Consumer surplus and consumer welfare

Continue…

- The definition of a measure of economic welfare for the consumer has been one of the most controversial subjects in economics.
- Unlike the producer’s case, where observable measures of well-being, such as profit, can be clearly determined, no equally appealing *observable* measure exists for a utility-maximizing consumer.
- That is, the criterion of the consumer – utility – is not observable.
- In most practical situations, the applied welfare economist can, at best, observe income and consumption decisions at various prices and then, on the basis of these economic transactions, try to compute some money-based measure of welfare effects.

Continue…

- A source of confusion in deriving measures of consumer welfare lies in not distinguishing between ‘cardinal’ and ‘ordinal’ analysis.
- Generally, the analysis was largely ordinal in that only consumer indifference curves were used.
- No attempt was made to measure the intensity of change in satisfaction or utility the consumer derived when moving from one indifference curve to another.

Continue…

- Rather, only qualitative concerns were important (for example, which indifference curve was preferred to the other).
- In applied welfare economics, measures of consumer welfare are generally not *cardinal* in the strict usage of the term.
- They are money measures of welfare change where money reflects willingness to pay (WTP) on the part of consumers, which in turn is related to the 'utility function' of the consumer.
-

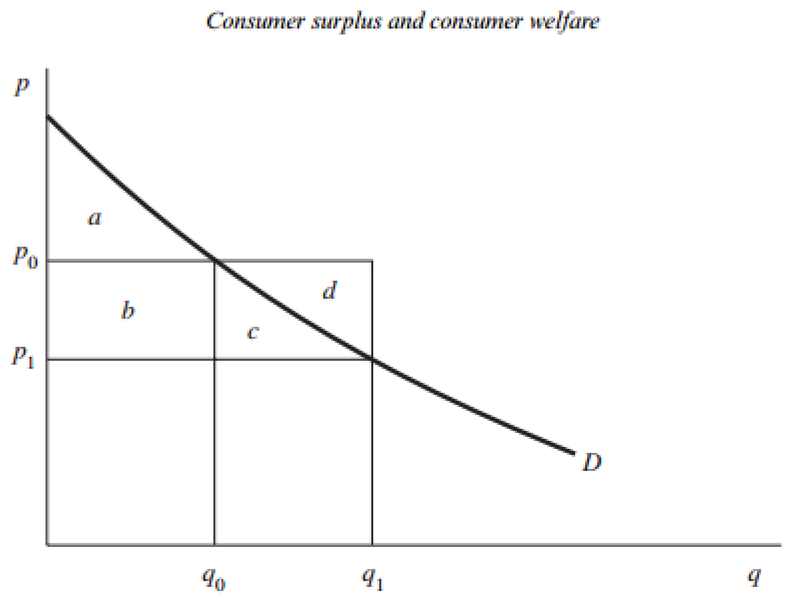
Continue…

- Thus, most measures do not seek to measure utility directly.
- Rather, they estimate a revealed WTP in terms of money.

- ‘Consumer surplus’ is the vehicle most often used in empirical work to measure consumer welfare.

THE NOTION OF CONSUMER SURPLUS

- Fig. 5.1



- Consumer gain, when p ↓s from p0 to p1

$$CG_{(p_1)} = area\,(b)|_{q_0}$$

- When p ↑s from p1 to p0

$$CL_{(p_0)} = area\,(b+c+d)\,|\,q_1$$

- This raises a paradox because the consumer apparently loses more with the price rise than is gained with the price fall.
  Intuition would suggest that the two changes should, on the contrary, exactly offset one another.

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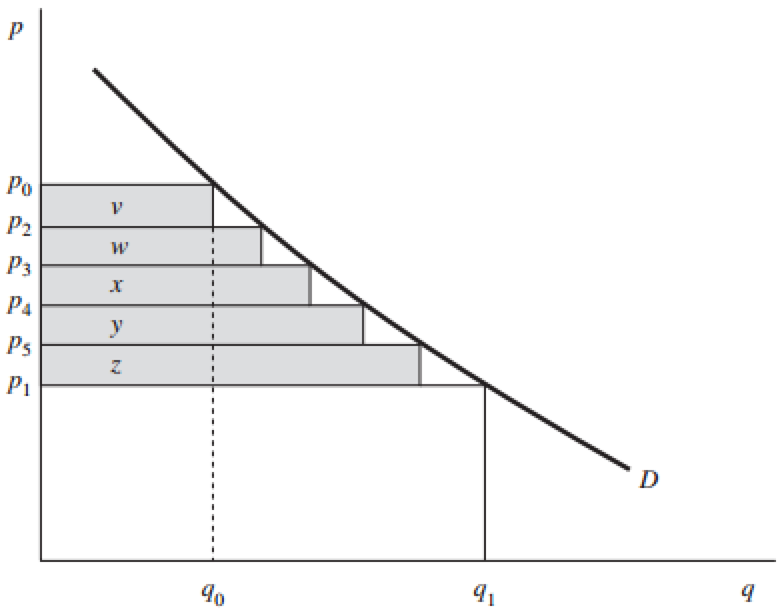
• 5.2



Obviously, as one divides the price change more finely, the shaded area begins to approach and become essentially synonymous with the area *b+ c* in Figure 5.1

- the price reduction is made in a series of small steps, as in Figure, from $p0$ to $p2$, then from $p2$ to $p3$, and so on until finally reaching price $p1$.
- The corresponding income equivalents in this context would be areas *v*, *w*, *x*, *y*, and *z*, respectively.
- Summing these effects over the entire price change, one would obtain the shaded area as a measure of the consumer's gain.

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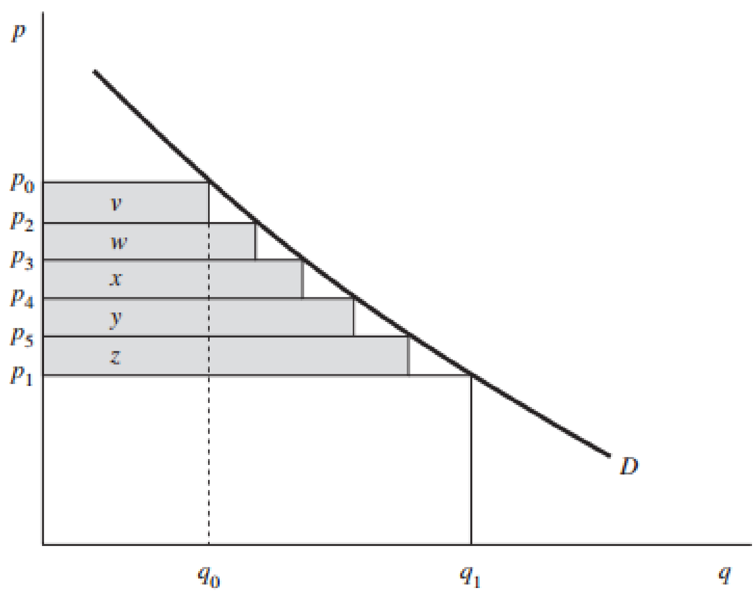
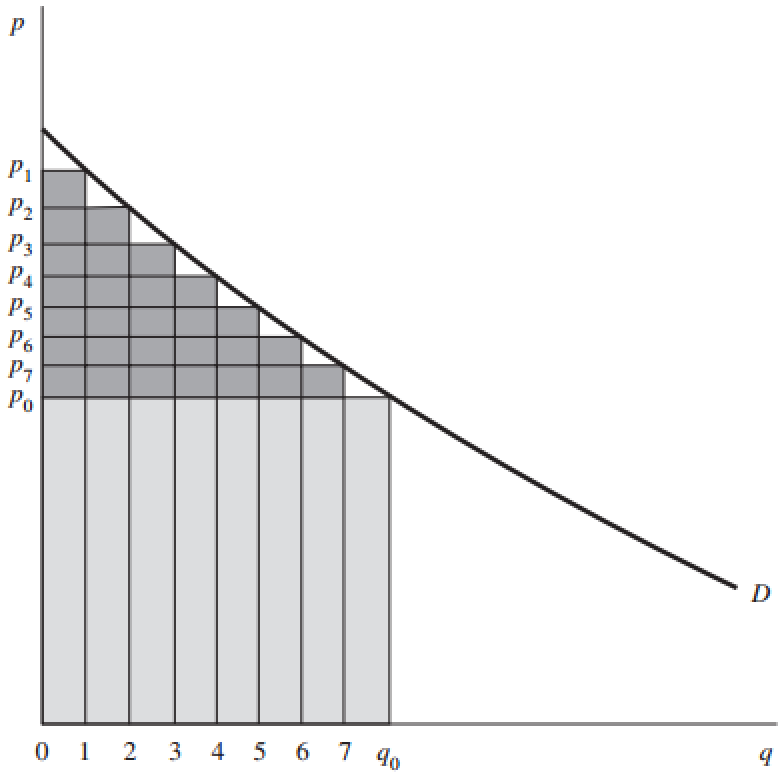


- Similarly, if one were to repeat this process in the case of a price rise, the corresponding income equivalent measure of loss for the consumer would also approach the area *b*+ *c* in Figure 5.1 as the number of price increments (adding up to *p*0 *p*1) becomes larger.
- The area *b*+*c* results from a summing of the cost differences in consumption bundles as price is continuously and incrementally varied from *p*0 to *p*1, or vice versa.
- Area *b*+ *c* is referred to as a change in consumer surplus,
- area *a*+ *b* +*c* is the total consumer surplus for price *p*1, and area *a* is the consumer surplus for price *p*0.
- That is, *consumer surplus is defined as the area under the demand curve and above the price line.*

Demand curve as marginal willing to pay (WTP) CURVE



Long ago, Jules Dupuit (1844), who actually coined the phrase ‘consumer surplus’, postulated that the price associated with any quantity on a consumer’s demand curve is the maximum price the consumer is willing to pay for the last unit consumed.

Hence, the demand curve is a marginal WTP curve.

Thus, in Figure 5.3, the consumer is viewed as willing to pay *p*1 for the first unit, *p*2 for the second, *p*3 for the third, and so on.

If the consumer actually pays only *p*0 for the entire quantity *q*0, then the consumer gains a ‘surplus’ of *p*1 - *p*0 on the first unit purchased because WTP exceeds what is actually paid by that amount.

The consumer gains similar, but declining, increments of surplus on each of the units purchased up to *q*0.

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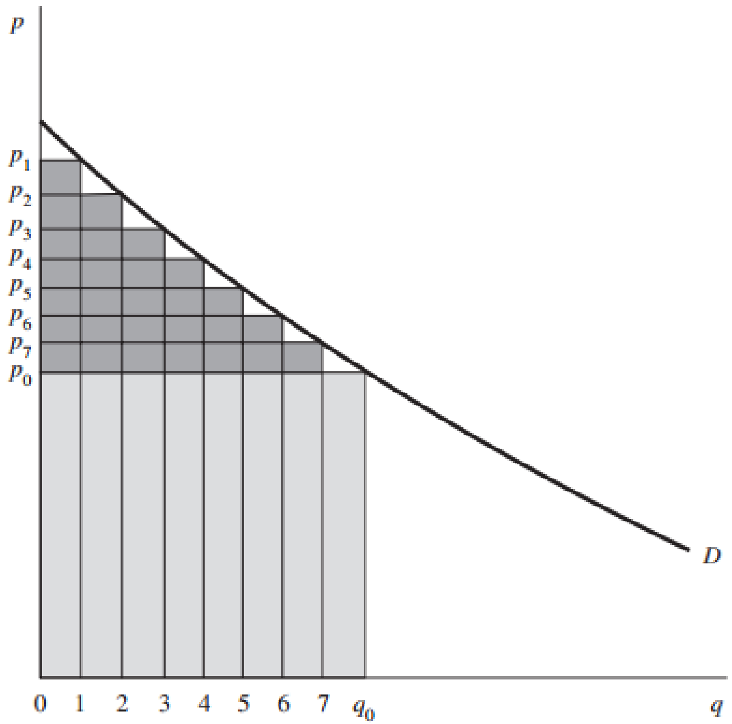

- In this context, the total area below the demand curve and left of the quantity consumed *q*0 (the lightly shaded area plus the heavily shaded area) is called *gross benefits*,
- while the area below the demand curve and above price *p*0 (the heavily shaded area), obtained by subtracting costs from gross benefits, is the net surplus accruing to the consumer from buying *q*0 at *p*0.
- Clearly, if the commodity *q* is perfectly divisible, the Dupuit surplus from buying *q*0 at *p*0 will simply be given by the triangle-like area above the price line *p*0 and to the left of, or behind, the demand curve.

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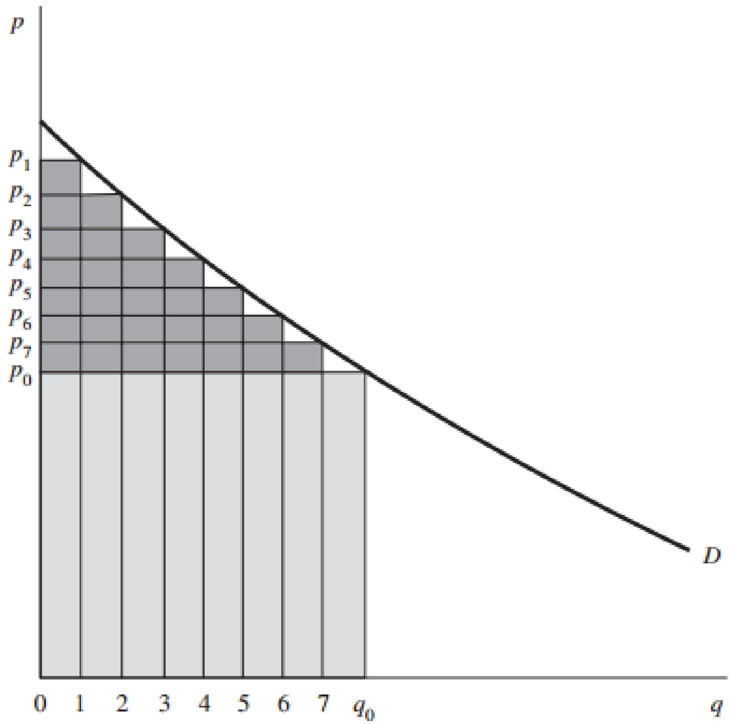
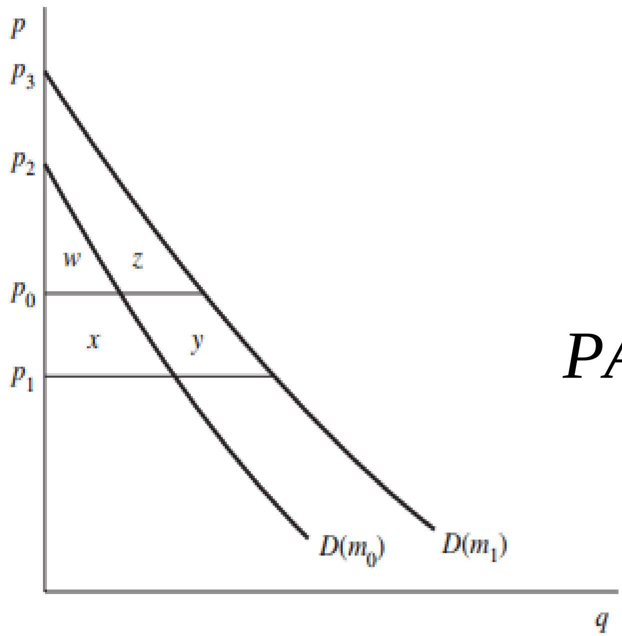


- Referring back to Figure 5.1, the benefits to the consumer of a price reduction from *p*0 to *p*1, following Dupuit, would be given by area *b*+ *c*, because this is the increase in the triangle-like area resulting from the price fall.
- Thus, on the surface, there are at least two reasons why the change in consumer surplus is appealing as a measure of consumer benefits:
- (1) it represents the sum of cost differences as price is continuously reduced from *p*0 to *p*1, and
- (2) it gives the change in what the consumer is willing to pay over that which is actually paid with the price change if the demand curve is a marginal WTP curve.

Continue…

- Consumer surplus appears to be a useful construct in consumer welfare measurement.
- It turns out, however, that the change in consumer surplus is not so well defined for the case where several prices change simultaneously, or where income changes together with price.
- In point of fact, the change in consumer surplus in the case of a multiple price change or a simultaneous price–income change depends on the order in which these price changes are considered or, more generally, on the path of adjustment.
- The associated problem is called the *path-dependence problem.*

PATH DEPENDENCE OF CONSUMER SURPLUS (**PRICE INCOME CHANGE CASE**)



$PATH-1$

- First P ↓, then m↑

$$\Delta CS = area(x)$$

$$\Delta m = m_1 - m_0$$

$$\Delta W_1 = m_1 - m_0 + area(x)$$

$PATH-2$

- First m ↑, then p↓

$$\Delta m = m_1 - m_0$$

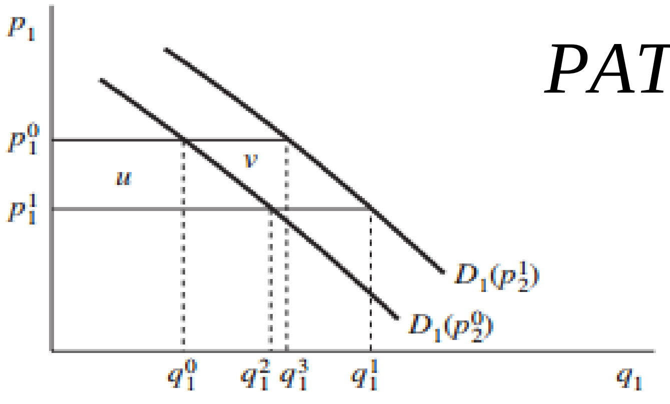
$$\Delta CS = area(x+y)$$

$$\Delta W_2 = m_1 - m_0 + area(x+y)$$

$$\Delta W_1 \neq \Delta W_2$$

$$m_1 - m_0 + area(x) \neq m_1 - m_0 + area(x+y)$$

PATH DEPENDENCE OF CONSUMER SURPLUS (PRICE-PRICE CHANGE CASE)

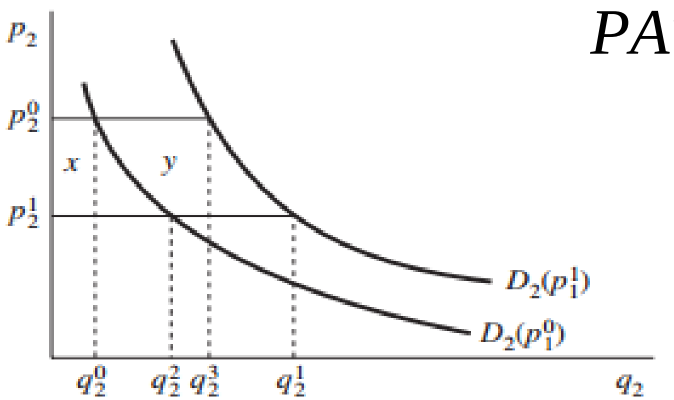


(b)

$PATH-1$

- First $P_1\downarrow$, then $P_2\downarrow$

$$\Delta CS_1 = area(u)$$
$$\Delta CS_2 = area(x+y)$$
$$\Delta W_1 = \Delta CS_1 + \Delta CS_2$$
$$\Delta W_1 = area(u) + area(x+y)$$
$$\Delta W_1 = area(u+x+y)$$



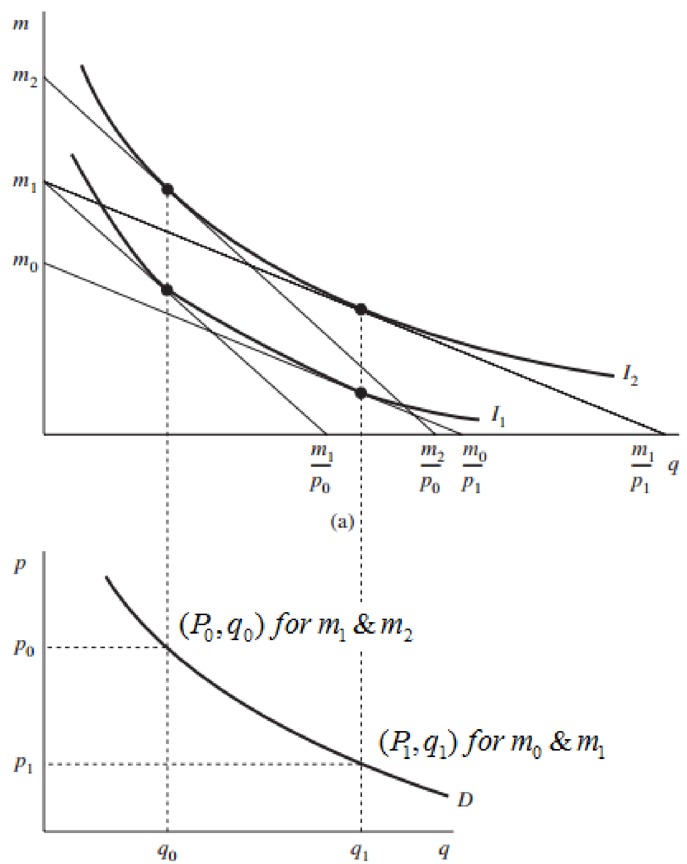
(c)

$PATH-2$

- First $P_2\downarrow$, then $P_1\downarrow$

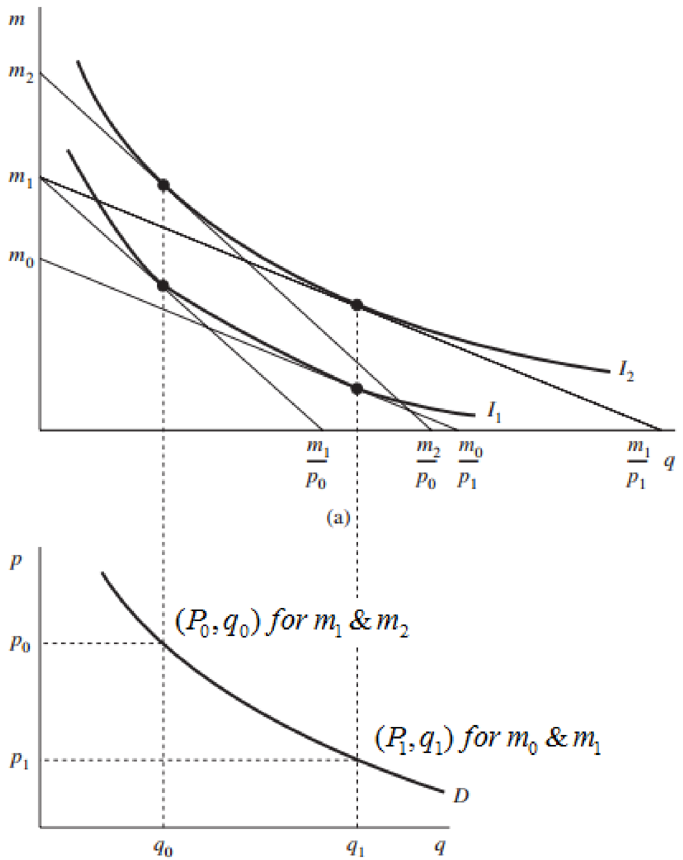
$$\Delta CS_1 = area(x)$$
$$\Delta CS_2 = area(u+v)$$
$$\Delta W_2 = \Delta CS_1 + \Delta CS_2$$
$$\Delta W_2 = area(x) + area(u+v)$$
$$\Delta W_2 = area(x+u+v)$$
$$\Delta W_1 \neq \Delta W_2$$
$$area(u+x+y) \neq \Delta W_2 = area(x+u+v)$$

UNIQUENESS OF CONSUMER SURPLUS (simultaneous price–income change)



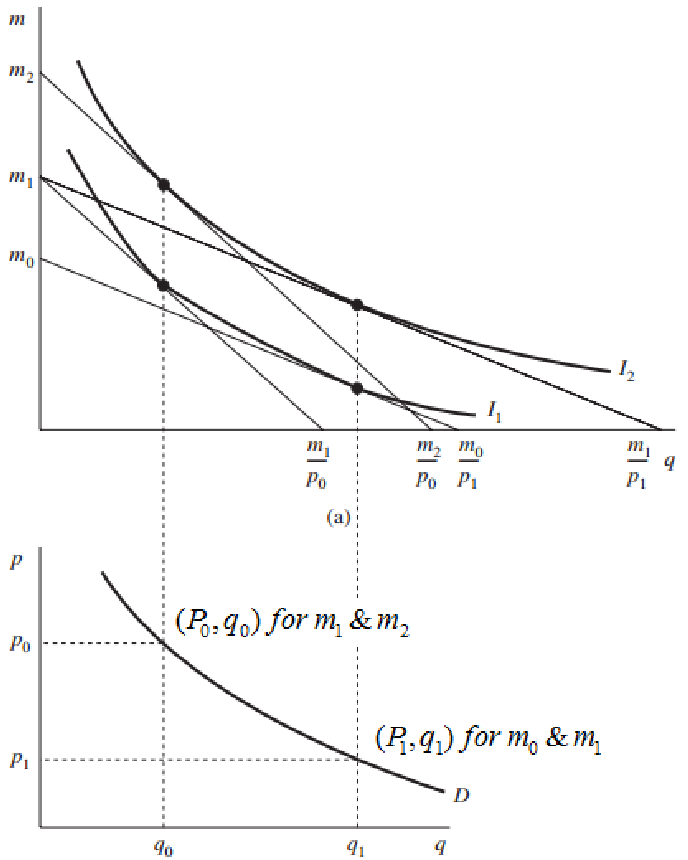
- when both price and income change,
- *the consumer surplus measure is unique*
- *if, and only if,*
- *the income effect is zero* –
- meaning that the change in quantity consumed, Δq, associated with a change in income, Δm (that is, $\Delta q/\Delta m$), is zero.

UNIQUENESS OF CONSUMER SURPLUS (simultaneous price–income change)



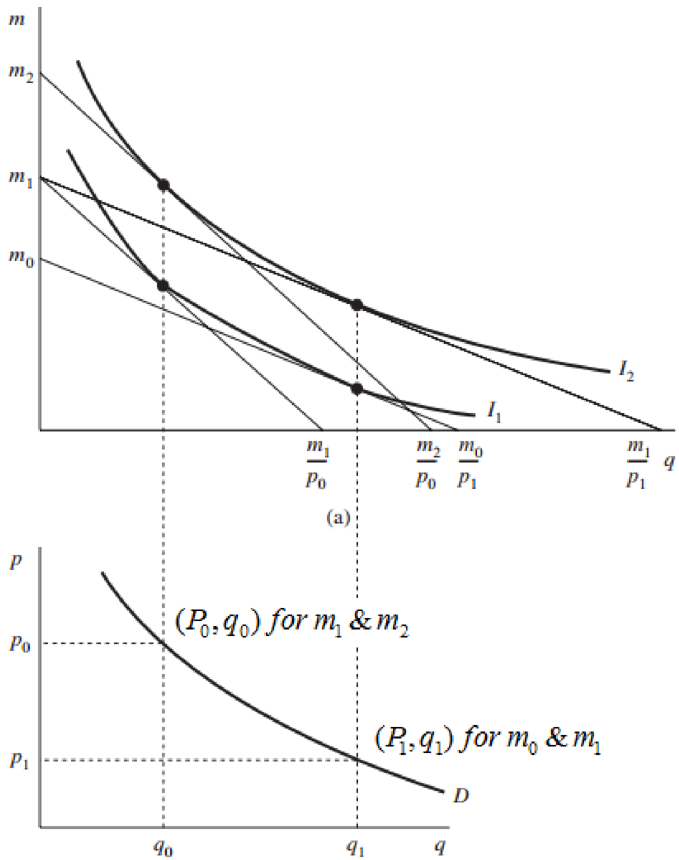
- Trivially, this condition is associated with zero income elasticity because income elasticity is defined by
- $\eta_m = (\Delta q/\Delta m) \cdot (m/q)$

UNIQUENESS OF CONSUMER SURPLUS
(simultaneous price–income change)



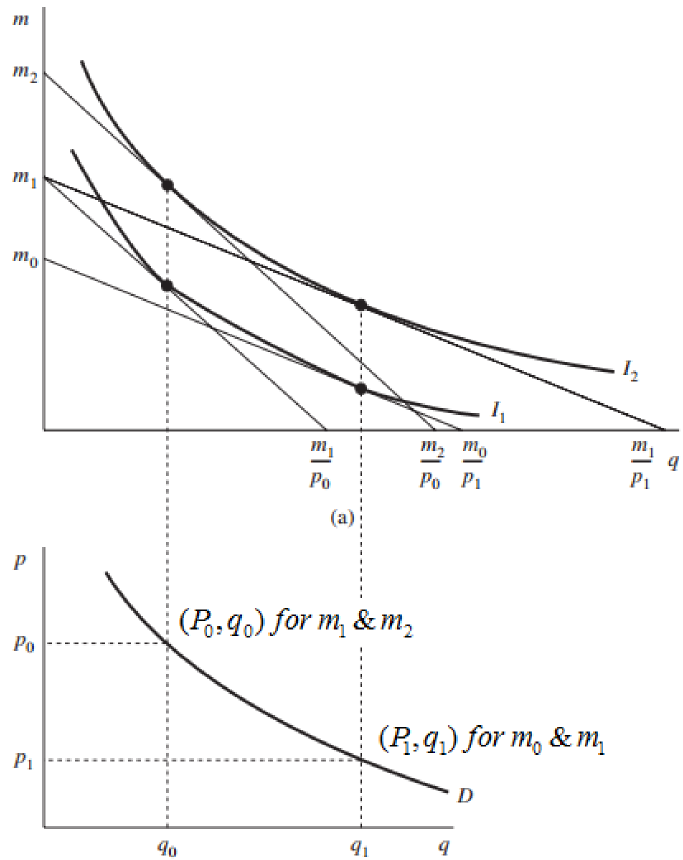
- It is also instructive to investigate the implications of uniqueness of the surplus measure in terms of the consumer's indifference map.
- The case where indifference curves lead to the same demand curve, regardless of income level, is demonstrated in Figure 5.6.

UNIQUENESS OF CONSUMER SURPLUS (simultaneous price–income change)



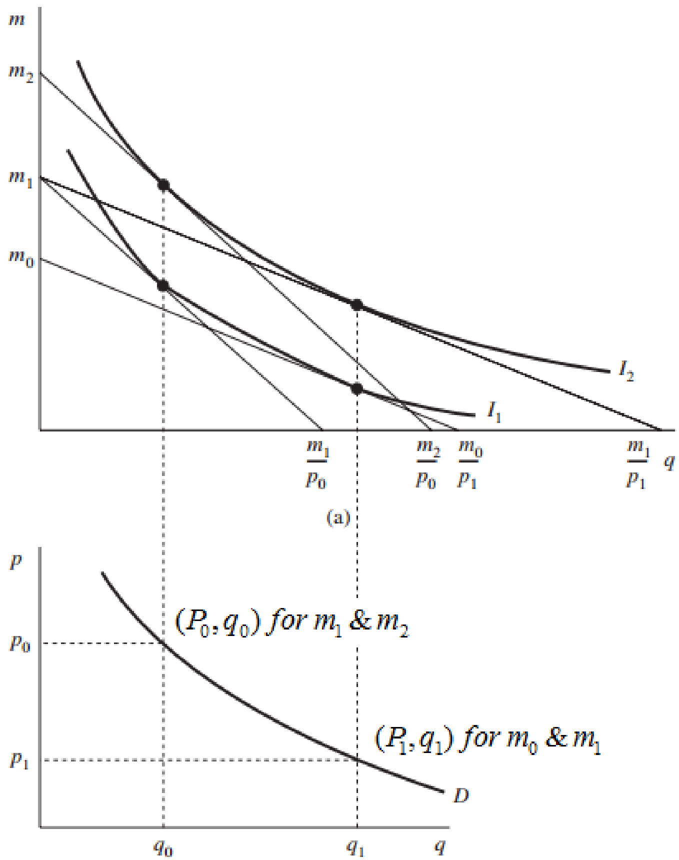
- The indifference curves in Figure 5.6(a) are *I*1 and *I*2.
- As income is changed from *m*0 to *m*1 at price *p*1, the quantity consumed remains at *q*1 because the associated tangencies of the budget lines with indifference curves lie directly above one another at a quantity of *q*1.

UNIQUENESS OF CONSUMER SURPLUS (simultaneous price–income change)



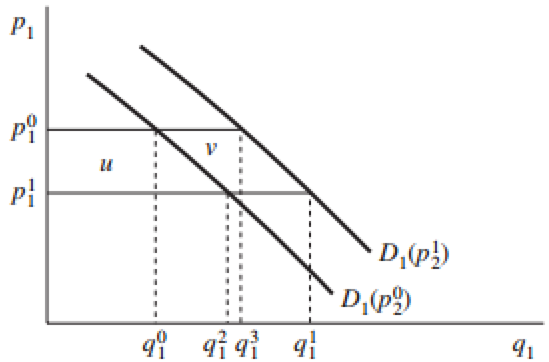
- Thus, the demand curves for both levels of income include the point ($p1$,$q1$).
- Similarly, the budget line/indifference curve tangencies all occur at quantity $q0$ when price is $p0$.
- In other words, the same demand curve *D* results in Figure 5.6(b) regardless of income level.

UNIQUENESS OF CONSUMER SURPLUS (simultaneous price–income change)

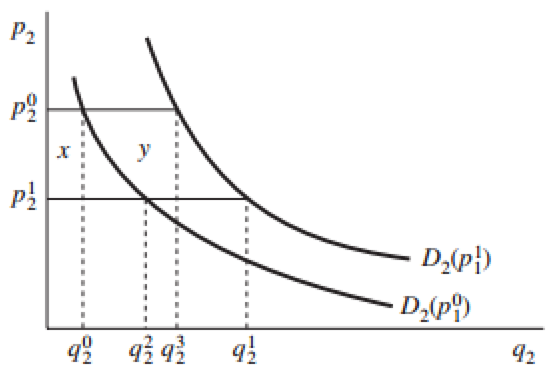


- Furthermore, because the budget lines for different income levels but the same price must be parallel, it is clear that coincidence of the demand curve at different income levels is obtained if, and only if, the consumer's indifference curves are vertically parallel.
- In the latter case, the income-expansion path is a vertical straight line for any set of prices.

UNIQUENESS OF CONSUMER SURPLUS (simultaneous price–price change)



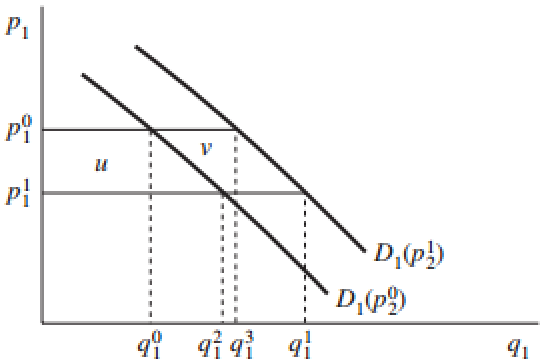
(b)



(c)

- In this case the two paths *L*1 and *L*2 lead to the same consumer surplus change
- if area $(u + v + x)$ = area $(u + x + y)$
- (that is, if area (v) =area (y).
- Under what conditions would these areas be equal?

UNIQUENESS OF CONSUMER SURPLUS (simultaneous price–price change)



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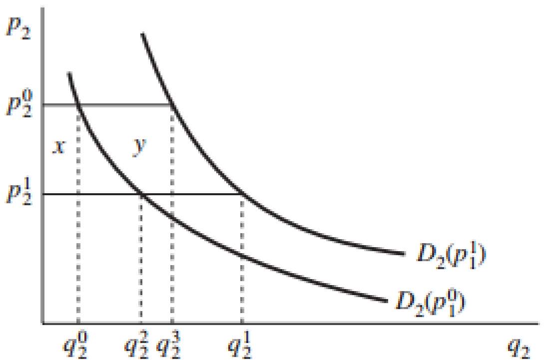
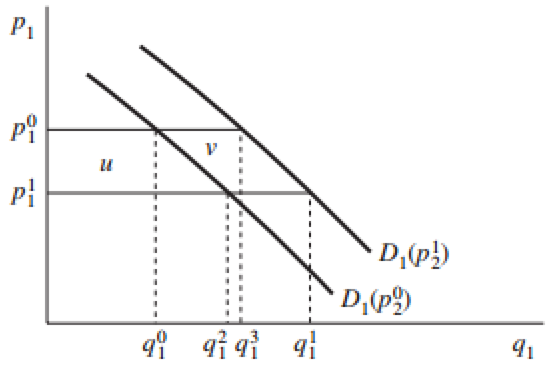


(c)

- To answer this question, consider arbitrary price changes, $\Delta P_1 = P_1^0 - P_1^1$

  $\Delta P_2 = P_2^0 - P_2^1$
- If these price changes become small,
- *v* and *y* are approximately parallelograms, in which case the corresponding areas are given by the product of the price changes and the respective quantity changes, $\Delta q_1 = q_1^3 - q_1^0 = q_1^1 - q_1^2$

UNIQUENESS OF CONSUMER SURPLUS (simultaneous price–price change)



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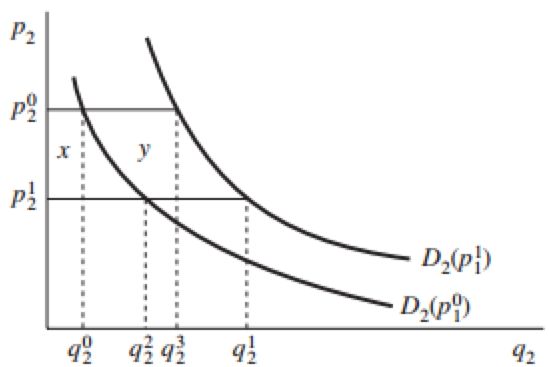

(c)

- Thus, the conditions of equality of areas *v* and *y* become

$$\Delta p_1 \times \Delta q_1 = \Delta p_2 \times \Delta q_2$$

- or

$$\frac{\Delta q_1}{\Delta p_2} = \frac{\Delta q_2}{\Delta p_1}$$

UNIQUENESS OF CONSUMER SURPLUS (simultaneous price–price change)

- In intuitive terms, this condition implies that demand must be such that the change in consumption of the first good associated with a small unit change in the price of the second good must be the same as the change in consumption of the second good associated with a small unit change in the price of the first.
- If area(*v)=* area (*y)* for all arbitrary sets of price changes,
- this condition must hold all along both sets of demand curves.

Economic Implications

- Consider, however, the economic implications of these conditions.
- As in the preceding case, an interesting implication can be developed by relating the conditions to a change in income.
- To do this, one can use the concept of *zero-degree homogeneity of demand in prices and income.*

Economic Implications

- This implies that a consumer's consumption bundle choice is not altered as all prices and income are adjusted proportionally (for example, consider redenominating the unit of currency).

Continue…

- In this context, suppose that all prices are adjusted proportionally so that $p_1^1 = \alpha p_1^0 \quad (1)$
  &
- $p_2^1 = \alpha p_2^0 \quad (2)$
- Changes in prices $\Delta p_1 = p_1^1 - p_1^0 \quad (3)$
  $\Delta p_2 = p_2^1 - p_2^0 \quad (4)$
  putting the alues of p_1^1 & p_2^1 *from* (1) & (2) *in* (3) & (4)
- *we get*
  $\Delta p_1 = \alpha p_1^0 - p_1^0$
  $\Delta p_1 = (\alpha - 1) p_1^0$
  $\Delta p_2 = \alpha p_2^0 - p_2^0$
  $\Delta p_2 = (\alpha - 1) p_2^0$

Continue…

- Using the path independence condition

$$\frac{\Delta q_1}{\Delta q_2} = \frac{\Delta p_2}{\Delta p_1} = \frac{(\alpha - 1)\, p_2^0}{(\alpha - 1)\, p_1^0} = \frac{p_2^0}{p_1^0}$$

- Hence, the ratio of adjustments of $q1$ and $q2$ corresponding to any proportional changes in prices is a constant (that is, the ratio determined by initial prices) no matter how much prices are adjusted.

Continue…

- Using homogeneity of demand, as shown above, indicates that this proportional change in prices is equivalent to an inversely proportional change in income.
- For example, doubling income has the same effect on the consumer (excluding wealth considerations) as cutting all prices by half.
- Hence, the foregoing arguments also imply that the ratio of consumption adjustments in response to an income adjustment is a constant determined completely by prices, regardless of income level.

Continue…

- Interpreting these results in the context of the consumer's indifference map thus implies straight-line income-expansion paths emanating from the origin, for example,
- $E(p_1^0, p_2^0)$ and $E(p_1^1, p_2^1)$ as in Figure 5.7.

Continue…

- Fig

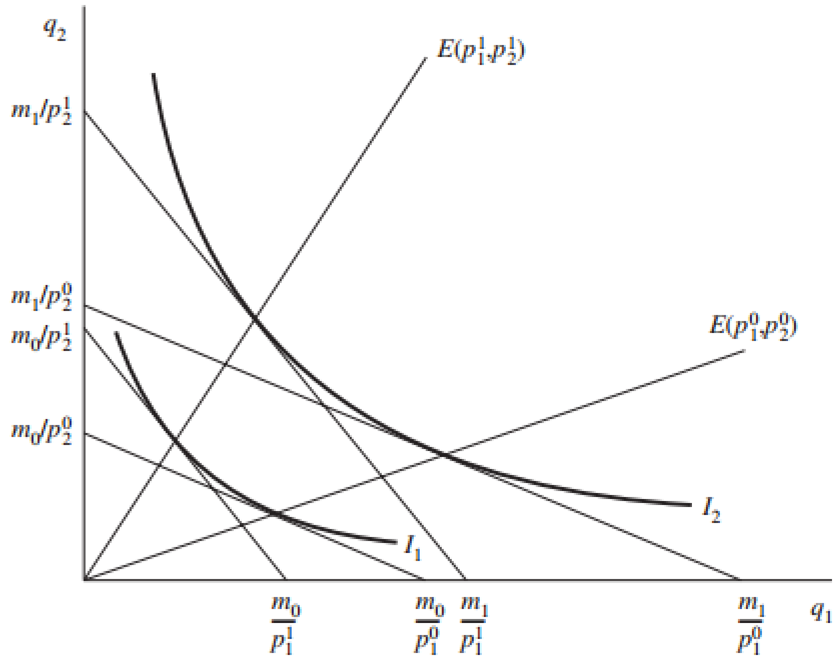

Figure 5.7

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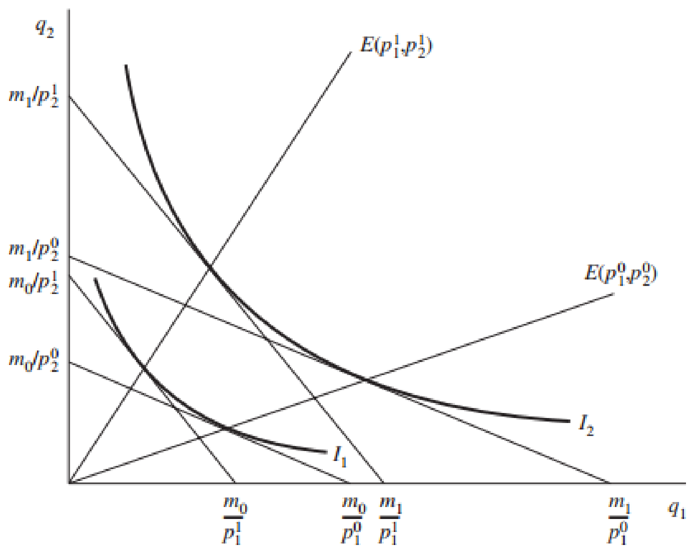

Figure 5.7

- As income is adjusted upward from zero with prices p_1^0 and p_2^0, the changes in quantities are always proportional.
- Thus, the ratio between quantities is a constant
- $\dfrac{p_2^0}{p_1^0}$.
-

- An indifference map with these properties is generally called *homothetic*.

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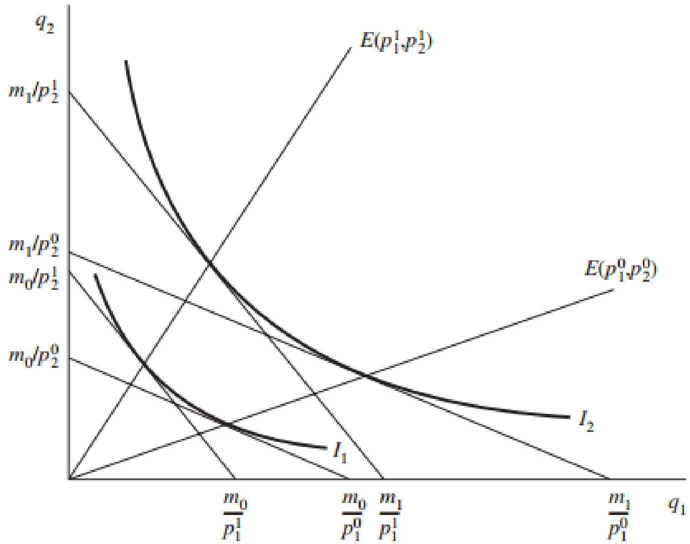

Figure 5.7

- Geometrically, from Figure 5.7, homotheticity clearly implies that any percentage change in income (holding prices fixed) leads to an equal percentage change in all quantities consumed. Hence, all income elasticities of demand must be 1.