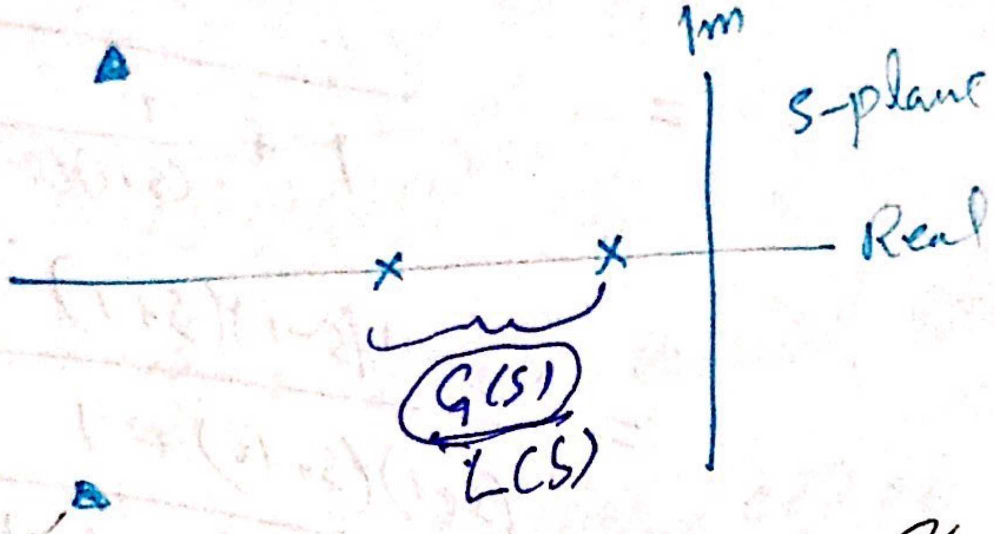
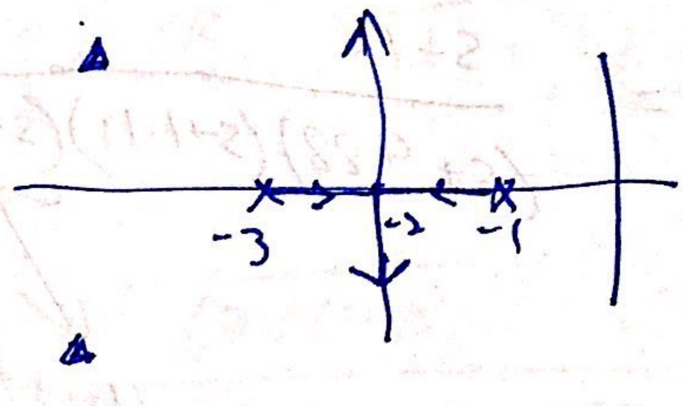
R/L Design ~~Example~~



desired closed loop pole location.

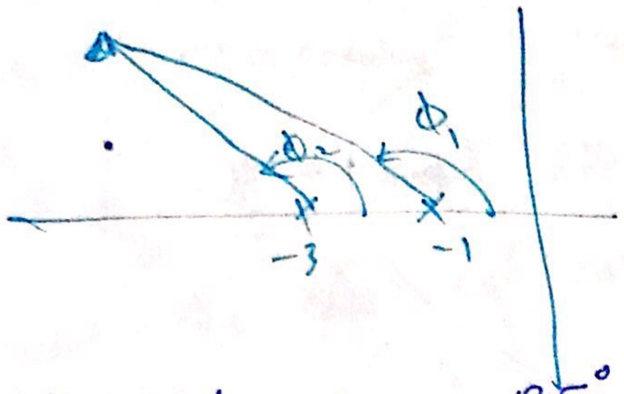
→ To design a controller s.t ▲ are our c/L pole location.



can a kp do this job??

For ▲ to be on the locus we've to satisfy phase and magnitude condition at ▲.

Now what is the phase at ▲ ??



assume $\phi_1 = 135^\circ$
$\phi_2 = 120^\circ$

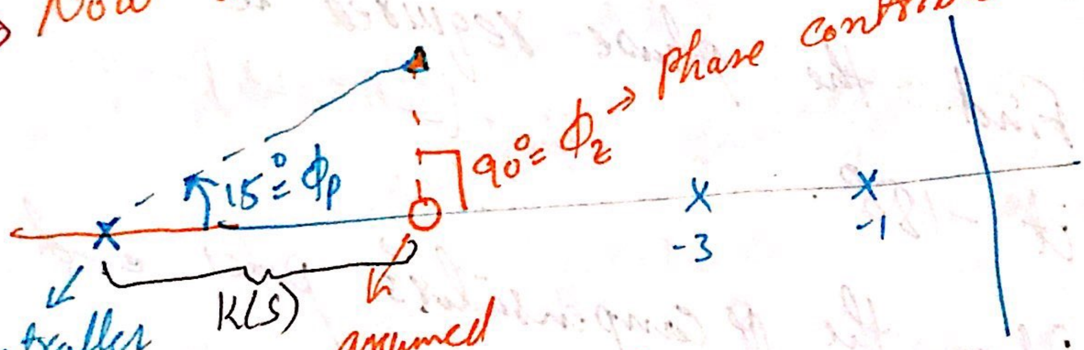
So phase at ▲ $= -135^\circ - 120^\circ = -255^\circ$

but we need it to -180°.

So we need a phase lead of $-180^\circ + 255^\circ = 75^\circ$, so that $-255^\circ + 75^\circ = -180^\circ$

→ So the controller has to provide 75° phase.

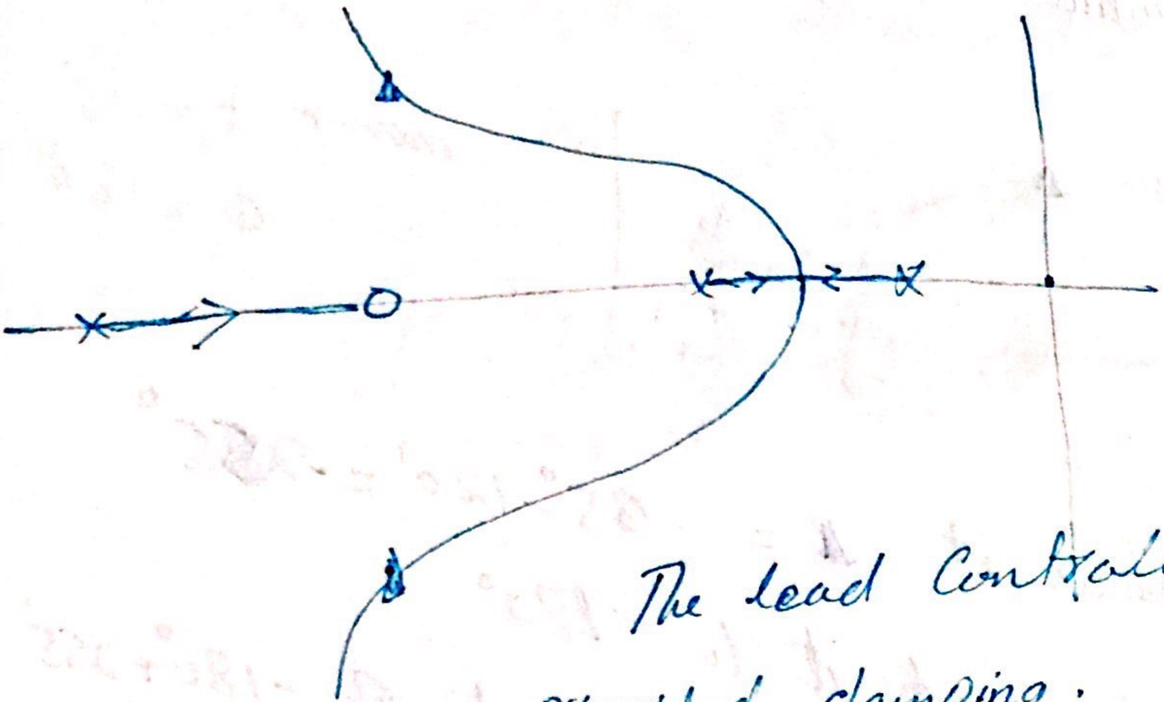
→ Now we've to build a controller



↓
lead Controller.

$\phi_z - \phi_p = 90^\circ - 15^\circ = 75^\circ$

So in this case ▲ will be on the locus.



The lead Controller has provided damping.

Steps

① You calculate the phase contribution at the desired location, from the O/L poles and zeros. $\sum\angle\psi_i - \sum\angle\phi_i$

② Find the phase required to make it -180°. $\theta_{req} + [\sum\angle\psi_i - \sum\angle\phi_i] = -180^\circ$

③ Place the compensator's poles and zero to get that required phase.

④ gain K_0 = ? s.t the locus is stoped at ▲.

Observations :-> Now we've freedom to put compensator pole/zero pair any where we want. It just has to ensure the desired phase.

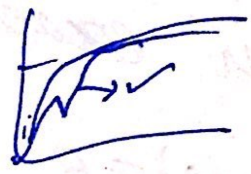
So what will be a good location for the zero and pole???

① if controller zero is in RHP then for some high gain a pole will approach the zero and the sys will become unstable.

② if the zero is exactly at the origin then $K(0)=0$. i.e in steady state we are breaking/opening the loop. Although we need $K(0)$ to be bigger for good steady state tracking.

③ If zero is very near to the origin then ~~will be distu~~ there will be a very slow pole, disturbing the dominance of the desired pole pair.

④ If zero is put too much on the left then there will be a very fast pole which may saturate the actuator.



⑤ So a good option is to put the zero in the vicinity of the ~~of the~~ desired pole.
↳ real part of the

(4-28)

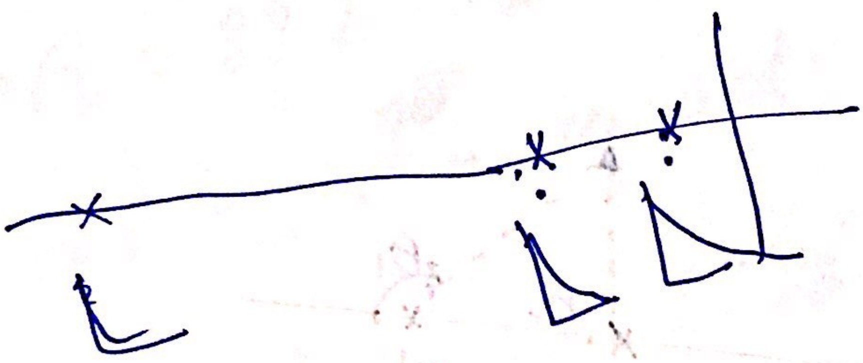
Lead Compensator (Design Trade-offs).

① Compensator zero not to be in the RHP (Can cause instability.)

② " " " " on the origin (Large SS error)

③ " " " " too close to the origin on the left side (A slow mode in the response is possible)

④ " " " " too far on the left (3rd pole may become too fast ⇒ actuator saturation)



R/L Design Example

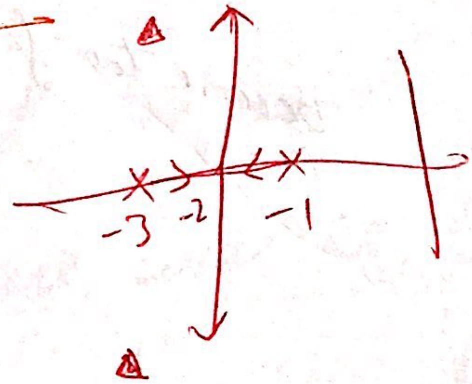
$$G(s) = \frac{1}{(s+1)(s+3)}$$

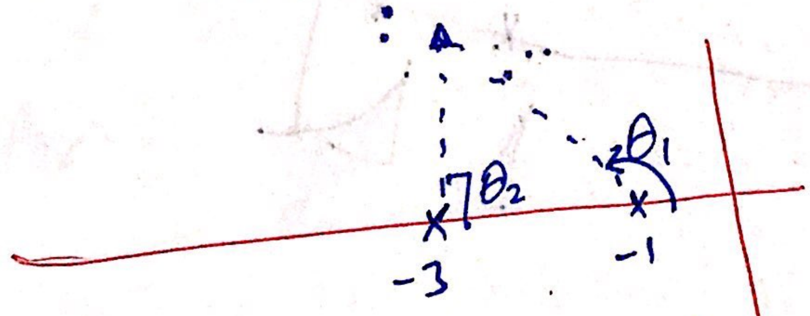
Desired Specs: ① c/L poles : $-3 \pm j3$ ↗ Transient Resp specs

② $e_{ss} < 5\%$ ↳ Tracking Specs.

Solution

A simple K_p will not work.





$$\theta_1 = 180^\circ - \tan^{-1}\left(\frac{3}{2}\right) = 123.7^\circ$$

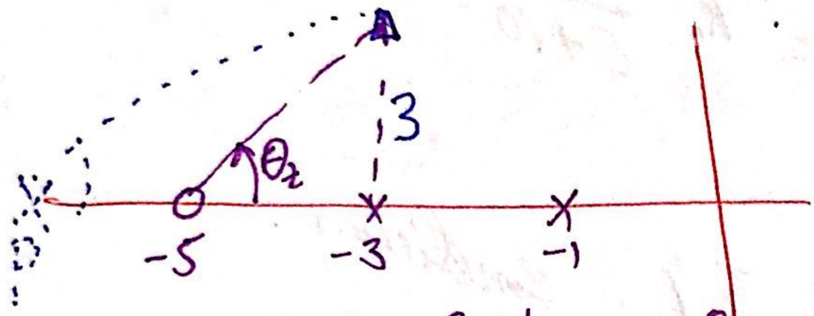
$$\theta_2 = 90^\circ$$

$$\text{Phase} = -90^\circ - 123.7^\circ = -213.7^\circ.$$

$\Rightarrow$ ϕ_{lead} required $= -180 + 213.7^\circ = 33.7^\circ$

to satisfy the phase condition at $-3 \pm 3j$

$\rightarrow$ So let $K_{lead}(s) = \dfrac{s+5}{s+p}$



$\theta_z = \tan^{-1}(3/2) = 56.3^\circ$

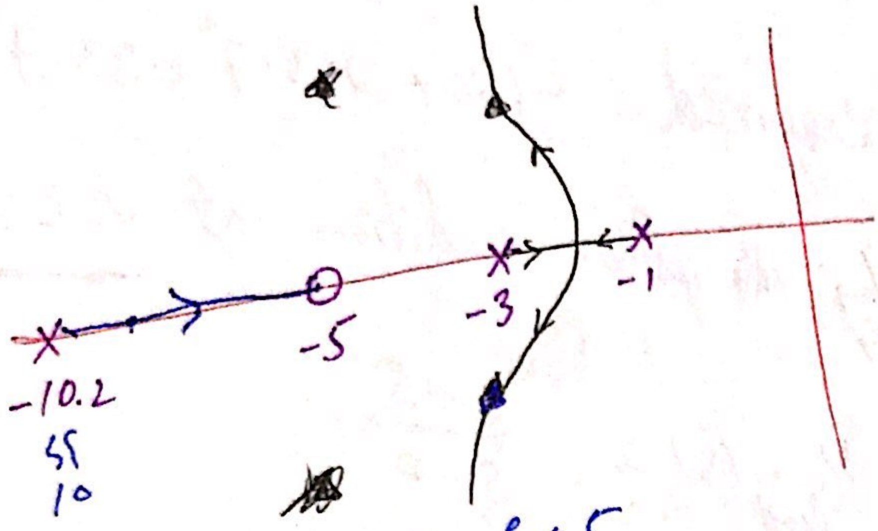
$\theta_z - \theta_p = 33.7^\circ$

$-\theta_p = 33.7 - 56.3^\circ = -22.6$

$\theta_p = 22.6^\circ$ is required.

$22.6^\circ = \tan^{-1}\left(\frac{3}{y}\right) \Rightarrow \frac{3}{y} = \tan 22.6^\circ = 0.4163$

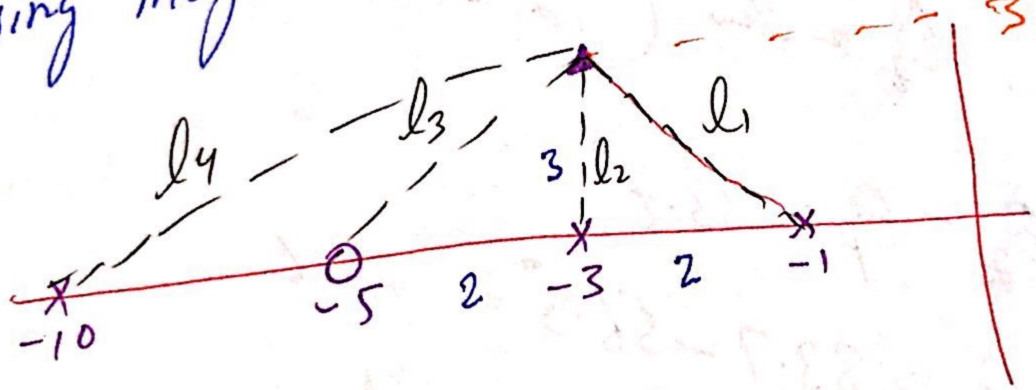
$y = \dfrac{3}{0.4163} \approx 7.2$ which means the compensator pole is at $-3 - 7.2 = -10.2$



So $K_{lead} = K_0 \dfrac{s+5}{s+10}$

Now $K_0 = ??$

Using Magnitude condition.



$$\left| K(s)\, G(s) \right| = 1$$

$$\left| K_0 \frac{s+5}{s+10} \frac{1}{(s+1)(s+3)} \right| = 1$$

$$\left| K_0 \frac{l_3}{l_4} \cdot \frac{1}{l_1\, l_2} \right| = 1$$

$$K \frac{3.6}{83.3} = 1 \qquad \boxed{K_0 = 22.9}$$

$l_1 = \sqrt{2^2 + 3^2} \simeq$

$l_2 = 3$

$l_3 = \simeq 3.6$

$l_4 = 7.62$

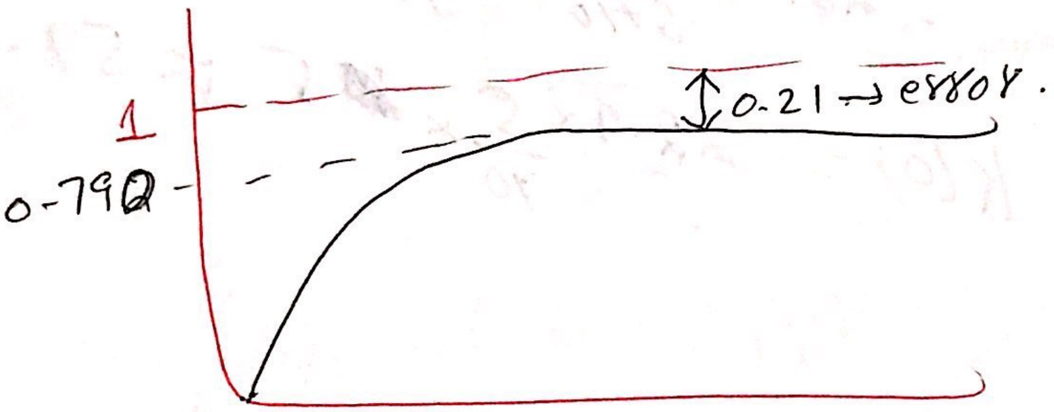
$$K_{lead}(s) = 22.9 \frac{s+5}{s+10}$$

Now the tracking specs.

$$E_{ss} = \frac{1}{1+K(0)G(0)} = \frac{1}{1+22.9\frac{5}{10}\cdot\frac{1}{3}}$$

due to ref: step

$$= \frac{1}{4.82} \simeq 0.21$$



We want

$$E_{ss} = \frac{1}{1+G(0)K(0)} \leq 5\% \leq 0.05$$

$$= \frac{1}{1+0.33K(0)} \leq 0.05$$

$$\Rightarrow K(0)\times 0.33 \geq \frac{1}{0.05} - 1 \Rightarrow 20 - 1 = 19$$

$$K(0) \geq \frac{19}{0.33} \geq 57.58$$

So let us choose

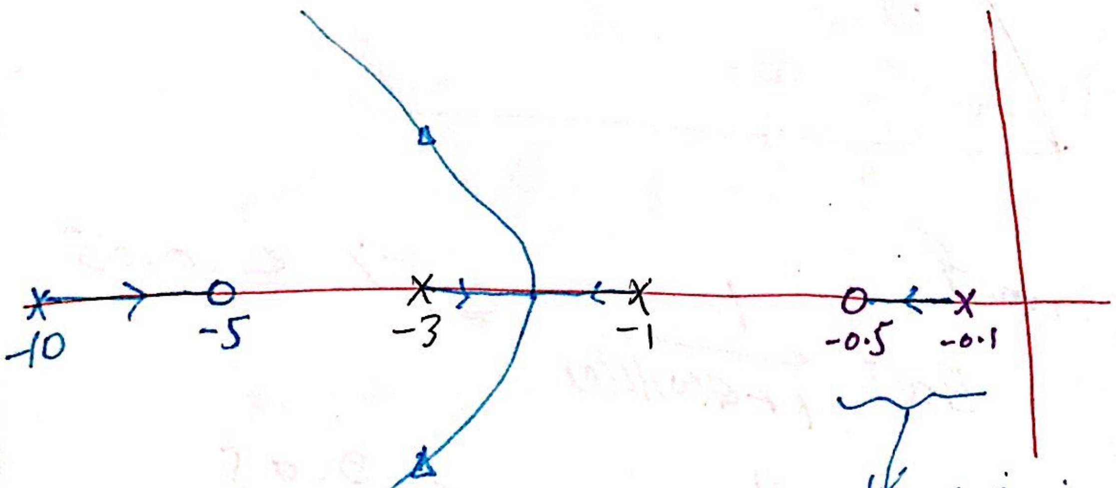
$$K_{lag}(s) = \frac{s+0.5}{s+0.1}$$

↓ to increase the DC gain

$$K(s) = K_{lead}(s)\, K_{lag}(s)$$

$$= 22.9\, \frac{s+5}{s+10} \cdot \frac{s+0.5}{s+0.1}$$

$$K(0) = 22.9 \times \frac{5}{10} \times 5 \simeq 57.25$$



what is it going to do

do we still have that angle condition satisfied at $-3 \pm 3j$. (K_{lag} behave opposite to K_{lead})

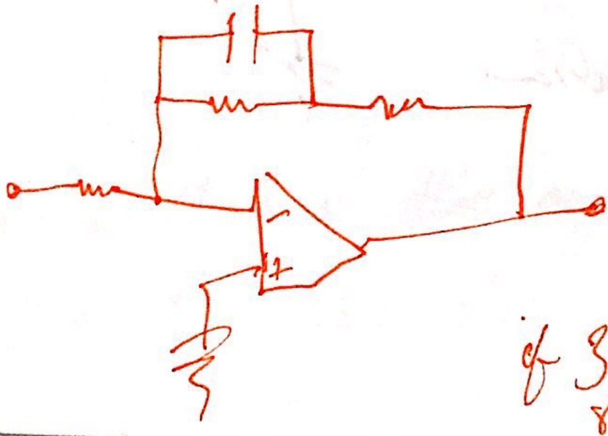
→ If the ~~lead and then~~ lag controller's pole and zero too much far away from the desired location then their respective angles are almost the same. ~~and~~ However a lag will still destroy the damping.

~~Phase~~ Lead Controller
- → Provides Phase lead
- → Increases damping
- → Pulls Locus leftwards
- → Used for improving transient resp.
- → Move centre of assymptotes to the left.

Practical form of PD

→ Poles move with different speeds along the locus.

Lag Controller
- → Gives phase lag
- → Reduces damping/Deteriorates transient resp.
- → Pushes the locus rightwards
- → Used for reducing e_{ss}, ref. tracking and dist. rejection.
- → Moves the α rightwards



→ Used in place of PI control if zero e_{ss} is not actually required for step sign.

R/L design guidlines

① First try simple k_p

② If it doesn't meet the specs, shape the locus by putting in a dynamic compensator (Lead or lag)

③ For Lead Controller: Lead zero is usually chosen close to the desired C/L pole location, lead pole is then put at a factor of 2~85 away depending on the amount of phase lead required.

④ Compensator pole & zero are chosen to satisfy the phase condition at the desired C/L poles.

(5) Calculate the K_o for which the roots are at the desired location.

If more phase lead is required, a double lead can also be used

$$K(s) \; K_o \cdot \frac{s+a_1}{s+b_1} \; \frac{s+a_2}{s+b_2}$$

(6) For better tracking a lag compensator is used

$$K(s) = K_o \frac{s+a}{s+b} \qquad a>b$$

(7) The lag zero + pole are usually chosen small enough so as not to disturb the C/L pole locations adjusted by the lead controller.

(8) If too slow the error will dicrease very slowly bcz of the lag compensator.

(9) Final Controller

$$K(s) = K_{lead}(s)\, K_{lag}(s)$$

$$= K_0 \frac{s + a_{lead}}{s + b_{lead}} \frac{s + a_{lag}}{s + b_{lag}}$$