## Solution.

The power loss due to corona for 3 phases is given by :

$$
P=3 \times \frac{242 \cdot 2(f+25)}{\delta} \sqrt{\frac{r}{d}}\left(V-V_{c}\right)^{2} \times 10^{-5} \mathrm{~kW} / \mathrm{km}
$$

As $f, \delta, r$ and $d$ are the same for the two cases,

$$
\therefore \quad P \propto\left(V-V_{c}\right)^{2}
$$

For first case, $P=53 \mathrm{~kW}$ and $V=106 / \sqrt{3}=61.2 \mathrm{kV}$
For second case, $P=98 \mathrm{~kW}$ and $V=110 \cdot 9 / \sqrt{3}=64 \mathrm{kV}$

$$
\begin{equation*}
\therefore \quad 53 \propto\left(61 \cdot 2-V_{c}\right)^{2} \tag{i}
\end{equation*}
$$

$$
\begin{equation*}
\text { and } \quad 98 \propto\left(64-V_{c}\right)^{2} \tag{ii}
\end{equation*}
$$

Dividing [(ii)/(i)], we get,

$$
\frac{98}{53}=\frac{\left(64-V_{c}\right)^{2}}{\left(61 \cdot 2-V_{c}\right)^{2}}
$$

or

$$
V_{c}=54 \mathbf{k V}
$$

Let $W$ kilowatt be the power loss at 113 kV .

$$
\therefore \quad W \propto\left(\frac{113}{\sqrt{3}}-V_{c}\right)^{2}
$$

$$
\begin{equation*}
\propto(65 \cdot 2-54)^{2} \tag{iii}
\end{equation*}
$$

Dividing $[(i i i) /(i)]$, we get,

$$
\begin{array}{rlrl} 
& & \frac{W}{53} & =\frac{(65 \cdot 2-54)^{2}}{(61 \cdot 2-54)^{2}} \\
\therefore & W & =(11 \cdot 2 / 7 \cdot 2)^{2} \times 53=\mathbf{1 2 8} \mathbf{k W}
\end{array}
$$

## TUTORIAL PROBLEMS

1. Estimate the corona loss for a three-phase, $110 \mathrm{kV}, 50 \mathrm{~Hz}, 150 \mathrm{~km}$ long transmission line consisting of three conductors each of 10 mm diameter and spaced 2.5 m apart in an equilateral triangle formation. The temperature of air is $30^{\circ} \mathrm{C}$ and the atmospheric pressure is 750 mm of mercury. Take irregularity factor as 0.85 . Ionisation of air may be assumed to take place at a maximum voltage gradient of $30 \mathrm{kV} /$ cm .
[ 316.8 kW ]
2. Taking the dielectric strength of air to be $30 \mathrm{kV} / \mathrm{cm}$, calculate the disruptive critical voltage for a 3-phase line with conductors of 1 cm radius and spaced symmetrically 4 m apart.
[220 kV line voltage]
3. A 3-phase, $220 \mathrm{kV}, 50 \mathrm{~Hz}$ transmission line consists of 1.2 cm radius conductors spaced 2 m at the corners of an equilateral triangle. Calculate the corona loss per km of the line. The condition of the wire is smoothly weathered and the weather is fair with temperature of $20^{\circ} \mathrm{C}$ and barometric pressure of 72.2 cm of Hg .
[2.148 kW]

### 8.15 Sag in Overhead Lines

While erecting an overhead line, it is very important that conductors are under safe tension. If the conductors are too much stretched between supports in a bid to save conductor material, the stress in the conductor may reach unsafe value and in certain cases the conductor may break due to excessive tension. In order to permit safe tension in the conductors, they are not fully stretched but are allowed to have a dip or sag.

The difference in level between points of supports and the lowest point on the conductor is called sag.

Fig. 8.23. ( $i$ ) shows a conductor suspended between two equilevel supports $A$ and $B$. The conductor is not fully stretched but is allowed to have a dip. The lowest point on the conductor is $O$ and the sag is $S$. The following points may be noted :


Fig. 8.23
(i) When the conductor is suspended between two supports at the same level, it takes the shape of catenary. However, if the sag is very small compared with the span, then sag-span curve is like a parabola.
(ii) The tension at any point on the conductor acts tangentially. Thus tension $T_{O}$ at the lowest point $O$ acts horizontally as shown in Fig. 8.23. (ii).
(iii) The horizontal component of tension is constant throughout the length of the wire.
(iv) The tension at supports is approximately equal to the horizontal tension acting at any point on the wire. Thus if $T$ is the tension at the support $B$, then $T=T_{O}$.
Conductor sag and tension. This is an important consideration in the mechanical design of overhead lines. The conductor sag should be kept to a minimum in order to reduce the conductor material required and to avoid extra pole height for sufficient clearance above ground level. It is also desirable that tension in the conductor should be low to avoid the mechanical failure of conductor and to permit the use of less strong supports. However, low conductor tension and minimum sag are not possible. It is because low sag means a tight wire and high tension, whereas a low tension means a loose wire and increased sag. Therefore, in actual practice, a compromise in made between the two.

### 8.16 Calculation of Sag

In an overhead line, the sag should be so adjusted that tension in the conductors is within safe limits. The tension is governed by conductor weight, effects of wind, ice loading and temperature variations. It is a standard practice to keep conductor tension less than $50 \%$ of its ultimate tensile strength i.e., minimum factor of safety in respect of conductor tension should be 2 . We shall now calculate sag and tension of a conductor when (i) supports are at equal levels and (ii) supports are at unequal levels.
(i) When supports are at equal levels. Con- A sider a conductor between two equilevel supports $A$ and $B$ with $O$ as the lowest point as shown in Fig. 8.24 . It can be proved that lowest point will be at the mid-span.

Let

$$
\begin{aligned}
l= & \text { Length of span } \\
w= & \text { Weight per unit length of con- } \\
& \text { ductor } \\
T= & \text { Tension in the conductor. }
\end{aligned}
$$



Fig. 8.24

Consider a point $P$ on the conductor. Taking the lowest point $O$ as the origin, let the co-ordinates of point $P$ be $x$ and $y$. Assuming that the curvature is so small that curved length is equal to its horizontal projection (i.e., $O P=x$ ), the two forces acting on the portion $O P$ of the conductor are :
(a) The weight $w x$ of conductor acting at a distance $x / 2$ from $O$.
(b) The tension $T$ acting at $O$.

Equating the moments of above two forces about point $O$, we get,
or

$$
\begin{aligned}
T y & =w x \times \frac{x}{2} \\
y & =\frac{w x^{2}}{2 T}
\end{aligned}
$$

The maximum dip (sag) is represented by the value of $y$ at either of the supports $A$ and $B$.
At support $A$,

$$
x=l / 2 \text { and } y=S
$$

$\therefore$
Sag, $\quad S=\frac{w(l / 2)^{2}}{2 T}=\frac{w l^{2}}{8 T}$
(ii) When supports are at unequal levels. In hilly areas, we generally come across conductors suspended between supports at unequal levels. Fig. 8.25 shows a conductor suspended between two supports $A$ and $B$ which are at different levels. The lowest point on the conductor is $O$.
Let
$l=$ Span length
$h=$ Difference in levels between two supports
$x_{1}=$ Distance of support at lower level (i.e., A) from $O$
$x_{2}=$ Distance of support at higher level (i.e. $B$ ) from $O$
$T=$ Tension in the conductor


Fig. 8.25
If $w$ is the weight per unit length of the conductor, then,

$$
\text { Sag } S_{1}=\frac{w x_{1}^{2^{2 *}}}{2 T}
$$

$$
\text { and } \quad \operatorname{Sag} S_{2}=\frac{w x_{2}^{2}}{2 T}
$$

$$
\begin{equation*}
x_{1}+x_{2}=l \tag{i}
\end{equation*}
$$

Also

$$
* y=\frac{w x^{2}}{2 T}
$$

At support $A, x=x_{1}$ and $y=S_{1}$.

$$
\therefore \quad S_{1}=\frac{w x_{1}^{2}}{2 T}
$$

Now

$$
S_{2}-S_{1}=\frac{w}{2 T}\left[x_{2}^{2}-x_{1}^{2}\right]=\frac{w}{2 T}\left(x_{2}+x_{1}\right)\left(x_{2}-x_{1}\right)
$$

$\therefore \quad S_{2}-S_{1}=\frac{w l}{2 T}\left(x_{2}-x_{1}\right) \quad\left[\because x_{1}+x_{2}=l\right]$
But

$$
S_{2}-S_{1}=h
$$

$\therefore \quad h=\frac{w l}{2 T}\left(x_{2}-x_{1}\right)$
or

$$
\begin{equation*}
x_{2}-x_{1}=\frac{2 T h}{w l} \tag{ii}
\end{equation*}
$$

Solving exps. (i) and (ii), we get,

$$
\begin{aligned}
& x_{1}=\frac{l}{2}-\frac{T h}{w l} \\
& x_{2}=\frac{l}{2}+\frac{T h}{w l}
\end{aligned}
$$

Having found $x_{1}$ and $x_{2}$, values of $S_{1}$ and $S_{2}$ can be easily calculated.
Effect of wind and ice loading. The above formulae for sag are true only in still air and at normal temperature when the conductor is acted by its weight only. However, in actual practice, a conductor may have ice coating and simultaneously subjected to wind pressure. The weight of ice acts vertically downwards i.e., in the same direction as the weight of conductor. The force due to the wind is assumed to act horizontally i.e., at right angle to the projected surface of the conductor. Hence, the total force on the conductor is the vector sum of horizontal and vertical forces as shown in Fig. 8.26 (iii).


Fig. 8.26
Total weight of conductor per unit length is
where

$$
w_{t}=\sqrt{\left(w+w_{i}\right)^{2}+\left(w_{w}\right)^{2}}
$$

$w=$ weight of conductor per unit length
$=$ conductor material density $\times$ volume per unit length
$w_{i}=$ weight of ice per unit length
$=$ density of ice $\times$ volume of ice per unit length
$=$ density of ice $\times \frac{\pi}{4}\left[(d+2 t)^{2}-d^{2}\right] \times 1$
$=$ density of ice $\times \pi t(d+t)^{*}$
$w_{w}=$ wind force per unit length
$=$ wind pressure per unit area $\times$ projected area per unit length
$=$ wind pressure $\times[(d+2 t) \times 1]$

[^0]When the conductor has wind and ice loading also, the following points may be noted :
(i) The conductor sets itself in a plane at an angle $\theta$ to the vertical where

$$
\tan \theta=\frac{w_{w}}{w+w_{i}}
$$

(ii) The sag in the conductor is given by:

$$
S=\frac{w_{t} l^{2}}{2 T}
$$

Hence $S$ represents the slant sag in a direction making an angle $\theta$ to the vertical. If no specific mention is made in the problem, then slant slag is calculated by using the above formula.
(iii) The vertical sag $=S \cos \theta$

Example 8.17. A 132 kV transmission line has the following data :

$$
\begin{array}{ll}
\text { Wt. of conductor }=680 \mathrm{~kg} / \mathrm{km} & ; \quad \\
\text { Ultimate strength }=3100 \mathrm{~kg} & ; \quad \text { Length of span }=260 \mathrm{~m} \\
\text { Safety factor }=2
\end{array}
$$

Calculate the height above ground at which the conductor should be supported. Ground clearance required is 10 metres.

## Solution.

Wt . of conductor/metre run, $w=680 / 1000=0.68 \mathrm{~kg}$
Working tension,

$$
T=\frac{\text { Ultimate strength }}{\text { Safety factor }}=\frac{3100}{2}=1550 \mathrm{~kg}
$$

Span length,

$$
l=260 \mathrm{~m}
$$

$$
\therefore \quad \text { Sag }=\frac{w l^{2}}{8 T}=\frac{0.68 \times(260)^{2}}{8 \times 1550}=3.7 \mathrm{~m}
$$

$\therefore$ Conductor should be supported at a height of $10+3.7=13 \cdot 7 \mathrm{~m}$
Example 8.18. A transmission line has a span of 150 m between level supports. The conductor has a cross-sectional area of $2 \mathrm{~cm}^{2}$. The tension in the conductor is 2000 kg . If the specific gravity of the conductor material is $9.9 \mathrm{gm} / \mathrm{cm}^{3}$ and wind pressure is $1.5 \mathrm{~kg} / \mathrm{m}$ length, calculate the sag. What is the vertical sag?

## Solution.

Span length, $l=150 \mathrm{~m} ; \quad$ Working tension, $T=2000 \mathrm{~kg}$
Wind force $/ \mathrm{m}$ length of conductor, $w_{w}=1.5 \mathrm{~kg}$
Wt. of conductor/m length, $w=$ Sp. Gravity $\times$ Volume of 1 m conductor

$$
=9.9 \times 2 \times 100=1980 \mathrm{gm}=1.98 \mathrm{~kg}
$$

Total weight of 1 m length of conductor is

$$
\begin{aligned}
w_{t} & =\sqrt{w^{2}+w_{w}^{2}}=\sqrt{(1.98)^{2}+(1.5)^{2}}=2.48 \mathrm{~kg} \\
\therefore \quad \text { Sag, } \quad S & =\frac{w_{t} l^{2}}{8 T}=\frac{2.48 \times(150)^{2}}{8 \times 2000}=3.48 \mathrm{~m}
\end{aligned}
$$

This is the value of slant sag in a direction making an angle $\theta$ with the vertical. Referring to Fig. 8.27, the value of $\theta$ is given by ;

$$
\begin{array}{rlrl} 
& & \tan \theta & =w_{w} / w=1.5 / 1.98=0.76 \\
\therefore & \theta & =\tan ^{-1} 0.76=37.23^{\circ} \\
\therefore & & \text { Vertical sag } & =S \cos \theta \\
& & =3.48 \times \cos 37.23^{\circ}=2.77 \mathrm{~m}
\end{array}
$$



Fig. 8.27

Example 8.19. A transmission line has a span of 200 metres between level supports. The conductor has a cross-sectional area of $1.29 \mathrm{~cm}^{2}$, weighs $1170 \mathrm{~kg} / \mathrm{km}$ and has a breaking stress of $4218 \mathrm{~kg} / \mathrm{cm}^{2}$. Calculate the sag for a safety factor of 5, allowing a wind pressure of 122 kg per square metre of projected area. What is the vertical sag?

## Solution.

Span length,

$$
l=200 \mathrm{~m}
$$

Wt. of conductor $/ \mathrm{m}$ length, $w=1170 / 1000=1 \cdot 17 \mathrm{~kg}$
Working tension, $\quad * T=4218 \times 1.29 / 5=1088 \mathrm{~kg}$
Diameter of conductor,

$$
d=\sqrt{\frac{4 \times \text { area }}{\pi}}=\sqrt{\frac{4 \times 1 \cdot 29}{\pi}}=1.28 \mathrm{~cm}
$$

Wind force/m length, $\quad w_{w}=$ Pressure $\times$ projected area in $\mathrm{m}^{2}$

$$
=(122) \times\left(1.28 \times 10^{-2} \times 1\right)=1.56 \mathrm{~kg}
$$

Total weight of conductor per metre length is

$$
\begin{aligned}
w_{t} & =\sqrt{w^{2}+w_{w}^{2}}=\sqrt{(1 \cdot 17)^{2}+(1 \cdot 56)^{2}}=1.95 \mathrm{~kg} \\
\therefore \quad \text { Slant sag, } \quad S & =\frac{w_{t} l^{2}}{8 T}=\frac{1 \cdot 95 \times(200)^{2}}{8 \times 1088}=8.96 \mathrm{~m}
\end{aligned}
$$

The slant sag makes an angle $\theta$ with the vertical where value of $\theta$ is given by :

$$
\theta=\tan ^{-1}\left(w_{w} / w\right)=\tan ^{-1}(1.56 / 1.17)=53.13^{\circ}
$$

$\therefore \quad$ Vertical sag $=S \cos \theta=8.96 \times \cos 53.13^{\circ}=5.37 \mathrm{~m}$
Example 8.20. A transmission line has a span of 275 m between level supports. The conductor has an effective diameter of 1.96 cm and weighs $0.865 \mathrm{~kg} / \mathrm{m}$. Its ultimate strength is 8060 kg . If the conductor has ice coating of radial thickness 1.27 cm and is subjected to a wind pressure of $3.9 \mathrm{gm} /$ $\mathrm{cm}^{2}$ of projected area, calculate sag for a safety factor of 2 . Weight of 1 c.c. of ice is 0.91 gm .

## Solution.

Span length, $\quad l=275 \mathrm{~m}$; Wt. of conductor $/ \mathrm{m}$ length, $w=0.865 \mathrm{~kg}$
Conductor diameter, $\quad d=1.96 \mathrm{~cm}$; Ice coating thickness, $t=1.27 \mathrm{~cm}$
Working tension, $\quad T=8060 / 2=4030 \mathrm{~kg}$
Volume of ice per metre (i.e., 100 cm ) length of conductor

$$
\begin{aligned}
& =\pi t(d+t) \times 100 \mathrm{~cm}^{3} \\
& =\pi \times 1.27 \times(1.96+1.27) \times 100=1288 \mathrm{~cm}^{3}
\end{aligned}
$$

Weight of ice per metre length of conductor is

$$
w_{i}=0.91 \times 1288=1172 \mathrm{gm}=1.172 \mathrm{~kg}
$$

Wind force/m length of conductor is

$$
\begin{aligned}
w_{w} & =[\text { Pressure }] \times[(d+2 t) \times 100] \\
& =[3.9] \times(1.96+2 \times 1.27) \times 100 \mathrm{gm}=1755 \mathrm{gm}=1.755 \mathrm{~kg}
\end{aligned}
$$

Total weight of conductor per metre length of conductor is

$$
\begin{aligned}
w_{t} & =\sqrt{\left(w+w_{i}\right)^{2}+\left(w_{w}\right)^{2}} \\
& =\sqrt{(0.865+1 \cdot 172)^{2}+(1.755)^{2}}=2.688 \mathrm{~kg}
\end{aligned}
$$

* Working stress $=\frac{\text { Ultimate Strength }}{\text { Safety factor }}=\frac{4218}{5}$
$\therefore \quad$ Working Tension, $T=$ Working stress $\times$ conductor area $=4218 \times 1.29 / 5$

$$
\therefore \quad \text { Sag }=\frac{w_{t} l^{2}}{8 T}=\frac{2 \cdot 688 \times(275)^{2}}{8 \times 4030}=6.3 \mathrm{~m}
$$

Example 8.21. A transmission line has a span of 214 metres between level supports. The conductors have a cross-sectional area of $3.225 \mathrm{~cm}^{2}$. Calculate the factor of safety under the following conditions :

$$
\text { Vertical sag }=2.35 \mathrm{~m} ; \quad \text { Wind pressure }=1.5 \mathrm{~kg} / \mathrm{m} \text { run }
$$

Breaking stress $=2540 \mathrm{~kg} / \mathrm{cm}^{2}$; Wt. of conductor $=1.125 \mathrm{~kg} / \mathrm{m}$ run
Solution.
Here, $\quad l=214 \mathrm{~m} ; w=1.125 \mathrm{~kg} ; \quad w_{w}=1.5 \mathrm{~kg}$
Total weight of one metre length of conductor is

$$
w_{t}=\sqrt{w^{2}+w_{w}^{2}}=\sqrt{(1 \cdot 125)^{2}+(1 \cdot 5)^{2}}=1.875 \mathrm{~kg}
$$

If $f$ is the factor of safety, then,
Working tension, $\quad T=\frac{\text { Breaking stress } \times \text { conductor area }}{\text { safety factor }}=2540 \times 3.225 / f=8191 / f \mathrm{~kg}$
Slant Sag, $\quad S=\frac{\text { Vertical sag }}{* \cos \theta}=\frac{2.35 \times 1.875}{1 \cdot 125}=3.92 \mathrm{~m}$
Now

$$
\mathrm{S}=\frac{w_{t} l^{2}}{8 T}
$$

or

$$
T=\frac{w_{t} l^{2}}{8 S}
$$

$$
\therefore \quad \frac{8191}{f}=\frac{1.875 \times(214)^{2}}{8 \times 3.92}
$$

$$
\text { or } \text { Safety factor, } \quad f=\frac{8191 \times 8 \times 3.92}{1.875 \times(214)^{2}}=3
$$

Example 8.22. An overhead line has a span of 150 m between level supports. The conductor has a cross-sectional area of $2 \mathrm{~cm}^{2}$. The ultimate strength is $5000 \mathrm{~kg} / \mathrm{cm}^{2}$ and safety factor is 5 . The specific gravity of the material is $8.9 \mathrm{gm} / \mathrm{cc}$. The wind pressure is $1.5 \mathrm{~kg} / \mathrm{m}$. Calculate the height of the conductor above the ground level at which it should be supported if a minimum clearance of 7 m is to be left between the ground and the conductor.

## Solution.

Span length, $l=150 \mathrm{~m} ; \quad$ Wind force $/ \mathrm{m}$ run, $w_{w}=1.5 \mathrm{~kg}$
Wt. of conductor/m run, $\quad w=$ conductor area $\times 100 \mathrm{~cm} \times$ sp. gravity

$$
=2 \times 100 \times 8.9=1780 \mathrm{gm}=1.78 \mathrm{~kg}
$$

Working tension,

$$
T=5000 \times 2 / 5=2000 \mathrm{~kg}
$$

Total weight of one metre length of conductor is

$$
\begin{aligned}
w_{t} & =\sqrt{w^{2}+w_{w}^{2}}=\sqrt{(1.78)^{2}+(1.5)^{2}}=2.33 \mathrm{~kg} \\
\text { Slant sag, } \quad S & =\frac{w_{t} l^{2}}{8 T}=\frac{2.33 \times(150)^{2}}{8 \times 2000}=3.28 \mathrm{~m} \\
\text { Vertical sag } & =S \cos \theta=3.28 \times w / w_{t}=3.28 \times 1.78 / 2.33=2.5 \mathrm{~m}
\end{aligned}
$$

Conductor should be supported at a height of $7+2.5=9.5 \mathrm{~m}$

[^1]Example 8.23. The towers of height 30 m and 90 m respectively support a transmission line conductor at water crossing. The horizontal distance betwen the towers is 500 m . If the tension in the conductor is 1600 kg , find the minimum clearance of the conductor and water and clearance mid-way between the supports. Weight of conductor is $1.5 \mathrm{~kg} / \mathrm{m}$. Bases of the towers can be considered to be at water level.

Solution. Fig. 8.28 shows the conductor suspended between two supports $A$ and $B$ at different levels with $O$ as the lowest point on the conductor.

Here, $l=500 \mathrm{~m} ; \quad w=1.5 \mathrm{~kg} ; T=1600 \mathrm{~kg}$.
Difference in levels between supports, $h=90-30=60 \mathrm{~m}$. Let the lowest point $O$ of the conductor be at a distance $x_{1}$ from the support at lower level (i.e., support $A$ ) and at a distance $x_{2}$ from the support at higher level (i.e., support $B$ ).

Obviously,

$$
\begin{equation*}
x_{1}+x_{2}=500 \mathrm{~m} \tag{i}
\end{equation*}
$$



Fig. 8.28

Now
Sag $S_{1}=\frac{w x_{1}^{2}}{2 T} \quad$ and $\quad$ Sag $S_{2}=\frac{w x_{2}^{2}}{2 T}$
$\therefore \quad h=S_{2}-S_{1}=\frac{w x_{2}^{2}}{2 T}-\frac{w x_{1}^{2}}{2 T}$
or
$60=\frac{w}{2 T}\left(x_{2}+x_{1}\right)\left(x_{2}-x_{1}\right)$
$\therefore \quad x_{2}-x_{1}=\frac{60 \times 2 \times 1600}{1.5 \times 500}=256 \mathrm{~m}$
Solving exps. (i) and (ii), we get, $x_{1}=122 \mathrm{~m} ; x_{2}=378 \mathrm{~m}$
Now,

$$
S_{1}=\frac{w x_{1}^{2}}{2 T}=\frac{1.5 \times(122)^{2}}{2 \times 1600}=7 \mathrm{~m}
$$

Clearance of the lowest point $O$ from water level

$$
=30-7=23 \mathrm{~m}
$$

Let the mid-point $P$ be at a distance $x$ from the lowest point $O$.
Clearly,

$$
x=250-x_{1}=250-122=128 \mathrm{~m}
$$

Sag at mid-point $P, \quad S_{\text {mid }}=\frac{w x^{2}}{2 T}=\frac{1.5 \times(128)^{2}}{2 \times 1600}=7.68 \mathrm{~m}$

Clearance of mid-point $P$ from water level

$$
=23+7 \cdot 68=30 \cdot 68 \mathrm{~m}
$$

Example 8.24. An overhead transmission line conductor having a parabolic configuration weighs 1.925 kg per metre of length. The area of $X$-section of the conductor is $2.2 \mathrm{~cm}^{2}$ and the ultimate strength is $8000 \mathrm{~kg} / \mathrm{cm}^{2}$. The supports are 600 m apart having 15 m difference of levels. Calculate the sag from the taller of the two supports which must be allowed so that the factor of safety shall be 5. Assume that ice load is 1 kg per metre run and there is no wind pressure.

Solution. Fig. 8.29. shows the conductor suspended between two supports at $A$ and $B$ at different levels with $O$ as the lowest point on the conductor.

Here,

$$
\begin{aligned}
l & =600 \mathrm{~m} ; w_{i}=1 \mathrm{~kg} ; h=15 \mathrm{~m} \\
w & =1.925 \mathrm{~kg} ; T=8000 \times 2 \cdot 2 / 5=3520 \mathrm{~kg}
\end{aligned}
$$

Total weight of 1 m length of conductor is

$$
w_{t}=w+w_{i}=1.925+1=2.925 \mathrm{~kg}
$$

Let the lowest point $O$ of the conductor be at a distance $x_{1}$ from the support at lower level (i.e., $A$ ) and at a distance $x_{2}$ from the support at higher level (i.e., $B$ ).

Clearly,

$$
\begin{equation*}
x_{1}+x_{2}=600 \mathrm{~m} \tag{i}
\end{equation*}
$$

Now,

$$
h=S_{2}-S_{1}=\frac{w_{t} x_{2}^{2}}{2 T}-\frac{w_{t} x_{1}^{2}}{2 T}
$$

or

$$
15=\frac{w_{t}}{2 T}\left(x_{2}+x_{1}\right)\left(x_{2}-x_{1}\right)
$$

$$
\begin{equation*}
\therefore \quad x_{2}-x_{1}=\frac{2 \times 15 \times 3520}{2 \cdot 925 \times 600}=60 \mathrm{~m} \tag{ii}
\end{equation*}
$$

Solving exps. (i) and (ii), we have, $x_{1}=270 \mathrm{~m}$ and $x_{2}=330 \mathrm{~m}$


Fig. 8.29
Sag from the taller of the two towers is

$$
S_{2}=\frac{w_{t} x_{2}^{2}}{2 T}=\frac{2.925 \times(330)^{2}}{2 \times 3520}=45.24 \mathrm{~m}
$$

Example 8.25. An overhead transmission line at a river crossing is supported from two towers at heights of 40 m and 90 m above water level, the horizontal distance between the towers being 400 $m$. If the maximum allowable tension is 2000 kg , find the clearance between the conductor and water at a point mid-way between the towers. Weight of conductor is $1 \mathrm{~kg} / \mathrm{m}$.

Solution. Fig. 8.30 shows the whole arrangement.


Fig. 8.30
Here,

$$
\begin{aligned}
h & =90-40=50 \mathrm{~m} ; & l & =400 \mathrm{~m} \\
T & =2000 \mathrm{~kg} ; & w & =1 \mathrm{~kg} / \mathrm{m}
\end{aligned}
$$

Obviously,

$$
\begin{equation*}
x_{1}+x_{2}=400 \mathrm{~m} \tag{i}
\end{equation*}
$$

Now
or

$$
\begin{equation*}
\therefore \quad x_{2}-x_{1}=\frac{50 \times 2 \times 2000}{400}=500 \mathrm{~m} \tag{ii}
\end{equation*}
$$

Solving exps. (i) and (ii), we get, $x_{2}=450 \mathrm{~m}$ and $x_{1}=-50 \mathrm{~m}$
Now $x_{2}$ is the distance of higher support $B$ from the lowest point $O$ on the conductor, whereas $x_{1}$ is that of lower support $A$. As the span is 400 m , therefore, point $A$ lies on the same side of $O$ as $B$ (see Fig. 8.30).

Horizontal distance of mid-point $P$ from lowest point $O$ is

$$
x=\text { Distance of } A \text { from } O+400 / 2=50+200=250 \mathrm{~m}
$$

$\therefore \quad$ Sag at point $P, \quad S_{\text {mid }}=\frac{w x^{2}}{2 T}=\frac{1 \times(250)^{2}}{2 \times 2000}=15.6 \mathrm{~m}$
Now

$$
\text { Sag } \quad S_{2}=\frac{w x_{2}^{2}}{2 T}=\frac{1 \times(450)^{2}}{2 \times 2000}=50.6 \mathrm{~m}
$$

Height of point $B$ above mid-point $P$

$$
=S_{2}-S_{m i d}=50 \cdot 6-15 \cdot 6=35 \mathrm{~m}
$$

$\therefore \quad$ Clearance of mid-point $P$ above water level

$$
=90-35=55 \mathrm{~m}
$$

Example 8.26. A transmission line over a hillside where the gradient is $1: 20$, is supported by two 22 m high towers with a distance of 300 m between them. The lowest conductor is fixed 2 m below the top of each tower. Find the clearance of the conductor from the ground. Given that conductor weighs $1 \mathrm{~kg} / \mathrm{m}$ and the allowable tension is 1500 kg .

Solution. The conductors are supported between towers $A D$ and $B E$ over a hillside having gradient of 1:20 as shown in Fig. 8.31. The lowest point on the conductor is $O$ and $\sin \theta=1 / 20$.

Effective height of each tower ( $A D$ or $B E$ )

$$
=22-2=20 \mathrm{~m}
$$

Vertical distance between towers is

$$
h=E C=D E \sin \theta=300 \times 1 / 20=15 \mathrm{~m}
$$

Horizontal distance between two towers is

$$
D C=\sqrt{D E^{2}-E C^{2}}=\sqrt{(300)^{2}-(15)^{2}} \simeq 300 \mathrm{~m}
$$

or

$$
\begin{equation*}
x_{1}+x_{2}=300 \mathrm{~m} \tag{i}
\end{equation*}
$$

Now

$$
h=\frac{w x_{2}^{2}}{2 T}-\frac{w x_{1}^{2}}{2 T}=\frac{w}{2 T}\left(x_{2}+x_{1}\right)\left(x_{2}-x_{1}\right)
$$

or

$$
\begin{equation*}
x_{2}-x_{1}=\frac{2 T h}{w\left(x_{2}+x_{1}\right)}=\frac{2 \times 1500 \times 15}{1 \times 300}=150 \mathrm{~m} \tag{ii}
\end{equation*}
$$



Fig. 8.31
Solving exps. (i) and (ii), we have, $x_{1}=75 \mathrm{~m}$ and $x_{2}=225 \mathrm{~m}$

Now

$$
\text { Sag } S_{2}=\frac{w x_{2}^{2}}{2 T}=\frac{1 \times(225)^{2}}{2 \times 1500}=16.87 \mathrm{~m}
$$

Clearance of the lowest point $O$ from the ground is

$$
\begin{aligned}
O G & =H F-S_{2}-G F \\
& =B C-S_{2}-G F
\end{aligned}
$$

$$
(\because B C=H F)
$$

$$
\begin{aligned}
{[\text { Now } G F} & \left.=x_{1} \tan \theta=75 \times 0.05=3.75 \mathrm{~m}\right] \\
& =35-16.87-3.75=\mathbf{1 4 . 3 8} \mathbf{~ m}
\end{aligned}
$$

Example 8.27. A transmission tower on a level ground gives a minimum clearance of 8 metres for its lowest conductor with a sag of 10 m for a span of 300 m . If the same tower is to be used over a slope of 1 in 15, find the minimum ground clearance obtained for the same span, same conductor and same weather conditions.

Solution. On level ground

$$
\text { Sag, } S=\frac{w l^{2}}{8 T}
$$

$$
\therefore \quad \frac{w}{T}=\frac{8 S}{l^{2}}=\frac{8 \times 10}{(300)^{2}}=\frac{8}{9 \times 10^{3}}
$$

$$
\text { Height of tower }=\text { Sag }+ \text { Clearance }=10+8=18 \mathrm{~m}
$$

On sloping ground. The conductors are supported between towers $A D$ and $B E$ over a sloping ground having a gradient 1 in 15 as shown in Fig. 8.32. The height of each tower $(A D$ or $B E)$ is 18 m .

Vertical distance between the two towers is

$$
h=E C=* D E \sin \theta=300 \times 1 / 15=20 \mathrm{~m}
$$

Now

$$
\begin{equation*}
x_{1}+x_{2}=300 \mathrm{~m} \tag{i}
\end{equation*}
$$

Also
$h=\frac{w x_{2}^{2}}{2 T}-\frac{w x_{1}^{2}}{2 T}=\frac{w}{2 T}\left(x_{2}+x_{1}\right)\left(x_{2}-x_{1}\right)$
$\therefore \quad x_{2}-x_{1}=\frac{2 T h}{w\left(x_{2}+x_{1}\right)}=\frac{2 \times 9 \times 10^{3} \times 20}{8 \times 300}=150 \mathrm{~m}$


Fig. 8.32
Solving exps. (i) and (ii), we have, $x_{1}=75 \mathrm{~m}$ and $x_{2}=225 \mathrm{~m}$
Now

$$
\begin{aligned}
& S_{1}=\frac{w x_{1}^{2}}{2 T}=\frac{8 \times(75)^{2}}{2 \times 9 \times 10^{3}}=2.5 \mathrm{~m} \\
& S_{2}=\frac{w x_{2}^{2}}{2 T}=\frac{8 \times(225)^{2}}{2 \times 9 \times 10^{3}}=22.5 \mathrm{~m}
\end{aligned}
$$

Clearance of point $O$ from the ground is

$$
\begin{aligned}
& O G=B C-S_{2}-G F=38-22 \cdot 5-5=10 \cdot 5 \mathrm{~m} \\
& {\left[\because G F=x_{1} \tan \theta=75 \times 1 / 15=5 \mathrm{~m}\right]}
\end{aligned}
$$

Since $O$ is the origin, the equation of slope of ground is given by :
Here $\quad m=1 / 15$ and $A=O G=-10 \cdot 5 \mathrm{~m}$
$\therefore \quad y=\frac{x}{15}-10.5$
$\therefore \quad$ Clearance $C$ from the ground at any point $x$ is

* $D E \simeq D C=300 \mathrm{~m}$

$$
\begin{aligned}
C & =\text { Equation of conductor curve }-y=\left(\frac{w x^{2}}{2 T}\right)-\left(\frac{x}{15}-10 \cdot 5\right) \\
& =\frac{8 x^{2}}{2 \times 9 \times 10^{3}}-\left(\frac{x}{15}-10 \cdot 5\right) \\
\therefore \quad C & =\frac{x^{2}}{2250}-\frac{x}{15}+10 \cdot 5
\end{aligned}
$$

Clearance will be minimum when $d C / d x=0$ i.e.,
or

$$
\begin{aligned}
\frac{d}{d x}\left[\frac{x^{2}}{2250}-\frac{x}{15}+10 \cdot 5\right] & =0 \\
\frac{2 x}{2250}-\frac{1}{15} & =0 \\
x & =\frac{1}{15} \times \frac{2250}{2}=75 \mathrm{~m}
\end{aligned}
$$

or
i.e., minimum clearance will be at a point 75 m from $O$.

$$
\begin{aligned}
\text { Minimum clearance } & =\frac{x^{2}}{2250}-\frac{x}{15}+10 \cdot 5=(75)^{2} / 2250-75 / 15+10 \cdot 5 \\
& =2 \cdot 5-5+10 \cdot 5=8 \mathrm{~m}
\end{aligned}
$$

## TUTORIAL PROBLEMS

1. A transmission line conductor is supported from two towers at heights of 70 m above water level. The horizontal distance between the towers is 300 m . If the tension in the conductors is 1500 kg , find the clearance at a point mid-way between the towers. The size of the conductor is $0.9 \mathrm{~cm}^{2}$ and density of conductor material is $8.9 \mathrm{gm} / \mathrm{cm}^{3}$.
[64 m]
2. An overhead line has a span of 260 m , the weight of the line conductor is 0.68 kg per metre run. Calculate the maximum sag in the line. The maximum allowable tension in the line is $1550 \mathrm{~kg} . \quad[3.7 \mathrm{~m}]$
3. A transmission line has a span of 150 m between level supports. The cross-sectional area of the conductor is $1.25 \mathrm{~cm}^{2}$ and weighs 100 kg per 100 m . The breaking stress is $4220 \mathrm{~kg} / \mathrm{cm}^{2}$. Calculate the factor of safety if the sag of the line is 3.5 m . Assume a maximum wind pressure of 100 kg per sq. metre. [4]
4. A transmission line has a span of 150 m between the level supports. The conductor has a cross-sectional area of $2 \mathrm{~cm}^{2}$. The ultimate strength is $5000 \mathrm{~kg} / \mathrm{cm}^{2}$. The specific gravity of the material is $8.9 \mathrm{gm} / \mathrm{cm}^{3}$. If the wind pressure is $1.5 \mathrm{~kg} / \mathrm{m}$ length of conductor, calculate the sag at the centre of the conductor if factor of safety is 5 .
[ 3.28 m ]
5. A transmission line has a span of 250 m between supports, the supports being at the same level. The conductor has a cross-sectional area of $1.29 \mathrm{~cm}^{2}$. The ultimate strength is $4220 \mathrm{~kg} / \mathrm{cm}^{2}$ and factor of safety is 2 . The wind pressure is $40 \mathrm{~kg} / \mathrm{cm}^{2}$. Calculate the height of the conductor above ground level at which it should be supported if a minimum clearance of 7 m is to be kept between the ground and the conductor.
[10.24 m]
6. A transmission line has a span of 150 m between level supports. The conductor has a cross-sectional area of $2 \mathrm{~cm}^{2}$. The ultimate strength is $5000 \mathrm{~kg} / \mathrm{cm}^{2}$. The specific gravity of the material is $8.9 \mathrm{gm} / \mathrm{cm}^{3}$. If the wind pressure is $1.5 \mathrm{~kg} / \mathrm{m}$ length of the conductor, calculate the sag if factor of safety is $5 . \quad[3.5 \mathrm{~m}$ ]
7. Two towers of height 40 m and 30 m respectively support a transmission line conductor at water crossing. The horizontal distance between the towers is 300 m . If the tension in the conductor is 1590 kg , find the clearance of the conductor at a point mid-way between the supports. Weight of conductor is $0.8 \mathrm{~kg} / \mathrm{m}$. Bases of the towers can be considered to be at the water level.
[59 m]
8. An overhead transmission line at a river crossing is supported from two towers at heights of 50 m and 100 m above the water level. The horizontal distance between the towers is 400 m . If the maximum allowable tension is 1800 kg and the conductor weighs $1 \mathrm{~kg} / \mathrm{m}$, find the clearance between the conductor and water at a point mid-way between the supports.
[63.8 m]

## Mechanical Design of Overhead Lines

### 8.17 Some Mechanical Principles

Mechanical factors of safety to be used in transmission line design should depend to some extent on the importance of continuity of operation in the line under consideration. In general, the strength of the line should be such as to provide against the worst probable weather conditions. We now discuss some important points in the mechanical design of overhead transmission lines.
(i) Tower height : Tower height depends upon the length of span. With long spans, relatively few towers are required but they must be tall and correspondingly costly. It is not usually possible to determine the tower height and span length on the basis of direct construction costs because the lightning hazards increase greatly as the height of the conductors above ground is increased. This is one reason that horizontal spacing is favoured inspite of the wider right of way required.
(ii) Conductor clearance to ground : The conductor clearance to ground at the time of greatest sag should not be less than some specified distance (usually between 6 and 12 m ), depending on the voltage, on the nature of the country and on the local laws. The greatest sag may occur on the hottest day of summer on account of the expansion of the wire or it may occur in winter owing to the formation of a heavy coating of ice on the wires. Special provisions must be made for melting ice from the power lines.
(iii) Sag and tension : When laying overhead transmission lines, it is necessary to allow a reasonable factor of safety in respect of the tension to which the conductor is subjected. The tension is governed by the effects of wind, ice loading and temperature variations. The relationship between tension and sag is dependent on the loading conditions and temperature variations. For example, the tension increases when the temperature decreases and there is a corresponding decrease in the sag. Icing-up of the line and wind loading will cause stretching of the conductor by an amount dependent on the line tension.

In planning the sag, tension and clearance to ground of a given span, a maximum stress is selected. It is then aimed to have this stress developed at the worst probable weather conditions (i.e. minimum expected temperature, maximum ice loading and maximum wind). Wind loading increases the sag in the direction of resultant loading but decreases the vertical component. Therefore, in clearance calculations, the effect of wind should not be included unless horizontal clearance is important.
(iv) Stringing charts: For use in the field work of stringing the conductors, temperature-sag and temperaturetension charts are plotted for the given conductor and loading conditions. Such curves are called stringing charts (see Fig. 8.33). These charts are very helpful while stringing overhead lines.
(v) Conductor spacing : Spacing of conductors should be such so as to provide safety against flash-over when the wires are swinging in the wind. The proper spacing is a function of span length, voltage and weather conditions. The use of horizontal spacing eliminates the danger caused by unequal ice loading. Small wires or wires of


Fig. 8.33 light material are subjected to more swinging by the wind than heavy conductors. Therefore, light wires should be given greater spacings.
(vi) Conductor vibration : Wind exerts pressure on the exposed surface of the conductor. If the wind velocity is small, the swinging of conductors is harmless provided the clearance is sufficiently large so that conductors do not approach within the sparking distance of each other. A completely different type of vibration, called dancing, is caused by the action of fairly strong wind on a
wire covered with ice, when the ice coating happens to take a form which makes a good air-foil section. Then the whole span may sail up like a kite until it reaches the limit of its slack, stops with a jerk and falls or sails back. The harmful effects of these vibrations occur at the clamps or supports where the conductor suffers fatigue and breaks eventually. In order to protect the conductors, dampers are used.

## SELF-TEST

1. Fill in the blanks by inserting appropriate words/figures.
(i) Cross-arms are used on poles or towers to provide $\qquad$ to the insulators.
(ii) The most commonly used material for insulators of overhead lines is $\qquad$
$\qquad$ capacitance.
(iii) The potential across the various discs of suspension string is different because of . .. to the conductor.
(iv) In a string of suspension insulators, the maximum voltage appears across the unit.
(v) If the string efficiency is $100 \%$, it means that $\qquad$ ..
(vi) If shunt capacitance is reduced, then string efficiency is . $\qquad$ ..
(vii) If the spacing between the conductors is increased, then corona effect is $\qquad$
(viii) If sag in an overhead line increases, tension in the line $\qquad$ ....
(ix) By using a guard ring, string efficiency is $\qquad$ ...
(x) Shunt capacitance in suspension insulators can be decreased by increasing the distance of . $\qquad$ from $\qquad$
2. Pick up the correct words/figures from the brackets and fill in the blanks.
(i) The insulator is so designed that it should fail only by $\qquad$ .. (flash-over, puncture)
(ii) Suspension type insulators are used for voltages beyond $\qquad$ ( $33 \mathrm{kV}, 400 \mathrm{~V}, 11 \mathrm{kV}$ )
(iii) In a string of suspension insulators, if the unit nearest to the conductor breaks down, then other units will $\qquad$ (also breakdown, remain intact)
(iv) A shorter string has $\qquad$ string efficiency than a larger one. (less, more)
(v) Corona effect is .................. pronounced in stormy weather as compared to fair weather. (more, less)
(vi) If the conductor size is increased, the corona effect is $\qquad$ (increased, decreased)
(vii) The longer the cross arm, the $\qquad$ the string efficiency.
(greater, lesser)
(viii) The discs of the strain insulators are used in $\qquad$ plane.
(vertical, horizontal)
(ix) Sag is provided in overhead lines so that $\qquad$
(Safe tension is not exceeded, repair can be done)
(x) When an insulator breaks down by puncture, it is $\qquad$ damaged.
(permanently, only partially)

## ANSWERS

1. (i) support (ii) porcelain (iii) shunt (iv) nearest ( $v$ ) potential across each disc is the same (vi) increased (vii) reduced (viii) decreases (ix) increased ( $x$ ) conductor, tower.
2. (i) flash-over (ii) 33 kV (iii) also breakdown (iv) more (v) more (vi) decreased (vii) greater (viii) vertical (ix) safe tension is not exceeded $(x)$ permanently.

## CHAPTER REVIEW TOPICS

1. Name the important components of an overhead transmission line.
2. Discuss the various conductor materials used for overhead lines. What are their relative advantages and disadvantages?
3. Discuss the various types of line supports.
4. Why are insulators used with overhead lines? Discuss the desirable properties of insulators.

Mechanical Design of Overhead Lines
5. Discuss the advantages and disadvantages of (i) pin-type insulators (ii) suspension type insulators.
6. Explain how the electrical breakdown can occur in an insulator.
7. What is a strain insulator and where is it used ? Give a sketch to show its location.
8. Give reasons for unequal potential distribution over a string of suspension insulators.
9. Define and explain string efficiency. Can its value be equal to $100 \%$ ?
10. Show that in a string of suspension insulators, the disc nearest to the conductor has the highest voltage across it.
11. Explain various methods of improving string efficiency.
12. What is corona? What are the factors which affect corona?
13. Discuss the advantages and disadvantages of corona.
14. Explain the following terms with reference to corona :
(i) Critical disruptive voltage
(ii) Visual critical voltage
(iii) Power loss due to corona
15. Describe the various methods for reducing corona effect in an overhead transmission line.
16. What is a sag in overhead lines? Discuss the disadvantages of providing too small or too large sag on a line.
17. Deduce an approximate expression for sag in overhead lines when
(i) supports are at equal levels
(ii) supports are at unequal levels.

## DISCUSSION QUESTIONS

1. What is the need for stranding the conductors?
2. Is sag a necessity or an evil ? Discuss.
3. String efficiency for a d.c. system is $100 \%$ ? Discuss.
4. Can string efficiency in an a.c. system be $100 \%$ ?
5. Why are suspension insulators preferred for high voltage power transmission ?
6. Give reasons for the following :
(i) A.C.S.R. conductors are preferred for transmission and distribution lines.
(ii) Conductors are not fully stretched between supports.

[^0]:    * Volume of ice per unit length $=\frac{\pi}{4}\left[(d+t)^{2}-d^{2}\right] \times 1=\frac{\pi}{4}\left[4 d t+4 t^{2}\right]=\pi t(d+t)$

[^1]:    * The slant sag makes an angle $\theta$ with the vertical.

    $$
    \therefore \quad \cos \theta=w / w_{t}=1 \cdot 125 / 1 \cdot 875
    $$

