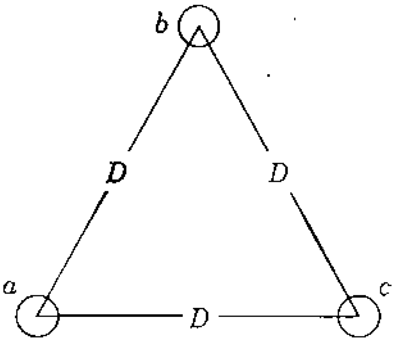

FIGURE 4.10
Cross-sectional view of the equilaterally spaced conductors of a three-phase line.

4.10 INDUCTANCE OF THREE-PHASE LINES WITH EQUILATERAL SPACING

So far in our discussion we have considered only single-phase lines. The equations we have developed are quite easily adapted, however, to the calculation of the inductance of three-phase lines. Figure 4.10 shows the conductors of a three-phase line spaced at the corners of an equilateral triangle. If we assume that there is no neutral wire, or if we assume balanced three-phase phasor currents, $I_a + I_b + I_c = 0$. Equation (4.36) determines the flux linkages of conductor a:

$$\lambda_a = 2 \times 10^{-7}\left(I_a \ln \frac{1}{D_s} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D}\right) \text{ Wbt/m} \tag{4.48}$$

Since $I_a = -(I_b + I_c)$, Eq. (4.48) becomes

$$\lambda_a = 2 \times 10^{-7}\left(I_a \ln \frac{1}{D_s} - I_a \ln \frac{1}{D}\right) = 2 \times 10^{-7} I_a \ln \frac{D}{D_s} \text{ Wbt/m} \tag{4.49}$$

and $$L_a = 2 \times 10^{-7} \ln \frac{D}{D_s} \text{ H/m} \tag{4.50}$$

Equation (4.50) is the same in form as Eq. (4.25) for a single-phase line except that D_s replaces r'. Because of symmetry, the inductances of conductors b and c are the same as the inductance of conductor a. Since each phase consists of only one conductor, Eq. (4.50) gives the inductance per phase of the three-phase line.

4.11 INDUCTANCE OF THREE-PHASE LINES WITH UNSYMMETRICAL SPACING

When the conductors of a three-phase line are not spaced equilaterally, the problem of finding the inductance becomes more difficult. The flux linkages and inductance of each phase are not the same. A different inductance in each

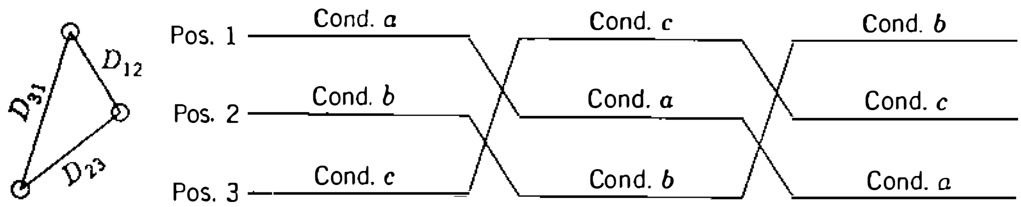

FIGURE 4.11
Transposition cycle.

phase results in an unbalanced circuit. Balance of the three phases can be restored by exchanging the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of conductor positions is called *transposition*. A complete transposition cycle is shown in Fig. 4.11. The phase conductors are designated a, b, and c, and the positions occupied are numbered 1, 2, and 3, respectively. Transposition results in each conductor having the same average inductance over the whole cycle.

Modern power lines are usually not transposed at regular intervals although an interchange in the positions of the conductors may be made at switching stations in order to balance the inductance of the phases more closely. Fortunately, the dissymmetry between the phases of an untransposed line is small and neglected in most calculations of inductance. If the dissymmetry is neglected, the inductance of the untransposed line is taken as equal to the average value of the inductive reactance of one phase of the same line correctly transposed. The derivations to follow are for transposed lines.

To find the average inductance of one conductor of a transposed line, we first determine the flux linkages of a conductor for each position it occupies in the transposition cycle and then determine the average flux linkages. Applying Eq. (4.36) to conductor a of Fig. 4.11 to find the phasor expression for the flux linkages of a in position 1 when b is in position 2 and c is in position 3, we obtain

$$\lambda_{a1} = 2 \times 10^{-7}\left(I_a \ln \frac{1}{D_s} + I_b \ln \frac{1}{D_{12}} + I_c \ln \frac{1}{D_{31}}\right) \text{Wbt/m} \tag{4.51}$$

With a in position 2, b in position 3, and c in position 1,

$$\lambda_{a2} = 2 \times 10^{-7}\left(I_a \ln \frac{1}{D_s} + I_b \ln \frac{1}{D_{23}} + I_c \ln \frac{1}{D_{12}}\right) \text{Wbt/m} \tag{4.52}$$

and, with a in position 3, b in position 1, and c in position 2,

$$\lambda_{a3} = 2 \times 10^{-7}\left(I_a \ln \frac{1}{D_s} + I_b \ln \frac{1}{D_{31}} + I_c \ln \frac{1}{D_{23}}\right) \text{Wbt/m} \tag{4.53}$$

The average value of the flux linkages of a is

$$\lambda_a = \frac{\lambda_{a1} + \lambda_{a2} + \lambda_{a3}}{3}$$

$$= \frac{2 \times 10^{-7}}{3}\left(3I_a \ln \frac{1}{D_s} + I_b \ln \frac{1}{D_{12}D_{23}D_{31}} + I_c \ln \frac{1}{D_{12}D_{23}D_{31}}\right) \quad (4.54)$$

With the restriction that $I_a = -(I_b + I_c)$,

$$\lambda_a = \frac{2 \times 10^{-7}}{3}\left(3I_a \ln \frac{1}{D_s} - I_a \ln \frac{1}{D_{12}D_{23}D_{31}}\right)$$

$$= 2 \times 10^{-7} I_a \ln \frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{D_s} \text{ Wbt/m} \quad (4.55)$$

and the *average* inductance per phase is

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \text{ H/m} \quad (4.56)$$

where

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} \quad (4.57)$$

and D_s is the GMR of the conductor. D_{eq}, the geometric mean of the three distances of the unsymmetrical line, is the equivalent equilateral spacing, as may be seen by a comparison of Eq. (4.56) with Eq. (4.50). We should note the similarity between all the equations for the inductance of a conductor. If the inductance is in henrys per meter, the factor 2×10^{-7} appears in all the equations, and the denominator of the logarithmic term is always the GMR of the conductor. The numerator is the distance between wires of a two-wire line, the mutual GMD between sides of a composite-conductor single-phase line, the distance between conductors of an equilaterally spaced line, or the equivalent equilateral spacing of an unsymmetrical line.

Example 4.4. A single-circuit three-phase line operated at 60 Hz is arranged, as shown in Fig. 4.12. The conductors are ACSR *Drake*. Find the inductive reactance per mile per phase.

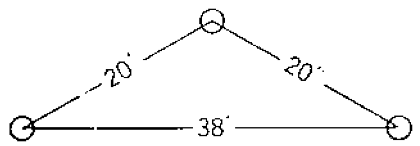

FIGURE 4.12
Arrangement of conductors for Example 4.4.

Solution. From Table A.3

$$D_s = 0.0373 \text{ ft} \qquad D_{eq} = \sqrt[3]{20 \times 20 \times 38} = 24.8 \text{ ft}$$

$$L = 2 \times 10^{-7} \ln \frac{24.8}{0.0373} = 13.00 \times 10^{-7} \text{ H/m}$$

$$X_L = 2\pi 60 \times 1609 \times 13.00 \times 10^{-7} = 0.788 \ \Omega/\text{mi per phase}$$

Equation (4.46) may be used also, or from Tables A.3 and A.4

$$X_a = 0.399$$

and by interpolation for 24.8 ft

$$X_d = 0.3896$$

$$X_L = 0.399 + 0.3896 = 0.7886 \ \Omega/\text{mi per phase}$$

4.12 INDUCTANCE CALCULATIONS FOR BUNDLED CONDUCTORS

At extra-high voltages (EHV), that is, voltages above 230 kV, corona with its resultant power loss and particularly its interference with communications is excessive if the circuit has only one conductor per phase. The high-voltage gradient at the conductor in the EHV range is reduced considerably by having two or more conductors per phase in close proximity compared with the spacing between phases. Such a line is said to be composed of *bundled* conductors. The bundle consists of two, three, or four conductors. Figure 4.13 shows the arrangements. The current will not divide exactly between the conductors of the bundle unless there is a transposition of the conductors within the bundle, but the difference is of no practical importance, and the GMD method is accurate for calculations.

Reduced reactance is the other equally important advantage of bundling. Increasing the number of conductors in a bundle reduces the effects of corona and reduces the reactance. The reduction of reactance results from the increased GMR of the bundle. The calculation of GMR is, of course, exactly the same as that of a stranded conductor. Each conductor of a two-conductor bundle, for instance, is treated as one strand of a two-strand conductor. If we let D_s^b indicate the GMR of a bundled conductor and D_s the GMR of the

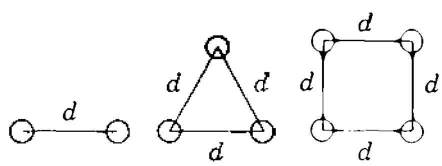

FIGURE 4.13
Bundle arrangements.

individual conductors composing the bundle, we find, referring to Fig. 4.13:

For a two-strand bundle

$$D_s^b = \sqrt[4]{(D_s \times d)^2} = \sqrt{D_s \times d} \tag{4.58}$$

For a three-strand bundle

$$D_s^b = \sqrt[9]{(D_s \times d \times d)^3} = \sqrt[3]{D_s \times d^2} \tag{4.59}$$

For a four-strand bundle

$$D_s^b = \sqrt[16]{\left(D_s \times d \times d \times \sqrt{2}\,d\right)^4} = 1.09\sqrt[4]{D_s \times d^3} \tag{4.60}$$

In computing inductance using Eq. (4.56), D_s^b of the bundle replaces D_s of a single conductor. To compute D_{eq}, the distance from the center of one bundle to the center of another bundle is sufficiently accurate for D_{ab}, D_{bc}, and D_{ca}. Obtaining the actual GMD between conductors of one bundle and those of another would be almost indistinguishable from the center-to-center distances for the usual spacing.

Example 4.5. Each conductor of the bundled-conductor line shown in Fig. 4.14 is ACSR, 1,272,000-cmil *Pheasant*. Find the inductive reactance in ohms per kilometer (and per mile) per phase for $d = 45$ cm. Also, find the per-unit series reactance of the line if its length is 160 km and the base is 100 MVA, 345 kV.

Solution. From Table A.3 $D_s = 0.0466$ ft, and we multiply feet by 0.3048 to convert to meters.

$$D_s^b = \sqrt{0.0466 \times 0.3048 \times 0.45} = 0.080 \text{ m}$$

$$D_{eq} = \sqrt[3]{8 \times 8 \times 16} = 10.08 \text{ m}$$

$$X_L = 2\pi 60 \times 2 \times 10^{-7} \times 10^3 \ln \frac{10.08}{0.08}$$

$$= 0.365\ \Omega/\text{km per phase}$$

$$= 0.365 \times 1.609 = 0.587\ \Omega/\text{mi per phase}$$

$$\text{Base } Z = \frac{(345)^2}{100} = 1190\ \Omega$$

$$X = \frac{0.365 \times 160}{1190} = 0.049 \text{ per unit}$$

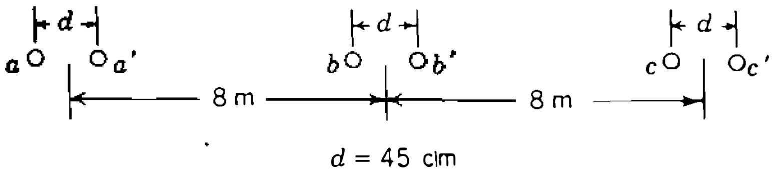

FIGURE 4.14
Spacing of conductors of a bundled-conductor line.

4.13 SUMMARY

Although computer programs are usually available or written rather easily for calculating inductance of all kinds of lines, some understanding of the development of the equations used is rewarding from the standpoint of appreciating the effect of variables in designing a line. However, tabulated values such as those in Tables A.3 and A.4 make the calculations quite simple except for parallel-circuit lines. Table A.3 also lists resistance.

The important equation for inductance per phase of single-circuit three-phase lines is given here for convenience:

$$L = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \text{ H/m per phase} \tag{4.61}$$

Inductive reactance in ohms per kilometer at 60 Hz is found by multiplying inductance in henrys per meter by $2\pi 60 \times 1000$:

$$X_L = 0.0754 \times \ln \frac{D_{eq}}{D_s} \ \Omega\text{/km per phase} \tag{4.62}$$

or

$$X_L = 0.1213 \times \ln \frac{D_{eq}}{D_s} \ \Omega\text{/mi per phase} \tag{4.63}$$

Both D_{eq} and D_s must be in the same units, usually feet. If the line has one conductor per phase, D_s is found directly from tables. For bundled conductors D_s^b, as defined in Sec. 4.12, is substituted for D_s. For both single-conductor and bundled-conductor lines

$$D_{eq} = \sqrt[3]{D_{ab} D_{bc} D_{ca}} \tag{4.64}$$

For bundled-conductor lines D_{ab}, D_{bc}, and D_{ca} are distances between the centers of the bundles of phases a, b, and c.

For lines with one conductor per phase it is convenient to determine X_L from tables by adding X_a for the conductor as found in Table A.3 to X_d as found in Table A.4 corresponding to D_{eq}.

PROBLEMS

4.1. The all-aluminum conductor (AAC) identified by the code word *Bluebell* is composed of 37 strands, each having a diameter of 0.1672 in. Tables of characteristics of AACs list an area of 1,033,500 cmil for this conductor (1 cmil = $(\pi/4) \times 10^{-6}$ in^2). Are these values consistent with each other? Find the overall area of the strands in square millimeters.

4.2. Determine the dc resistance in ohms per km of *Bluebell* at 20°C by Eq. (4.2) and the information in Prob. 4.1, and check the result against the value listed in tables of 0.01678 Ω per 1000 ft. Compute the dc resistance in ohms per kilometer at 50°C and compare the result with the ac 60-Hz resistance of 0.1024 Ω/mi listed in tables for this conductor at 50°C. Explain any difference in values. Assume that the increase in resistance due to spiraling is 2%.

4.3. An AAC is composed of 37 strands, each having a diameter of 0.333 cm. Compute the dc resistance in ohms per kilometer at 75°C. Assume that the increase in resistance due to spiraling is 2%.

4.4. The energy density (that is, the energy per unit volume) at a point in a magnetic field can be shown to be $B^2/2\mu$, where B is the flux density and μ is the permeability. Using this result and Eq. (4.10), show that the total magnetic field energy stored within a unit length of solid circular conductor carrying current I is given by $\mu I^2/16\pi$. Neglect skin effect, and thus verify Eq. (4.15).

4.5. The conductor of a single-phase 60-Hz line is a solid round aluminum wire having a diameter of 0.412 cm. The conductor spacing is 3 m. Determine the inductance of the line in millihenrys per mile. How much of the inductance is due to internal flux linkages? Assume skin effect is negligible.

4.6. A single-phase 60-Hz overhead power line is symmetrically supported on a horizontal crossarm. Spacing between the centers of the conductors (say, a and b) is 2.5 m. A telephone line is also symmetrically supported on a horizontal crossarm 1.8 m directly below the power line. Spacing between the centers of these conductors (say, c and d) is 1.0 m.

(*a*) Using Eq. (4.36), show that the mutual inductance per unit length between circuit a-b and circuit c-d is given by

$$4 \times 10^{-7} \ln \sqrt{\frac{D_{ad} D_{bc}}{D_{ac} D_{bd}}} \quad \text{H/m}$$

where, for example, D_{ad} denotes the distance in meters between conductors a and d.

(*b*) Hence, compute the mutual inductance per kilometer between the power line and the telephone line.

(*c*) Find the 60-Hz voltage per kilometer induced in the telephone line when the power line carries 150 A.

4.7. If the power line and the telephone line described in Prob. 4.6 are in the same horizontal plane and the distance between the nearest conductors of the two lines is 18 m, use the result of Prob. 4.6(*a*) to find the mutual inductance between the power and telephone circuits. Also, find the 60-Hz voltage per kilometer induced in the telephone line when 150 A flows in the power line.

4.8. Find the GMR of a three-strand conductor in terms of r of an individual strand.

4.9. Find the GMR of each of the unconventional conductors shown in Fig. 4.15 in terms of the radius r of an individual strand.

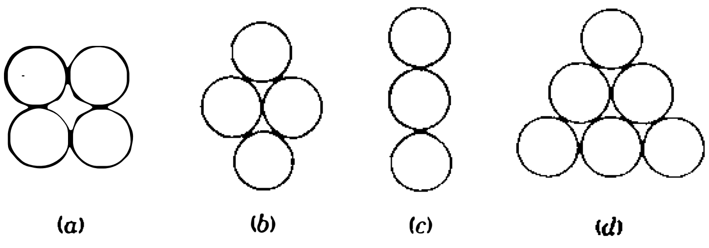

FIGURE 4.15
Cross-sectional view of unconventional conductors for Prob. 4.9.

4.10. The distance between conductors of a single-phase line is 10 ft. Each of its conductors is composed of six strands symmetrically placed around one center strand so that there are seven equal strands. The diameter of each strand is 0.1 in. Show that D_s of each conductor is 2.177 times the radius of each strand. Find the inductance of the line in mH/mi.

4.11. Solve Example 4.2 for the case where side Y of the single-phase line is identical to side X and the two sides are 9 m apart, as shown in Fig. 4.9.

4.12. Find the inductive reactance of ACSR *Rail* in ohms per kilometer at 1-m spacing.

4.13. Which conductor listed in Table A.3 has an inductive reactance at 7-ft spacing of 0.651 Ω/mi?

4.14. A three-phase line has three equilaterally spaced conductors of ACSR *Dove*. If the conductors are 10 ft apart, determine the 60-Hz per-phase reactance of the line in Ω/km.

4.15. A three-phase line is designed with equilateral spacing of 16 ft. It is decided to build the line with horizontal spacing ($D_{13} = 2D_{12} = 2D_{23}$). The conductors are transposed. What should be the spacing between adjacent conductors in order to obtain the same inductance as in the original design?

4.16. A three-phase 60-Hz transmission line has its conductors arranged in a triangular formation so that two of the distances between conductors are 25 ft and the third distance is 42 ft. The conductors are ACSR *Osprey*. Determine the inductance and inductive reactance per phase per mile.

4.17. A three-phase 60-Hz line has flat horizontal spacing. The conductors have a GMR of 0.0133 m with 10 m between adjacent conductors. Determine the inductive reactance per phase in ohms per kilometer. What is the name of this conductor?

4.18. For short transmission lines if resistance is neglected, the maximum power which can be transmitted per phase is equal to

$$\frac{|V_S| \times |V_R|}{|X|}$$

where V_S and V_R are the line-to-neutral voltages at the sending and receiving ends of the line and X is the inductive reactance of the line. This relationship will become apparent in the study of Chap. 6. If the magnitudes of V_S and V_R are held constant, and if the cost of a conductor is proportional to its cross-sectional area, find the conductor in Table A.3 which has the maximum power-handling capacity per cost of conductor at a given geometric mean spacing.

4.19. A three-phase underground distribution line is operated at 23 kV. The three conductors are insulated with 0.5-cm solid black polyethylene insulation and lie flat, side by side, directly next to each other in a dirt trench. The conductor is circular in cross section and has 33 strands of aluminum. The diameter of the conductor is 1.46 cm. The manufacturer gives the GMR as 0.561 cm and the cross section of the conductor as 1.267 cm^2. The thermal rating of the line buried in normal soil whose maximum temperature is 30°C is 350 A. Find the dc and ac resistance at 50°C and the inductive reactance in ohms per kilometer. To decide whether to consider skin effect in calculating resistance, determine the percent skin effect at 50°C in the ACSR conductor of the size nearest that of the underground conductor. Note that the series impedance of the distribution line is dominated by R rather than X_L because of the very low inductance due to the close spacing of the conductors.

4.20. The single-phase power line of Prob. 4.6 is replaced by a three-phase line on a horizontal crossarm in the same position as that of the original single-phase line. Spacing of the conductors of the power line is $D_{13} = 2D_{12} = 2D_{23}$, and equivalent equilateral spacing is 3 m. The telephone line remains in the position described in Prob. 4.6. If the current in the power line is 150 A, find the voltage per kilometer induced in the telephone line. Discuss the phase relation of the induced voltage with respect to the power-line current.

4.21. A 60-Hz three-phase line composed of one ACSR *Bluejay* conductor per phase has flat horizontal spacing of 11 m between adjacent conductors. Compare the inductive reactance in ohms per kilometer per phase of this line with that of a line using a two-conductor bundle of ACSR 26/7 conductors having the same total cross-sectional area of aluminum as the single-conductor line and 11-m spacing measured from the center of the bundles. The spacing between conductors in the bundle is 40 cm.

4.22. Calculate the inductive reactance in ohms per kilometer of a bundled 60-Hz three-phase line having three ACSR *Rail* conductors per bundle with 45 cm between conductors of the bundle. The spacing between bundle centers is 9, 9, and 18 m.