CHAPTER
5

CAPACITANCE OF TRANSMISSION LINES

As we discussed briefly at the beginning of Chap. 4, the shunt admittance of a transmission line consists of conductance and capacitive reactance. We have also mentioned that conductance is usually neglected because its contribution to shunt admittance is very small. For this reason this chapter has been given the title of capacitance rather than shunt admittance.

Capacitance of a transmission line is the result of the potential difference between the conductors; it causes them to be charged in the same manner as the plates of a capacitor when there is a potential difference between them. The capacitance between conductors is the charge per unit of potential difference. Capacitance between parallel conductors is a constant depending on the size and spacing of the conductors. For power lines less than about 80 km (50 mi) long, the effect of capacitance can be slight and is often neglected. For longer lines of higher voltage capacitance becomes increasingly important.

An alternating voltage impressed on a transmission line causes the charge on the conductors at any point to increase and decrease with the increase and decrease of the instantaneous value of the voltage between conductors at the point. The flow of charge is current, and the current caused by the alternate charging and discharging of a line due to an alternating voltage is called the ***charging current*** of the line. Since capacitance is a shunt between conductors, charging current flows in a transmission line even when it is open-circuited. It affects the voltage drop along the lines as well as efficiency and power factor of the line and the stability of the system of which the line is a part.

The basis of our analysis of capacitance is Gauss's law for electric fields. The law states that the total electric charge within a closed surface equals the total electric flux emerging from the surface. In other words, the total charge within the closed surface equals the integral over the surface of the normal component of the electric flux density.

The lines of electric flux originate on positive charges and terminate on negative charges. Charge density normal to a surface is designated D_f and equals kE, where k is the permittivity of the material surrounding the surface and E is the electric field intensity.[1]

5.1 ELECTRIC FIELD OF A LONG, STRAIGHT CONDUCTOR

If a long, straight cylindrical conductor lies in a uniform medium such as air and is isolated from other charges so that the charge is uniformly distributed around its periphery, the flux is radial. All points equidistant from such a conductor are points of equipotential and have the same electric flux density. Figure 5.1 shows such an isolated conductor. The electric flux density at x meters from the conductor can be computed by imagining a cylindrical surface concentric with the conductor and x meters in radius. Since all parts of the surface are equidistant from the conductor, the cylindrical surface is a surface of equipotential and the electric flux density on the surface is equal to the flux leaving the conductor per meter of length divided by the area of the surface in an axial length of 1 m. The electric flux density is

$$D_f = \frac{q}{2\pi x} \text{ C/m}^2 \tag{5.1}$$

where q is the charge on the conductor in coulombs per meter of length and x is the distance in meters from the conductor to the point where the electric flux density is computed. The electric field intensity, or the negative of the potential gradient, is equal to the electric flux density divided by the permittivity of the medium. Therefore, the electric field intensity is

$$E = \frac{q}{2\pi xk} \text{ V/m} \tag{5.2}$$

E and q both may be instantaneous, phasor, or dc expressions.

---

[1] In SI units the permittivity of free space k_0 is 8.85×10^{-12} F/m (farads per meter). Relative permittivity k_r is the ratio of the actual permittivity k of a material of the permittivity of free space. Thus, $k_r = k/k_0$. For dry air k_r is 1.00054 and is assumed equal to 1.0 in calculations for overhead lines.

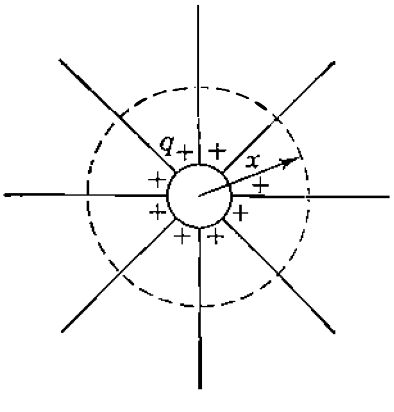


FIGURE 5.1
Lines of electric flux originating on the positive charges uniformly distributed over the surface of an isolated cylindrical conductor.

5.2 THE POTENTIAL DIFFERENCE BETWEEN TWO POINTS DUE TO A CHARGE

The potential difference between two points in volts is numerically equal to the work in joules per coulomb necessary to move a coulomb of charge between the two points. The electric field intensity is a measure of the force on a charge in the field. The electric field intensity in volts per meter is equal to the force in newtons per coulomb on a coulomb of charge at the point considered. Between two points the line integral of the force in newtons acting on a coulomb of positive charge is the work done in moving the charge from the point of lower potential to the point of higher potential and is numerically equal to the potential difference between the two points.

Consider a long, straight wire carrying a positive charge of q C/m, as shown in Fig. 5.2. Points P_1 and P_2 are located at distances D_1 and D_2 meters, respectively, from the center of the wire. The wire is an equipotential surface and the uniformly distributed charge on the wire is equivalent to a charge concentrated at the center of the wire for calculating flux external to the wire. The positive charge on the wire will exert a repelling force on a positive charge placed in the field. For this reason and because D_2 in this case is greater than

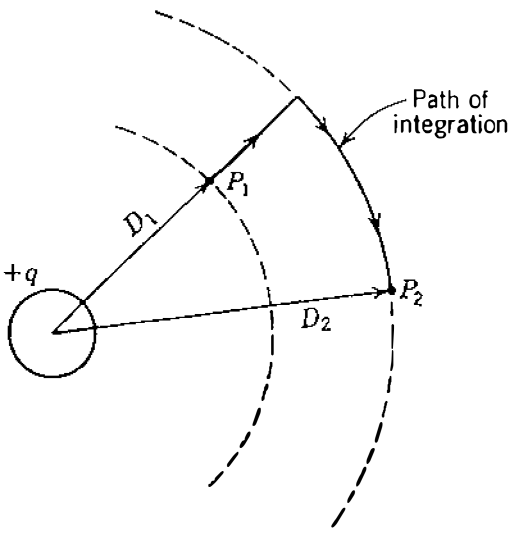


FIGURE 5.2
Path of integration between two points external to a cylindrical conductor having a uniformly distributed positive charge.

D_1, work must be done on a positive charge to move it from P_2 to P_1, and P_1 is at a higher potential than P_2. The difference in potential is the amount of work done per coulomb of charge moved. On the other hand, if the one coulomb of charge moves from P_1 to P_2, it expends energy, and the amount of work, or energy, in newton-meters is the voltage *drop* from P_1 to P_2. The potential difference is independent of the path followed. The simplest way to compute the voltage drop between two points is to compute the voltage between the equipotential surfaces passing through P_1 and P_2 by integrating the field intensity over a *radial* path between the equipotential surfaces. Thus, the instantaneous voltage drop between P_1 and P_2 is

$$v_{12} = \int_{D_1}^{D_2} E\,dx = \int_{D_1}^{D_2} \frac{q}{2\pi kx}\,dx = \frac{q}{2\pi k} \ln \frac{D_2}{D_1} \text{ V} \tag{5.3}$$

where q is the instantaneous charge on the wire in coulombs per meter of length. Note that the voltage drop between two points, as given by Eq. (5.3), may be positive or negative depending on whether the charge causing the potential difference is positive or negative and on whether the voltage drop is computed from a point near the conductor to a point farther away, or vice versa. The sign of q may be either positive or negative, and the logarithmic term is either positive or negative depending on whether D_2 is greater or less than D_1.

5.3 CAPACITANCE OF A TWO-WIRE LINE

Capacitance between the conductors of a two-wire line is defined as the charge on the conductors per unit of potential difference between them. In the form of an equation capacitance per unit length of the line is

$$C = \frac{q}{v} \text{ F/m} \tag{5.4}$$

where q is the charge on the line in coulombs per meter and v is the potential difference between the conductors in volts. Hereafter, for convenience, we refer to *capacitance per unit length* as *capacitance* and indicate the correct dimensions for the equations derived. The capacitance between two conductors can be found by substituting in Eq. (5.4) the expression for v in terms of q from Eq. (5.3). The voltage v_{ab} between the two conductors of the two-wire line shown in Fig. 5.3 can be found by determining the potential difference between the two conductors of the line, first by computing the voltage drop due to the charge q_a on conductor a and then by computing the voltage drop due to the charge q_b on conductor b. By the principle of superposition the voltage drop from conductor a to conductor b due to the charges on both conductors is the sum of the voltage drops caused by each charge alone.

The charge q_a on conductor a of Fig. 5.3 causes surfaces of equipotential in the vicinity of conductor b, which are shown in Fig. 5.4. We avoid the

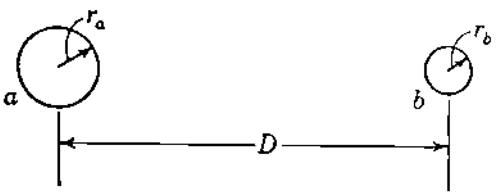


FIGURE 5.3
Cross section of a parallel-wire line.

distorted equipotential surfaces by integrating Eq. (5.3) along the alternate rather than the direct path of Fig. 5.4. In determining v_{ab} due to q_a, we follow the path through the undistorted region and see that distance D_1 of Eq. (5.3) is the radius r_a of conductor a and distance D_2 is the center-to-center distance between conductors a and b. Similarly, in determining v_{ab} due to q_b, we find that the distances D_2 and D_1 are r_b and D, respectively. Converting to phasor notation (q_a and q_b become phasors), we obtain

$$V_{ab} = \underbrace{\frac{q_a}{2\pi k} \ln \frac{D}{r_a}}_{\text{due to } q_a} + \underbrace{\frac{q_b}{2\pi k} \ln \frac{r_b}{D}}_{\text{due to } q_b} \text{ V} \tag{5.5}$$

and since $q_a = -q_b$ for a two-wire line,

$$V_{ab} = \frac{q_a}{2\pi k} \left(\ln \frac{D}{r_a} - \ln \frac{r_b}{D} \right) \text{ V} \tag{5.6}$$

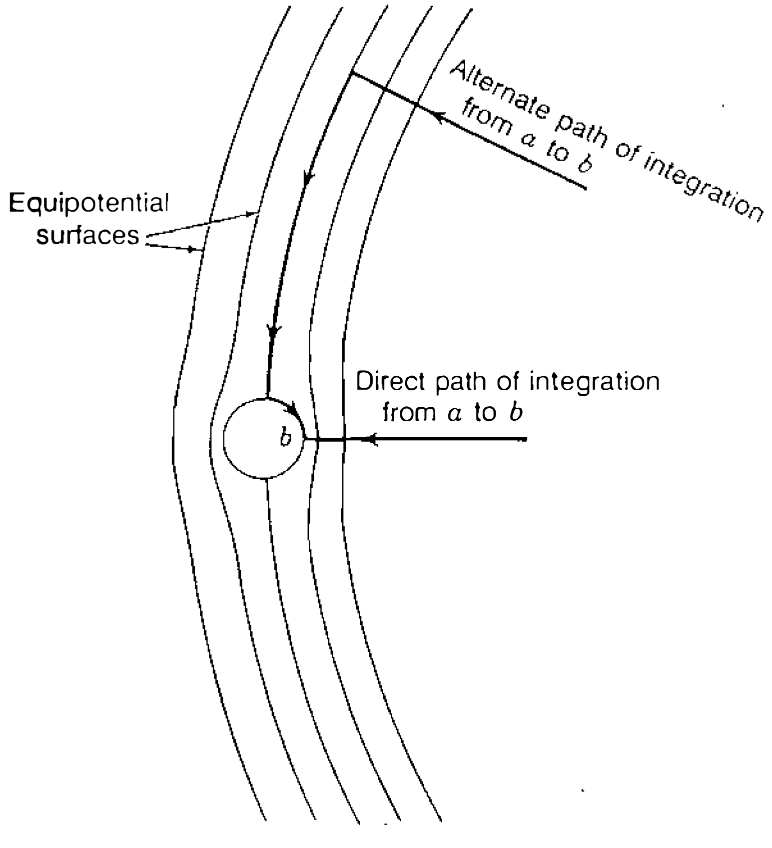


FIGURE 5.4
Equipotential surfaces of a portion of the electric field caused by a charged conductor a (not shown). Conductor b causes the equipotential surfaces to become distorted. Arrows indicate optional paths of integration between a point on the equipotential surface of conductor b and the conductor a, whose charge q_a creates the equipotential surfaces shown.

or by combining the logarithmic terms, we obtain

$$V_{ab} = \frac{q_a}{2\pi k} \ln \frac{D^2}{r_a r_b} \text{ V} \tag{5.7}$$

The capacitance between conductors is

$$C_{ab} = \frac{q_a}{V_{ab}} = \frac{2\pi k}{\ln(D^2/r_a r_b)} \text{ F/m} \tag{5.8}$$

If $r_a = r_b = r$,

$$C_{ab} = \frac{\pi k}{\ln(D/r)} \text{ F/m} \tag{5.9}$$

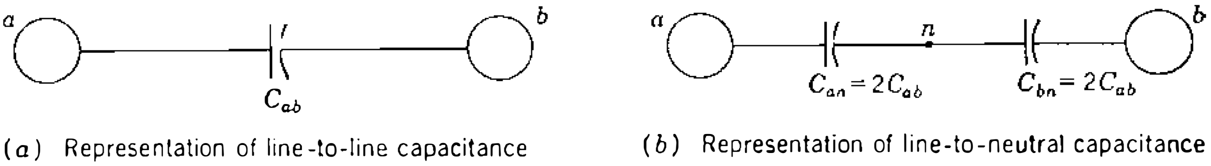
Equation (5.9) gives the capacitance between the conductors of a two-wire line. If the line is supplied by a transformer having a grounded center tap, the potential difference between each conductor and ground is half the potential difference between the two conductors and the *capacitance to ground*, or *capacitance to neutral*, is

$$C_n = C_{an} = C_{bn} = \frac{q_a}{V_{ab}/2} = \frac{2\pi k}{\ln(D/r)} \text{ F/m to neutral} \tag{5.10}$$

The concept of capacitance to neutral is illustrated in Fig. 5.5.

Equation (5.10) corresponds to Eq. (4.25) for inductance. One difference between the equations for capacitance and inductance should be noted carefully. The radius in the equation for capacitance is the *actual outside radius* of the conductor and not the geometric mean ratio (GMR) of the conductor, as in the inductance formula.

Equation (5.3), from which Eqs. (5.5) through (5.10) were derived, is based on the assumption of uniform charge distribution over the surface of the conductor. When other charges are present, the distribution of charge on the surface of the conductor is not uniform and the equations derived from Eq. (5.3) are not strictly correct. The nonuniformity of charge distribution, however, can



(a) Representation of line-to-line capacitance (b) Representation of line-to-neutral capacitance

FIGURE 5.5
Relationship between the concepts of line-to-line capacitance and line-to-neutral capacitance.

be neglected entirely in overhead lines since the error in Eq. (5.10) is only 0.01%, even for such a close spacing as that where the ratio $D/r = 50$.

A question arises about the value to be used in the denominator of the argument of the logarithm in Eq. (5.10) when the conductor is a stranded cable because the equation was derived for a solid round conductor. Since electric flux is perpendicular to the surface of a perfect conductor, the electric field at the surface of a stranded conductor is not the same as the field at the surface of a cylindrical conductor. Therefore, the capacitance calculated for a stranded conductor by substituting the outside radius of the conductor for r in Eq. (5.10) will be slightly in error because of the difference between the field in the neighborhood of such a conductor and the field near a solid conductor for which Eq. (5.10) was derived. The error is very small, however, since only the field very close to the surface of the conductor is affected. The outside radius of the stranded conductor is used in calculating the capacitance.

After the capacitance to neutral has been determined, the capacitive reactance existing between one conductor and neutral for relative permittivity $k_r = 1$ is found by using the expression for C given in Eq. (5.10) to yield

$$X_C = \frac{1}{2\pi f C} = \frac{2.862}{f} \times 10^9 \ln \frac{D}{r} \; \Omega \cdot \text{m to neutral} \qquad (5.11)$$

Since C in Eq. (5.11) is in farads per meter, the proper units for X_C must be ohm-meters. We should also note that Eq. (5.11) expresses the reactance from line to neutral for 1 m of line. Since capacitance reactance is in parallel along the line, X_C in ohm-meters must be *divided* by the length of the line in meters to obtain the capacitive reactance in ohms to neutral for the entire length of the line.

When Eq. (5.11) is divided by 1609 to convert to ohm-miles, we obtain

$$X_C = \frac{1.779}{f} \times 10^6 \ln \frac{D}{r} \; \Omega \cdot \text{mi to neutral} \qquad (5.12)$$

Table A.3 lists the outside diameters of the most widely used sizes of ACSR. If D and r in Eq. (5.12) are in feet, *capacitive reactance at 1-ft spacing* X'_a is the first term and *capacitive reactance spacing factor* X'_d is the second term when the equation is expanded as follows:

$$X_C = \frac{1.779}{f} \times 10^6 \ln \frac{1}{r} + \frac{1.779}{f} \times 10^6 \ln D \; \Omega \cdot \text{mi to neutral} \qquad (5.13)$$

Table A.3 includes values of X'_a for common sizes of ACSR, and similar tables are readily available for other types and sizes of conductors. Table A.5 in the Appendix lists values of X'_d which, of course, is different from the synchronous machine transient reactance bearing the same symbol.

Example 5.1. Find the capacitive susceptance per mile of a single-phase line operating at 60 Hz. The conductor is *Partridge*, and spacing is 20 ft between centers.

Solution. For this conductor Table A.3 lists an outside diameter of 0.642 in, and so

$$r = \frac{0.642}{2 \times 12} = 0.0268 \text{ ft}$$

and from Eq. (5.12)

$$X_C = \frac{1.779}{60} \times 10^6 \ln \frac{20}{0.0268} = 0.1961 \times 10^6 \ \Omega \cdot \text{mi to neutral}$$

$$B_C = \frac{1}{X_C} = 5.10 \times 10^{-6} \text{ S/mi to neutral}$$

or in terms of capacitive reactance at 1-ft spacing and capacitive reactance spacing factor from Tables A.3 and A.5

$$X'_a = 0.1074 \text{ M}\Omega \cdot \text{mi}$$

$$X'_d = 0.0889 \text{ M}\Omega \cdot \text{mi}$$

$$X'_C = 0.1074 + 0.0889 = 0.1963 \text{ M}\Omega \cdot \text{mi per conductor}$$

Line-to-line capacitive reactance and susceptance are

$$X_C = 2 \times 0.1963 \times 10^6 = 0.3926 \times 10^6 \ \Omega \cdot \text{mi}$$

$$B_C = \frac{1}{X_C} = 2.55 \times 10^{-6} \text{ S/mi}$$

5.4 CAPACITANCE OF A THREE-PHASE LINE WITH EQUILATERAL SPACING

The three identical conductors of radius r of a three-phase line with equilateral spacing are shown in Fig. 5.6. Equation (5.5) expresses the voltage between two conductors due to the charges on each one if the charge distribution on the conductors can be assumed to be uniform. Thus, the voltage V_{ab} of the three-phase line due only to the charges on conductors a and b is

$$V_{ab} = \underbrace{\frac{1}{2\pi k}\left(q_a \ln \frac{D}{r} + q_b \ln \frac{r}{D}\right)}_{\text{due to } q_a \text{ and } q_b} \text{ V} \tag{5.14}$$

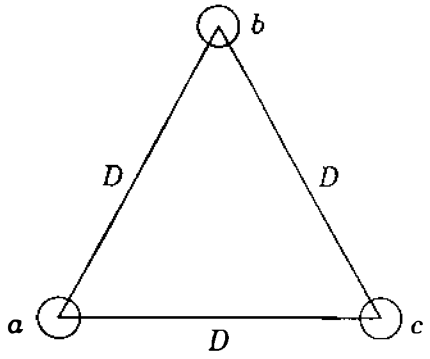


FIGURE 5.6
Cross section of a three-phase line with equilateral spacing.

Equation (5.3) enables us to include the effect of q_c since uniform charge distribution over the surface of a conductor is equivalent to a concentrated charge at the center of the conductor. Therefore, due only to the charge q_c,

$$V_{ab} = \frac{q_c}{2\pi k} \ln \frac{D}{D} \text{ V}$$

which is zero since q_c is equidistant from a and b. However, to show that we are considering all three charges, we can write

$$V_{ab} = \frac{1}{2\pi k}\left(q_a \ln \frac{D}{r} + q_b \ln \frac{r}{D} + q_c \ln \frac{D}{D}\right) \text{ V} \tag{5.15}$$

$$V_{ac} = \frac{1}{2\pi k}\left(q_a \ln \frac{D}{r} + q_b \ln \frac{D}{D} + q_c \ln \frac{r}{D}\right) \text{ V} \tag{5.16}$$

Adding Eqs. (5.15) and (5.16) gives

$$V_{ab} + V_{ac} = \frac{1}{2\pi k}\left[2q_a \ln \frac{D}{r} + (q_b + q_c)\ln \frac{r}{D}\right] \text{ V} \tag{5.17}$$

In deriving these equations, we have assumed that ground is far enough away to have negligible effect. Since the voltages are assumed to be sinusoidal and expressed as phasors, the charges are sinusoidal and expressed as phasors. If there are no other charges in the vicinity, the sum of the charges on the three conductors is zero and we can substitute $-q_a$ in Eq. (5.17) for $q_b + q_c$ and obtain

$$V_{ab} + V_{ac} = \frac{3q_a}{2\pi k} \ln \frac{D}{r} \text{ V} \tag{5.18}$$

Figure 5.7 is the phasor diagram of voltages. From this figure we obtain the following relations between the line voltages V_{ab} and V_{ac} and the voltage V_{an}

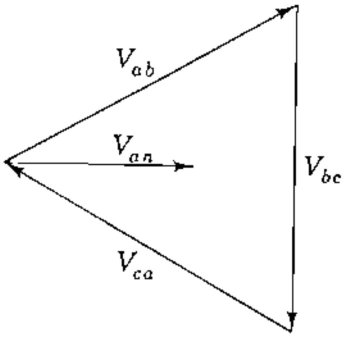


FIGURE 5.7
Phasor diagram of the balanced voltages of a three-phase line.

from line a to the neutral of the three-phase circuit:

$$V_{ab} = \sqrt{3}\,V_{an}\angle 30^\circ = \sqrt{3}\,V_{an}(0.866 + j0.5) \tag{5.19}$$

$$V_{ac} = -V_{ca} = \sqrt{3}\,V_{an}\angle -30^\circ = \sqrt{3}\,V_{an}(0.866 - j0.5) \tag{5.20}$$

Adding Eqs. (5.19) and (5.20) gives

$$V_{ab} + V_{ac} = 3V_{an} \tag{5.21}$$

Substituting $3V_{an}$ for $V_{ab} + V_{ac}$ in Eq. (5.18), we obtain

$$V_{an} = \frac{q_a}{2\pi k} \ln \frac{D}{r} \text{ V} \tag{5.22}$$

Since capacitance to neutral is the ratio of the charge on a conductor to the voltage between that conductor and neutral,

$$C_n = \frac{q_a}{V_{an}} = \frac{2\pi k}{\ln(D/r)} \text{ F/m to neutral} \tag{5.23}$$

Comparison of Eqs. (5.23) and (5.10) shows that the two are identical. These equations express the capacitance to neutral for single-phase and equilaterally spaced three-phase lines, respectively. Similarly, we recall that the equations for inductance per conductor are the same for single-phase and equilaterally spaced three-phase lines.

The term *charging current* is applied to the current associated with the capacitance of a line. For a *single-phase* circuit the charging current is the product of the line-to-line voltage and the line-to-line susceptance, or as a phasor,

$$I_{\text{chg}} = j\omega C_{ab} V_{ab} \tag{5.24}$$

For a three-phase line the charging current is found by multiplying the voltage to neutral by the capacitive susceptance to neutral. This gives the charging current per phase and is in accord with the calculation of balanced three-phase circuits on the basis of a single phase with neutral return. The phasor charging current in phase a is

$$I_{\text{chg}} = j\omega C_n V_{an} \text{ A/mi} \tag{5.25}$$

Since the rms voltage varies along the line, the charging current is not the same everywhere. Often the voltage used to obtain a value for charging current is the normal voltage for which the line is designed, such as 220 or 500 kV, which is probably not the actual voltage at either a generating station or a load.

5.5 CAPACITANCE OF A THREE-PHASE LINE WITH UNSYMMETRICAL SPACING

When the conductors of a three-phase line are not equilaterally spaced, the problem of calculating capacitance becomes more difficult. In the usual untransposed line the capacitances of each phase to neutral are unequal. In a transposed line the average capacitance to neutral of any phase for the complete transposition cycle is the same as the average capacitance to neutral of any other phase since each phase conductor occupies the same position as every other phase conductor over an equal distance along the transposition cycle. The dissymmetry of the untransposed line is slight for the usual configuration, and capacitance calculations are carried out as though all lines were transposed.

For the line shown in Fig. 5.8 three equations are found for V_{ab} for the three different parts of the transposition cycle. With phase a in position 1, b in position 2, and c in position 3,

$$V_{ab} = \frac{1}{2\pi k}\left(q_a \ln\frac{D_{12}}{r} + q_b \ln\frac{r}{D_{12}} + q_c \ln\frac{D_{23}}{D_{31}}\right) \text{V} \tag{5.26}$$

With phase a in position 2, b in position 3, and c in position 1,

$$V_{ab} = \frac{1}{2\pi k}\left(q_a \ln\frac{D_{23}}{r} + q_b \ln\frac{r}{D_{23}} + q_c \ln\frac{D_{31}}{D_{12}}\right) \text{V} \tag{5.27}$$

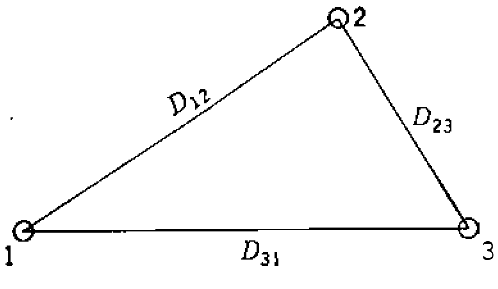


FIGURE 5.8
Cross section of a three-phase line with unsymmetrical spacing.

and with a in position 3, b in position 1, and c in position 2,

$$V_{ab} = \frac{1}{2\pi k}\left(q_a \ln \frac{D_{31}}{r} + q_b \ln \frac{r}{D_{31}} + q_c \ln \frac{D_{12}}{D_{23}}\right) \text{ V} \tag{5.28}$$

Equations (5.26) through (5.28) are similar to Eqs. (4.51) through (4.53) for the magnetic flux linkages of one conductor of a transposed line. However, in the equations for magnetic flux linkages we note that the current in any phase is the same in every part of the transposition cycle. In Eqs. (5.26) through (5.28), if we disregard the voltage drop along the line, the voltage to neutral of a phase in one part of a transposition cycle is equal to the voltage to neutral of that phase in any part of the cycle. Hence, the voltage between any two conductors is the same in all parts of the transposition cycle. It follows that the charge on a conductor must be different when the position of the conductor changes with respect to other conductors. A treatment of Eqs. (5.26) through (5.28) analogous to that of Eqs. (4.51) through (4.53) is not rigorous.

The rigorous solution for capacitances is too involved to be practical except perhaps for flat spacing with equal distances between adjacent conductors. With the usual spacings and conductors, sufficient accuracy is obtained by assuming that the charge per unit length on a conductor is the same in every part of the transposition cycle. When the above assumption is made with regard to charge, the voltage between a pair of conductors is different for each part of the transposition cycle. Then an average value of voltage between the conductors can be found and the capacitance calculated from the average voltage. We obtain the average voltage by adding Eqs. (5.26) through (5.28) and by dividing the result by 3. The average voltage between conductors a and b, assuming the same charge on a conductor regardless of its position in the transposition cycle, is

$$V_{ab} = \frac{1}{6\pi k}\left(q_a \ln \frac{D_{12}D_{23}D_{31}}{r^3} + q_b \ln \frac{r^3}{D_{12}D_{23}D_{31}} + q_c \ln \frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}\right)$$

$$= \frac{1}{2\pi k}\left(q_a \ln \frac{D_{eq}}{r} + q_b \ln \frac{r}{D_{eq}}\right) \tag{5.29}$$

where

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} \tag{5.30}$$

Similarly, the average voltage drop from conductor a to conductor c is

$$V_{ac} = \frac{1}{2\pi k}\left(q_a \ln \frac{D_{eq}}{r} + q_c \ln \frac{r}{D_{eq}}\right) \text{ V} \tag{5.31}$$

Applying Eq. (5.21) to find the voltage to neutral, we have

$$3V_{an} = V_{ab} + V_{ac} = \frac{1}{2\pi k}\left(2q_a \ln\frac{D_{eq}}{r} + q_b \ln\frac{r}{D_{eq}} + q_c \ln\frac{r}{D_{eq}}\right) \text{ V} \quad (5.32)$$

Since $q_a + q_b + q_c = 0$,

$$3V_{an} = \frac{3}{2\pi k} q_a \ln\frac{D_{eq}}{r} \text{ V} \quad (5.33)$$

and

$$C_n = \frac{q_a}{V_{an}} = \frac{2\pi k}{\ln(D_{eq}/r)} \text{ F/m to neutral} \quad (5.34)$$

Equation (5.34) for capacitance to neutral of a transposed three-phase line corresponds to Eq. (4.56) for the inductance per phase of a similar line. In finding capacitive reactance to neutral corresponding to C_n, we can split the reactance into components of capacitive reactance to neutral at 1-ft spacing X'_a and capacitive reactance spacing factor X'_d, as defined by Eq. (5.13).

Example 5.2. Find the capacitance and the capacitive reactance for 1 mi of the line described in Example 4.4. If the length of the line is 175 mi and the normal operating voltage is 220 kV, find capacitive reactance to neutral for the entire length of the line, the charging current per mile, and the total charging megavoltamperes.

Solution

$$r = \frac{1.108}{2 \times 12} = 0.0462 \text{ ft}$$

$$D_{eq} = 24.8 \text{ ft}$$

$$C_n = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln(24.8/0.0462)} = 8.8466 \times 10^{-12} \text{ F/m}$$

$$X_C = \frac{10^{12}}{2\pi \times 60 \times 8.8466 \times 1609} = 0.1864 \times 10^6 \ \Omega \cdot \text{mi}$$

or from tables

$$X'_a = 0.0912 \times 10^6 \qquad X'_d = 0.0953 \times 10^6$$

$$X_C = (0.0912 + 0.0953) \times 10^6 = 0.1865 \times 10^6 \ \Omega \cdot \text{mi to neutral}$$

For a length of 175 mi

$$\text{Capacitive reactance} = \frac{0.1865 \times 10^6}{175} = 1066\ \Omega \text{ to neutral}$$

$$|I_{\text{chg}}| = \frac{220{,}000}{\sqrt{3}} \frac{1}{X_C} = \frac{220{,}000 \times 10^{-6}}{\sqrt{3} \times 0.1865} = 0.681 \text{ A/mi}$$

or 0.681 × 175 = 119 A for the line. Reactive power is $Q = \sqrt{3} \times 220 \times 119 \times 10^{-3} = 43.5$ Mvar. This amount of reactive power absorbed by the distributed capacitance is negative in keeping with the convention discussed in Chap. 1. In other words, positive reactive power is being *generated* by the distributed capacitance of the line.

5.6 EFFECT OF EARTH ON THE CAPACITANCE OF THREE-PHASE TRANSMISSION LINES

Earth affects the capacitance of a transmission line because its presence alters the electric field of the line. If we assume that the earth is a perfect conductor in the form of a horizontal plane of infinite extent, we realize that the electric field of charged conductors above the earth is not the same as it would be if the equipotential surface of the earth were not present. The electric field of the charged conductors is forced to conform to the presence of the earth's surface. The assumption of a flat, equipotential surface is, of course, limited by the irregularity of terrain and the type of surface of the earth. The assumption enables us, however, to understand the effect of a conducting earth on capacitance calculations.

Consider a circuit consisting of a single overhead conductor with a return path through the earth. In charging the conductor, charges come from the earth to reside on the conductor, and a potential difference exists between the conductor and the earth. The earth has a charge equal in magnitude to that on the conductor but of opposite sign. The electric flux from the charges on the conductor to the charges on the earth is perpendicular to the earth's equipotential surface since the surface is assumed to be a perfect conductor. Let us imagine a fictitious conductor of the same size and shape as the overhead conductor lying directly below the original conductor at a distance equal to twice the distance of the conductor above the plane of the ground. The fictitious conductor is below the surface of the earth by a distance equal to the distance of the overhead conductor above the earth. If the earth is removed and a charge equal and opposite to that on the overhead conductor is assumed on the fictitious conductor, the plane midway between the original conductor and the fictitious conductor is an equipotential surface and occupies the same position as the equipotential surface of the earth. The electric flux between the overhead conductor and this equipotential surface is the same as that which existed between the conductor and the earth. Thus, for purposes of calculation of capacitance the earth may be replaced by a fictitious charged conductor below the surface of the earth by a distance equal to that of the overhead conductor above the earth. Such a conductor has a charge equal in magnitude and opposite in sign to that of the original conductor and is called the *image conductor*.

The method of calculating capacitance by replacing the earth by the image of an overhead conductor can be extended to more than one conductor. If we locate an image conductor for each overhead conductor, the flux between the original conductors and their images is perpendicular to the plane which replaces the earth, and that plane is an equipotential surface. The flux above the plane is the same as it is when the earth is present instead of the image conductors.

To apply the method of images to the calculation of capacitance for a three-phase line, refer to Fig. 5.9. We assume that the line is transposed and

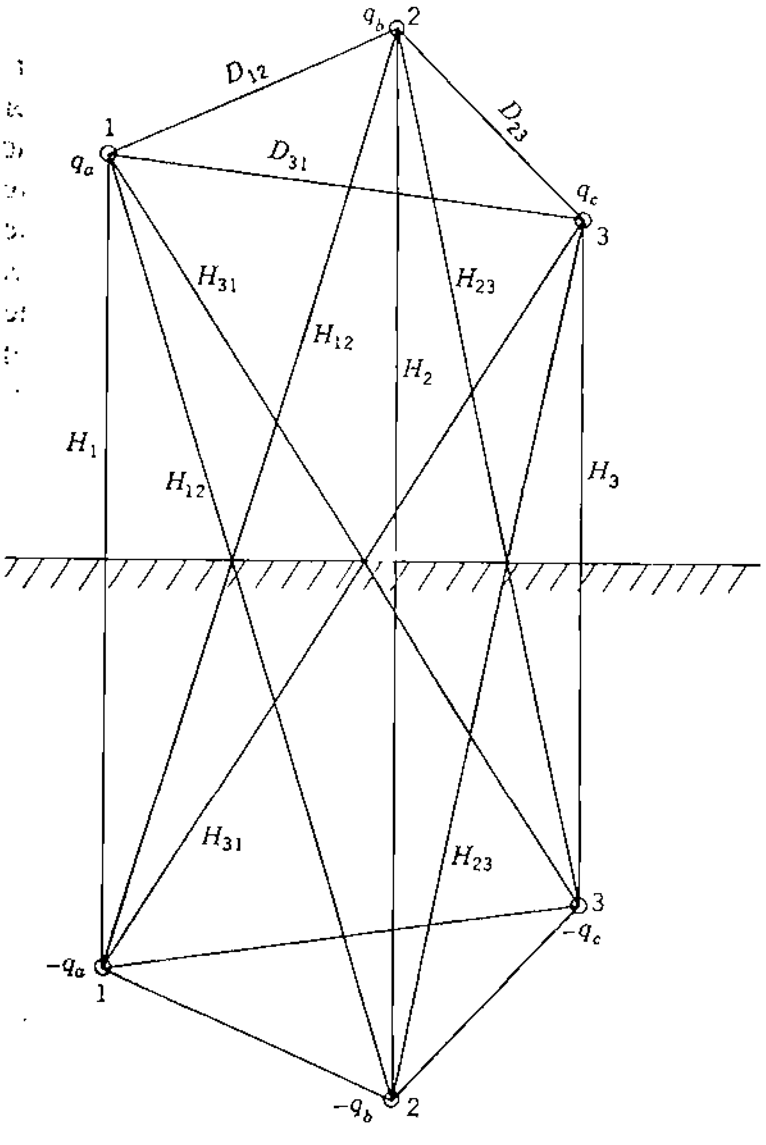


FIGURE 5.9
Three-phase line and its image.

that conductors, a, b, and c carry the charges q_a, q_b, and q_c and occupy positions 1, 2, and 3, respectively, in the first part of the transposition cycle. The plane of the earth is shown, and below it are the conductors with the image charges $-q_a$, $-q_b$, and $-q_c$. Equations for the three parts of the transposition cycle can be written for the voltage drop from conductor a to conductor b as determined by the three charged conductors and their images. With conductor a in position 1, b in position 2, and c in position 3, by Eq. (5.3)

$$V_{ab} = \frac{1}{2\pi k}\left[q_a\left(\ln\frac{D_{12}}{r} - \ln\frac{H_{12}}{H_1}\right) + q_b\left(\ln\frac{r}{D_{12}} - \ln\frac{H_2}{H_{12}}\right) + q_c\left(\ln\frac{D_{23}}{D_{31}} - \ln\frac{H_{23}}{H_{31}}\right)\right] \tag{5.35}$$

Similar equations for V_{ab} are written for the other parts of the transposition cycle. Accepting the approximately correct assumption of constant charge per unit length of each conductor throughout the transposition cycle allows us to obtain an average value of the phasor V_{ab}. The equation for the average value of the phasor V_{ac} is found in a similar manner, and $3V_{an}$ is obtained by adding the average values of V_{ab} and V_{ac}. Knowing that the sum of the charges is zero, we then find

$$C_n = \frac{2\pi k}{\ln\left(\dfrac{D_{eq}}{r}\right) - \ln\left(\dfrac{\sqrt[3]{H_{12}H_{23}H_{31}}}{\sqrt[3]{H_1H_2H_3}}\right)} \text{ F/m to neutral} \tag{5.36}$$

Comparison of Eqs. (5.34) and (5.36) shows that the effect of the earth is to increase the capacitance of a line. To account for the earth, the denominator of Eq. (5.34) must have subtracted from it the term

$$\ln\left(\frac{\sqrt[3]{H_{12}H_{23}H_{31}}}{\sqrt[3]{H_1H_2H_3}}\right)$$

If the conductors are high above ground compared with the distances between them, the diagonal distances in the numerator of the correction term are nearly equal to the vertical distances in the denominator, and the term is very small. This is the usual case, and the effect of ground is generally neglected for three-phase lines except for calculations by symmetrical components when the sum of the three line currents is not zero.

5.7 CAPACITANCE CALCULATIONS FOR BUNDLED CONDUCTORS

Figure 5.10 shows a bundled-conductor line for which we can write an equation for the voltage from conductor a to conductor b as we did in deriving Eq. (5.26), except that now we must consider the charges on all six individual conductors. The conductors of any one bundle are in parallel, and we can assume the charge per bundle divides equally between the conductors of the bundle since the separation between bundles is usually more than 15 times the spacing between the conductors of the bundle. Also, since D_{12} is much greater than d, we can use D_{12} in place of the distances $D_{12} - d$ and $D_{12} + d$ and make other similar substitutions of bundle separation distances instead of using the more exact expressions that occur in finding V_{ab}. The difference due to this approximation cannot be detected in the final result for usual spacings even when the calculation is carried to five or six significant figures.

If charge on phase a is q_a, each of conductors a and a' has the charge $q_a/2$; similar division of charge is assumed for phases b and c. Then,

$$V_{ab} = \frac{1}{2\pi k}\left[\frac{q_a}{2}\left(\underbrace{\ln\frac{D_{12}}{r}}_{a} + \underbrace{\ln\frac{D_{12}}{d}}_{a'}\right) + \frac{q_b}{2}\left(\underbrace{\ln\frac{r}{D_{12}}}_{b} + \underbrace{\ln\frac{d}{D_{12}}}_{b'}\right)\right.$$

$$\left. + \frac{q_c}{2}\left(\underbrace{\ln\frac{D_{23}}{D_{31}}}_{c} + \underbrace{\ln\frac{D_{23}}{D_{31}}}_{c'}\right)\right] \tag{5.37}$$

The letters under each logarithmic term indicate the conductor whose charge is accounted for by that term. Combining terms gives

$$V_{ab} = \frac{1}{2\pi k}\left(q_a \ln\frac{D_{12}}{\sqrt{rd}} + q_b \ln\frac{\sqrt{rd}}{D_{12}} + q_c \ln\frac{D_{23}}{D_{31}}\right) \tag{5.38}$$

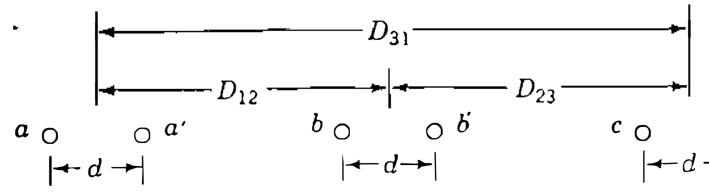


FIGURE 5.10
Cross section of a bundled-conductor three-phase line.

Equation (5.38) is the same as Eq. (5.26), except that $\sqrt{rd}$ has replaced r. It therefore follows that if we consider the line to be transposed, we find

$$C_n = \frac{2\pi k}{\ln\left(\dfrac{D_{eq}}{\sqrt{rd}}\right)} \text{ F/m to neutral} \tag{5.39}$$

The $\sqrt{rd}$ is the same as D_s^b for a two-conductor bundle, except that r has replaced D_s. This leads us to the very important conclusion that a modified geometric mean distance (GMD) method applies to the calculation of capacitance of a bundled-conductor three-phase line having two conductors per bundle. The modification is that we are using outside radius in place of the GMR of a single conductor.

It is logical to conclude that the modified GMD method applies to other bundling configurations. If we let D_{sC}^b stand for the modified GMR to be used in capacitance calculations to distinguish it from D_s^b used in inductance calculations, we have

$$C_n = \frac{2\pi k}{\ln\left(\dfrac{D_{eq}}{D_{sC}^b}\right)} \text{ F/m to neutral} \tag{5.40}$$

Then, for a two-strand bundle

$$D_{sC}^b = \sqrt[4]{(r \times d)^2} = \sqrt{rd} \tag{5.41}$$

for a three-strand bundle

$$D_{sC}^b = \sqrt[9]{(r \times d \times d)^3} = \sqrt[3]{rd^2} \tag{5.42}$$

and for a four-strand bundle

$$D_{sC}^b = \sqrt[16]{(r \times d \times d \times d \times \sqrt{2})^4} = 1.09\sqrt[4]{rd^3} \tag{5.43}$$

Example 5.3. Find the capacitive reactance to neutral of the line described in Example 4.5 in ohm-kilometers (and in ohm-miles) per phase.

Solution. Computed from the diameter given in Table A.3

$$r = \frac{1.382 \times 0.3048}{2 \times 12} = 0.01755 \text{ m}$$

$$D_{sC}^{b} = \sqrt{0.01755 \times 0.45} = 0.0889 \text{ m}$$

$$D_{\text{eq}} = \sqrt[3]{8 \times 8 \times 16} = 10.08 \text{ m}$$

$$C_m = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln\left(\dfrac{10.08}{0.0889}\right)} = 11.754 \times 10^{-12} \text{ F/m}$$

$$X_C = \frac{10^{12} \times 10^{-3}}{2\pi 60 \times 11.754} = 0.2257 \times 10^{6} \ \Omega \cdot \text{km per phase to neutral}$$

$$\left(X_C = \frac{0.2257 \times 10^{6}}{1.609} = 0.1403 \times 10^{6} \ \Omega \cdot \text{mi per phase to neutral} \right)$$

5.8 PARALLEL-CIRCUIT THREE-PHASE LINES

If two three-phase circuits that are identical in construction and operating in parallel are so close together that coupling exists between them, the GMD method can be used to calculate the inductive and capacitive reactances of their equivalent circuit.

Figure 5.11 shows a typical arrangement of parallel-circuit three-phase lines on the same tower. Although the line will probably not be transposed, we obtain practical values for inductive and capacitive reactances if transposition is assumed. Conductors a and a' are in parallel to compose phase a. Phases b and

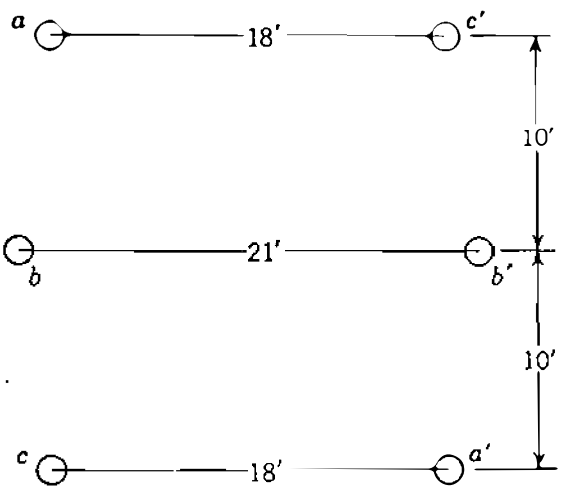


FIGURE 5.11
Typical arrangement of conductors of a parallel-circuit three-phase line.

c are similar. We assume that a and a' take the positions of b and b' and then of c and c' as those conductors are rotated similarly in the transposition cycle.

To calculate D_{eq} the GMD method requires that we use D_{ab}^p, D_{bc}^p, and D_{ca}^p, where the superscript indicates that these quantities are for parallel lines and where D_{ab}^p means the GMD between the conductors of phase a and those of phase b.

For *inductance* calculations D_s of Eq. (4.56) is replaced by D_s^p, which is the geometric mean of the GMR values of the two conductors occupying first the positions of a and a', then the positions of b and b', and finally the positions of c and c'.

Because of the similarity between inductance and capacitance calculations, we can assume that the D_{sC}^p for capacitance is the same as D_s^p for inductance, except that r is used instead of D_s of the individual conductor.

Following each step of Example 5.4 is possibly the best means of understanding the procedure.

Example 5.4. A three-phase double-circuit line is composed of 300,000-cmil 26/7 *Ostrich* conductors arranged as shown in Fig. 5.11. Find the 60-Hz inductive reactance and capacitive susceptance in ohms per mile per phase and siemens per mile per phase, respectively.

Solution. From Table A.3 for *Ostrich*

$$D_s = 0.0229 \text{ ft}$$

$$\text{Distance } a \text{ to } b\text{: original position} = \sqrt{10^2 + 1.5^2} = 10.1 \text{ ft}$$

$$\text{Distance } a \text{ to } b'\text{: original position} = \sqrt{10^2 + 19.5^2} = 21.9 \text{ ft}$$

The GMDs between phases are

$$D_{ab}^p = D_{bc}^p = \sqrt[4]{(10.1 \times 21.9)^2} = 14.88 \text{ ft}$$

$$D_{ca}^p = \sqrt[4]{(20 \times 18)^2} = 18.97 \text{ ft}$$

$$D_{eq} = \sqrt[3]{14.88 \times 14.88 \times 18.97} = 16.1 \text{ ft}$$

For inductance calculations the GMR for the parallel-circuit line is found after first obtaining the GMR values for the three positions. The actual distance from a

to a' is $\sqrt{20^2 + 18^2} = 26.9$ ft. Then, GMR of each phase is

$$\text{In position } a - a': \quad \sqrt{26.9 \times 0.0229} = 0.785 \text{ ft}$$

$$\text{In position } b - b': \quad \sqrt{21 \times 0.0229} = 0.693 \text{ ft}$$

$$\text{In position } c - c': \quad \sqrt{26.9 \times 0.0229} = 0.785 \text{ ft}$$

Therefore,

$$D_s^p = \sqrt[3]{0.785 \times 0.693 \times 0.785} = 0.753 \text{ ft}$$

$$L = 2 \times 10^{-7} \ln \frac{16.1}{0.753} = 6.13 \times 10^{-7} \text{ H/m per phase}$$

$$X_L = 2\pi 60 \times 1609 \times 6.13 \times 10^{-7} = 0.372\ \Omega/\text{mi per phase}$$

For capacitive calculations D_{sC}^p is the same as that of D_s^p, except that the outside radius of the *Ostrich* conductor is used instead of its GMR. The outside diameter of *Ostrich* is 0.680 in:

$$r = \frac{0.680}{2 \times 12} = 0.0283 \text{ ft}$$

$$D_{sC}^p = \left(\sqrt{26.9 \times 0.0283}\sqrt{21 \times 0.0283}\sqrt{26.9 \times 0.0283}\right)^{1/3}$$

$$= \sqrt{0.0283}\,(26.9 \times 21 \times 26.9)^{1/6} = 0.837 \text{ ft}$$

$$C_n = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln \dfrac{16.1}{0.837}} = 18.807 \times 10^{-12} \text{ F/m}$$

$$B_c = 2\pi \times 60 \times 18.807 \times 1609$$

$$= 11.41 \times 10^{-6} \text{ S/mi per phase to neutral}$$

5.9 SUMMARY

The similarity between inductance and capacitance calculations has been emphasized throughout our discussions. As in inductance calculations, computer programs are recommended if a large number of calculations of capacitance is required. Tables like A.3 and A.5 make the calculations quite simple, however, except for parallel-circuit lines.

The important equation for capacitance to neutral for a single-circuit, three-phase line is

$$C_n = \frac{2\pi k}{\ln \dfrac{D_{eq}}{D_{sC}}} \text{ F/m to neutral} \tag{5.44}$$

D_{sC} is the outside radius r of the conductor for a line consisting of one conductor per phase. For overhead lines k is 8.854×10^{-12} since k_r for air is 1.0. Capacitive reactance is ohm-meters is $1/2\pi fC$, where C is in farads per meter. So, at 60 Hz

$$X_C = 4.77 \times 10^4 \ln \frac{D_{eq}}{D_{sC}} \ \Omega \cdot \text{km to neutral} \tag{5.45}$$

or upon dividing by 1.609 km/mi, we have

$$X_C = 2.965 \times 10^4 \ln \frac{D_{eq}}{D_{sc}} \ \Omega \cdot \text{mi to neutral} \tag{5.46}$$

Values for capacitive susceptance in siemens per kilometer and siemens per mile are the reciprocals of Eqs. (5.45) and (5.46), respectively.

Both D_{eq} and D_{sC} must be in the same units, usually feet. For bundled conductors D_{sC}^b is substituted for D_{sC}. For both single- and bundled-conductor lines

$$D_{eq} = \sqrt[3]{D_{ab} D_{bc} D_{ca}} \tag{5.47}$$

For bundled-conductor lines D_{ab}, D_{bc}, and D_{ca} are distances between the centers of the bundles of phases a, b, and c.

For lines with one conductor per phase it is convenient to determine X_C by adding X_a' for the conductor as found in Table A.3 to X_d' as found in Table A.5 corresponding to D_{eq}.

Inductance, capacitance, and the associated reactances of parallel-circuit lines are found by following the procedure of Example 5.4.

PROBLEMS

5.1. A three-phase transmission line has flat horizontal spacing with 2 m between adjacent conductors. At a certain instant the charge on one of the outside conductors is 60 μC/km, and the charge on the center conductor and on the other outside conductor is −30 μC/km. The radius of each conductor is 0.8 cm. Neglect the effect of the ground and find the voltage drop between the two identically charged conductors at the instant specified.

5.2. The 60-Hz capacitive reactance to neutral of a solid conductor, which is one conductor of a single-phase line with 5-ft spacing, is 196.1 kΩ-mi. What value of reactance would be specified in a table listing the capacitive reactance in ohm-miles to neutral of the conductor at 1-ft spacing for 25 Hz? What is the cross-sectional area of the conductor in circular mils?

5.3. Solve Example 5.1 for 50-Hz operation and 10-ft spacing.

5.4. Using Eq. (5.23), determine the capacitance to neutral (in μF/km) of a three-phase line with three *Cardinal* ACSR conductors equilaterally spaced 20 ft apart. What is the charging current of the line (in A/km) at 60 Hz and 100 kV line to line?

5.5. A three-phase 60-Hz transmission line has its conductors arranged in a triangular formation so that two of the distances between conductors are 25 ft and the third is 42 ft. The conductors are ACSR *Osprey*. Determine the capacitance to neutral in microfarads per mile and the capacitive reactance to neutral in ohm-miles. If the line is 150 mi long, find the capacitance to neutral and capacitive reactance of the line.

5.6. A three-phase 60-Hz line has flat horizontal spacing. The conductors have an outside diameter of 3.28 cm with 12 m between conductors. Determine the capacitive reactance to neutral in ohm-meters and the capacitive reactance of the line in ohms if its length is 125 mi.

5.7. (*a*) Derive an equation for the capacitance to neutral in farads per meter of a single-phase line, taking into account the effect of ground. Use the same nomenclature as in the equation derived for the capacitance of a three-phase line where the effect of ground is represented by image charges.

(*b*) Using the derived equation, calculate the capacitance to neutral in farads per meter of a single-phase line composed of two solid circular conductors, each having a diameter of 0.229 in. The conductors are 10 ft apart and 25 ft above ground. Compare the result with the value obtained by applying Eq. (5.10).

5.8. Solve Prob. 5.6 while taking into account the effect of ground. Assume that the conductors are horizontally placed 20 m above ground.

5.9. A 60-Hz three-phase line composed of one ACSR *Bluejay* conductor per phase has flat horizontal spacing of 11 m between adjacent conductors. Compare the capacitive reactance in ohm-kilometers per phase of this line with that of a line using a two-conductor bundle of ACSR 26/7 conductors having the same total cross-sectional area of aluminum as the single-conductor line and the 11-m spacing measured between bundles. The spacing between conductors in the bundle is 40 cm.

5.10. Calculate the capacitive reactance in ohm-kilometers of a bundled 60-Hz three-phase line having three ACSR *Rail* conductors per bundle with 45 cm between conductors of the bundle. The spacing between bundle centers is 9, 9, and 18 m.

5.11. Six conductors of ACSR *Drake* constitute a 60-Hz double-circuit three-phase line arranged as shown in Fig. 5.11. The vertical spacing, however, is 14 ft; the longer horizontal distance is 32 ft; and the shorter horizontal distances are 25 ft. Find

(*a*) The inductance per phase (in H/mi) and the inductive reactance (in Ω/mi).

(*b*) The capacitive reactance to neutral (in $\Omega \cdot$ mi) and the charging current in A/mi per phase and per conductor at 138 kV.