to human capital when an individual passes primary school and again when the person obtains a secondary school diploma and so on. This is not because the last course taken conveys so much more knowledge than the ones preceding it but because the degree itself is what enables the individual to prove that an entire regimen of requirements has been met. Note that indivisibilities in amounts of investment imply a region of increasing returns to scale, as in the fixed costs of the big push model. Once again, increasing returns play a key role in generating multiple equilibria.[40] Empirically, many studies have found a negative impact of inequality on growth, especially for the period after 1980.[41]

4.5 Michael Kremer's O-Ring Theory of Economic Development

Another innovative and influential model that provides important insights into low-level equilibrium traps was provided by Michael Kremer.[42] The notion is that modern production (especially in contrast to traditional crafts production) requires that many activities be done well together in order for any of them to amount to a high value. This is a form of strong complementarity and is a natural way of thinking about specialization and the division of labor, which along with economies of scale is another hallmark of developed economies in general and industrial production in particular. The name for Kremer's model is taken from the 1986 *Challenger* disaster, in which the failure of one small, inexpensive part caused the space shuttle to explode. The O-ring theory is interesting in part because it explains not only the existence of poverty traps but also the reasons that countries caught in such traps may have such exceptionally low incomes compared with high-income countries.

The O-Ring Model

The key feature of the O-ring model is the way it models production with strong complementarities among inputs. We start by thinking of the model as describing what is going on inside a firm, but as we will see, this model also provides valuable insights into the impact of complementarities across firms or industrial (product) sectors of the economy.

Suppose that a production process is broken down into n tasks. There are many ways of carrying out these tasks, which for simplicity we order strictly by level of skill, q, required, where $0 \leq q \leq 1$. The higher the skill is, the higher the probability that the task will be "successfully completed" (which may mean, for example, that the part created in this task will not fail). Kremer's concept of q is quite flexible. Other interpretations may include a quality index for characteristics of the good: Consumers would be willing to pay more for higher-quality characteristics. For example, suppose that $q = 0.95$. Among other interpretations, this can mean (1) that there is a 95% chance that the task is completed perfectly, so the product keeps maximum value, and a 5% chance that it is completed so poorly that it has no value; (2) that the task is always completed well enough that it keeps 95% of its maximum value; or (3) that the product has a 50% chance of having full value and a 50% chance of an error reducing the value of the product to 90%. For simplicity, assume that

the probability of mistakes by different workers is strictly independent. The production function assumed is a simple one: Output is given by multiplying the q values of each of the n tasks together, in turn multiplied by a term, say, B, that depends on the characteristics of the firm and is generally larger with a larger number of tasks. Suppose also that each firm hires only two workers. Then the **O-ring production function** looks like this:[43]

O-ring production function
A production function with strong complementarities among inputs, based on the products (i.e., multiplying) of the input qualities.

$$BF(q_i q_j) = q_i q_j \tag{4.1}$$

That is, to make things simple, for this exposition we let the multiplier, B, equal 1. In addition to the form of the production function, we make three other significant types of simplifying assumptions: (1) Firms are risk-neutral, (2) labor markets are competitive, and (3) workers supply labor inelastically (i.e., they work regardless of the wage). If we consider capital markets, we assume that they are competitive as well. For now, we also assume that the economy is closed.

One of the most prominent features of this type of production function is what is termed *positive assortative matching*. This means that workers with high skills will work together and workers with low skills will work together. When we use the model to compare economies, this type of matching means that high-value products will be concentrated in countries with high-value skills. In this model, everyone will like to work with the more productive workers, because if your efforts are multiplied by those of someone else, as they are in Equation 4.1, you will be more productive when working with a more productive person. In competitive markets, your pay is based on how productive you are. A firm with a higher-productivity worker can more afford to pay a higher wage and has the incentive to bid higher to do so, because the value of output will be higher with two productive workers, say, than with one low- and one high-productivity worker. As a result, there will be a strong tendency for the most productive workers to work together.

This can be seen easily if we imagine a four-person economy. Suppose that this economy has two high-skill q_H workers and two low-skill q_L workers. The four workers can be arranged either as matched skill pairs or unmatched skill pairs. Total output will always be higher under a matching scheme because

$$q_H^2 + q_L^2 > 2q_H q_L \tag{4.2}$$

Recall that $(x - y)^2 > 0$ for any x that is not the same as y, so let x stand for q_H and y stand for q_L. Then $x^2 + y^2 > 2xy$, the same as in Equation 4.2. (Or try this by plugging in any values $q_H > q_L$.) This generalizes to larger numbers of workers in the firms and the economy; the result is that workers sort out by skill level.[44]

Because total value is higher when skill matching rather than skill mixing takes place, the firm that starts with high-productivity workers can afford to bid more to get additional high-productivity workers, and it is profitable to do so. Of course, every firm would like to hire the most productive worker, but it would be in that worker's interest to team up with other high-productivity workers. Think of firms being formed while workers try to determine for which firm they want to work. After the high-productivity workers pair off, they are out of the picture. The less productive workers are then stuck with each other. If there are many classes of skill or productivity, first the highest-skill workers

get together, then the next highest, and so on, such that skill matching results as a cascading process. For example, a symphony orchestra will be adversely affected as a whole by hiring one single poor performer. So an otherwise excellent orchestra has every incentive to bid the most for an outstanding performer to replace the poor performer. Similarly, the best jazz performers play and record together rather than each leading a group of poorer players. The restaurant with the very best chef also hires mature, highly trained, full-time waiters, while a fast-food restaurant does not hire a famous chef.

This sorting process is perhaps most vividly easy to remember by analogy to Nobel laureate Gary Becker's famous "marriage market" model, which is a somewhat different case[45] but offers some additional intuition. If prospective spouses care only about attractiveness, every man wants to marry the most attractive woman, and every woman wants to marry the most attractive man, so the most attractive man and woman will marry. They are now out of the picture, so next, the second most attractive man and woman marry. This process continues until the least attractive man and woman marry. Of course, beauty is in the eye of the beholder, and most people care about things besides attractiveness in a mate such as kindness, intelligence, wealth, beliefs, interests, commitment, and sense of humor; but the marriage model serves as a memorable analogy. The result in the business world is that some firms and workers, even an entire low-income economy, can fall into a trap of low skill and low productivity, while others escape into higher productivity.

Although this model may seem abstract, a numerical example can show how the firms with high-skill workers can and will pay more to get other high-skill workers or will have more incentive to upgrade skills among existing workers. Suppose that there are six workers; three have $q = 0.4$ and are grouped together in equilibrium, while the other three have $q = 0.8$. Now suppose that the q of one of the workers in the first firm rises from 0.4 to 0.5 (perhaps due to training). Similarly, suppose the q of one worker in the second firm rises from 0.8 to 1.0. In each case, we have a 25% increase in the quality of one worker. As you may expect, a 25% increase in the quality of one worker leads to a 25% increase in output quality. But starting from a higher level of quality, that 25% clearly translates into a much larger point increase: In the example, the first firm goes from $(0.4)(0.4)(0.4) = 0.064$ to $(0.4)(0.4)(0.5) = 0.080$; this is a difference of $0.080 - 0.064$, which is a point change of 0.016; and $0.016/0.064 = 0.25$, which is a 25% increase. For the second firm, we move from $(0.8)(0.8)(0.8) = 0.512$ to $(0.8)(0.8)(1.0) = 0.640$; the change in this case is 0.128, which is again 25%. However, the point value of the increase is much greater—eight times greater—for a doubled point-value investment (0.2 in the second firm versus 0.1 in the first firm). If a firm can increase quality in percentage terms at constant marginal cost or even a not too quickly rising cost, there is a virtuous circle in that the more the firm upgrades overall, the more value it obtains by doing so. Accordingly, *wages will increase at an increasing rate as skill is steadily raised*. As Kremer shows, the O-ring model is consistent with competitive equilibrium.

The O-ring result of positive assortative matching relies on some rather strong assumptions. How important are each of these, and how much can they be relaxed? Two points are crucial: (1) Workers must be sufficiently imperfect substitutes for each other, and (2) we must have sufficient complementarity of tasks. As long as these conditions hold, the basic results will follow.

To see why workers must be imperfect substitutes, suppose they were perfect substitutes. Specifically, suppose there are two skill levels, q_L and $q_H = 2q_L$, so every q_H worker can be replaced by two q_L workers with no other change. Thus q_H workers will be paid twice the amount that q_L workers are paid. We can draw no predictions about what combination of worker skill levels a firm—or an economy—will use, so we can learn nothing about low-skill-level equilibrium traps. In fact, there is empirical evidence for imperfect substitutability across worker types in firms.

To see why we must have complementarity of tasks, suppose that there were two tasks indexed by g and h but with no complementarity between them. To be specific, suppose that our q_H worker is hired for the g task, and a q_L worker is hired for the h task; then

$$F(q_H q_L) = g(q_H) + h(q_L)$$

Here skills are imperfect substitutes for each other, because only one type of worker can be hired for each task (i.e., no two-for-one type of substitution is possible here). However, because tasks are not complementary, the optimal choice of skill for the g task is independent of that of the h task, and again no strategic complementarities are present.[46]

Implications of the O-Ring Theory

The analysis has several important implications:

- Firms tend to employ workers with similar skills for their various tasks.
- Workers performing the same task earn higher wages in a high-skill firm than in a low-skill firm.
- Because wages increase in q at an increasing rate, wages will be more than proportionally higher in developed countries than would be predicted from standard measures of skill.
- If workers can improve their skill level and make such investments, and if it is in their interests to do so, they will consider the level of human capital investments made by other workers as a component of their own decision about how much skill to acquire. Put differently, when those around you have higher average skills, you have a greater *incentive* to acquire more skills. This type of complementarity should by now be a familiar condition in which multiple equilibria can emerge; it parallels issues raised in our analysis of the big push model. Kremer shows that a graph like Figure 4.1 can apply to choices about how much skill to acquire.
- One can get caught in economy-wide, low-production-quality traps. This will occur when there are (quite plausibly) O-ring effects across firms as well as within firms. Because there is an externality at work, there could thus be a case for an industrial policy to encourage quality upgrading, as some East Asian countries have undertaken in the past (see Chapter 12, section 12.6, and its end-of-chapter case study of South Korea). This could be relevant for a country trying to escape the middle-income trap.