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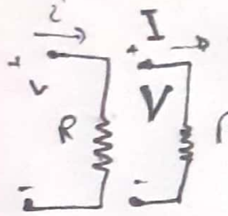
Phasor Relationship for circuit Elements

Resistor



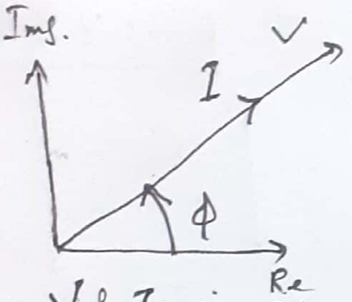
Time domain

$v = iR$

freq. domain

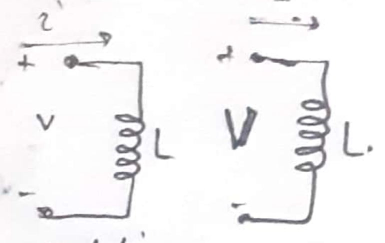
$V = IR$

No. Phase differen



V & I in Phase

Inductor



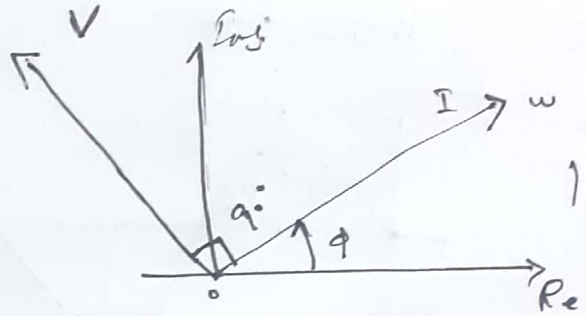
$v = L\frac{di}{dt}$

Time domain

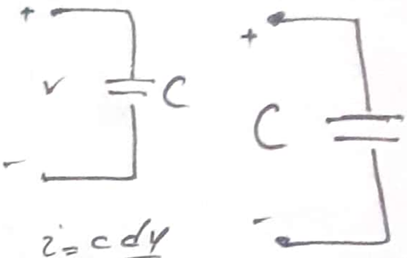
$V = j\omega L I$

$V = X_L I$

$V = j\omega L I$



Capacitor



$i = c\frac{dv}{dt}$

Time domain

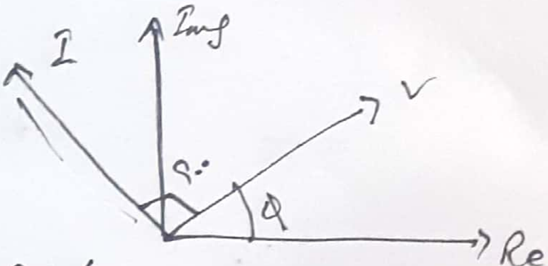
$I = C.V\, j\omega$

$V = \frac{I}{jC\omega}$

$V = \frac{I}{j\omega C}$

$V = \frac{-jI}{\omega c}$

"Phasor diagram"



Hint

Remember

ELI is ICE MAN

Voltage lead in Inductor

current Lag in Inductor

current lead in Capacitor

Voltage lag in capacitor

"OR"

$\boxed{\text{CIVIL}}$

In Inductor

In Capacitor

current Lead

Voltag "lag

Voltag lead

current Lag.

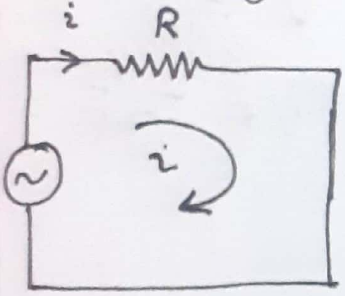
1

Summary of Voltage - current Relationship

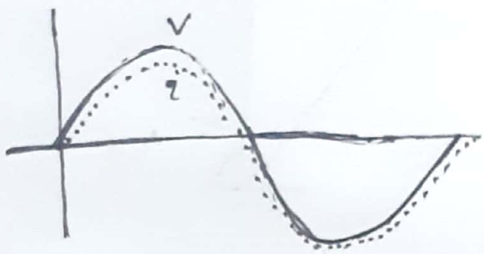
Element	Time domain	Frequency domain
R	$V = iR$	$\mathbf{V} = \mathbf{I}R$
L	$V_L = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\frac{dV}{dt}$	$\mathbf{I} = j\omega C\mathbf{V}$ $\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$

A.C Response of the Basic Elements

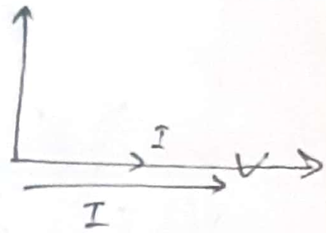
Circuit diagram



Wave form



Phasor Diagram



According to ohm's Law

$$i = \frac{V}{R} \qquad \because V(t) = V_m \sin(\omega t)$$

$$V = V_m \sin(\omega t)$$

$$i = \frac{V_m \sin(\omega t)}{R}$$

$\frac{V_m}{R} \rightarrow I_m$

As we know that voltage drop across Resistance $= V = iR$

$$i = I_m \sin(\omega t)$$

$$V = iR \qquad \text{Putting the value of } i$$

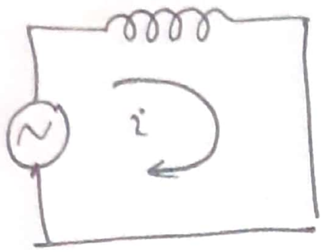
$$V_m \sin(\omega t) = I_m \sin(\omega t) \cdot R$$

$$V_m = I_m \cdot R$$

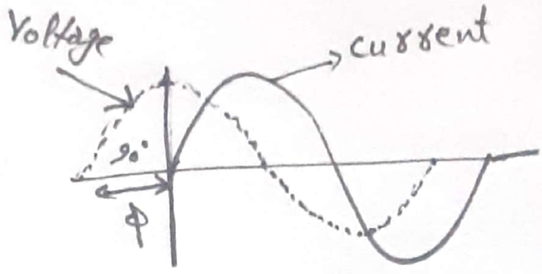
$$I_m = \frac{V_m}{R}$$

$\because$ Remember when the value of sine become unity, it mean that time the value of current i is its maximum value.

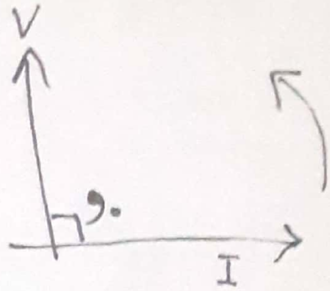
AC Response of the Inductor :-



(a) circuit diagram



(b) wave form



(c) Phasor diagram

⟹ back Emf due to AC, L/Inductor oppose the change in current

Assume ~~consider~~ current

$$i = I_m \sin(\omega t) \longrightarrow 1$$

The voltage Across the Inductor

$$V_L = L\frac{di}{dt}$$

(Putting the value of i from Equation 1)

$$V_L = L\frac{d}{dt}\left(I_m \sin\omega t\right)$$

$$V_L = L I_m \frac{d}{dt}\left(\sin(\omega t)\right)$$

$$V_L = L I_m \cos(\omega t)\frac{d}{dt}(\omega t)$$

$$V_L = \omega L I_m \cos(\omega t)$$

$$V_L = V_m \cos(\omega t)$$

$\therefore$ $V = IR$

$V_m = I_m R$

$V_m = I_m \cdot L$

$V_m = \omega L I_m$

As we know that

$$\cos(\omega t) = (\sin \omega t + 90°)$$

or

$$\sin(\omega t + 90°) = \cos\omega t$$

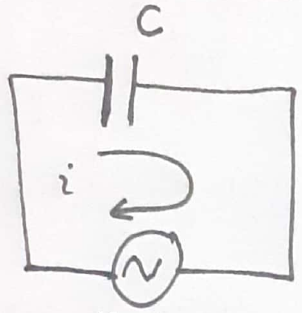
So voltage wave form is cosine where as current wave form is sine.

There fore Voltage lead 90° by current.

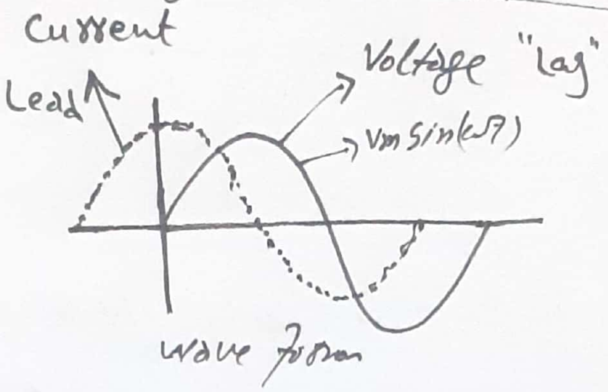
$\therefore$ So The Voltage wave form leads by current 90°.

$\therefore$ ———— .

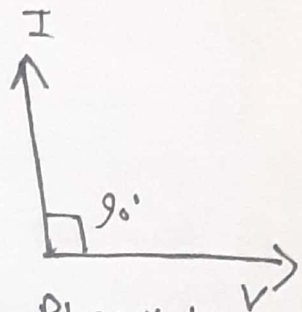
describe the AC Response of the capacitor alongwith circuit diagram, Phasor diagram & wave form :



circuit diagram



wave form



Phasor diagram

Capacitor don't like the abrut change in Voltage

$$V(t) = V_m \sin(\omega t) \qquad \therefore V = V_m \sin(\omega t)$$

The current through the capacitor is

$$i = C\frac{dV}{dt}$$

$$i = C\frac{d}{dt}\left(V_m \sin(\omega t)\right)$$

$$i = C V_m \frac{d}{dt}\left(\sin(\omega t)\right)$$

$$i = C V_m \cos(\omega t) \frac{d}{dt}(\omega t)$$

$$i = \omega C V_m \cos(\omega t)$$

$$\downarrow$$

$$i = I_m \cos(\omega t)$$

$\therefore$ Ohm's Law

$$i = V/R$$

$$i = \frac{V}{\frac{1}{C}}$$

$$i = CV$$

As we know that

$$\cos(\omega t) = \sin(\omega t + 90^\circ)$$

$$\therefore \quad i = I_m \sin(\omega t + 90^\circ)$$

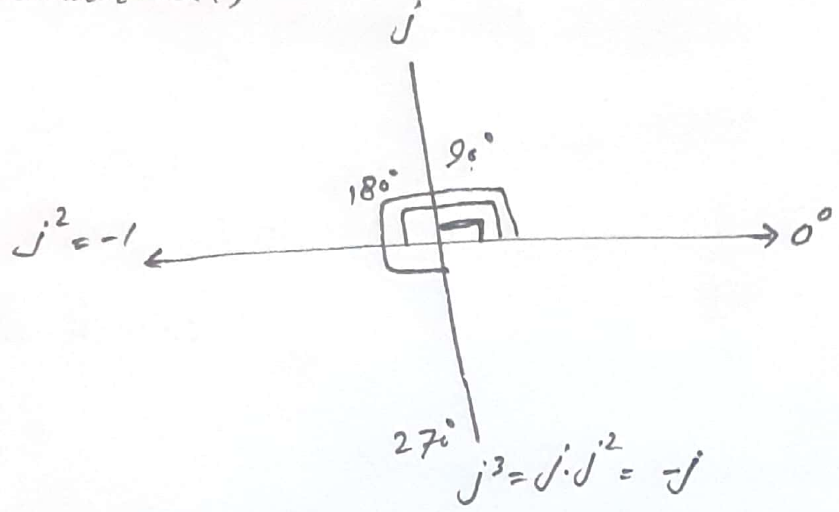
Therefore: we can say That the voltage wave form Lags The current wave form by 90°.

"OR"

current Lead 90° by voltage

Alternative concept of Phasor of Basic Elements

Consider axis

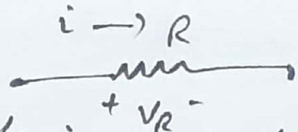


we can say, if Any term containing j, then that term Lead 90°.

$\Rightarrow$ if (−1) then 180° lead.

Application in Electrical circuits

① Consider Resistance (R)

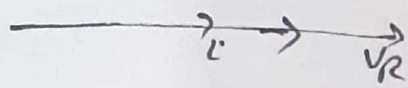


i is current flowing & voltage drop is V_R

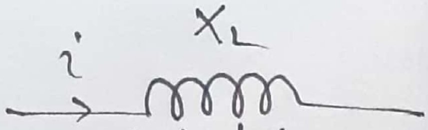
$\Rightarrow$ According to ohm's Law $\Rightarrow$ $V_R = iR$

Analyze there is no angle between them, so both voltage & current are in Phase.

Now



② Consider Inductor



Inductance of Inductor $= X_L$

Voltage Drop will be $= V_L$

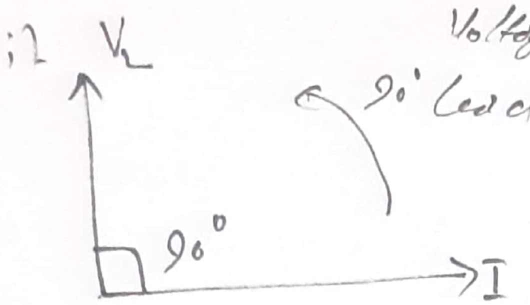
so $V_L = iX_L$ $\quad\therefore X_L = j\omega L$

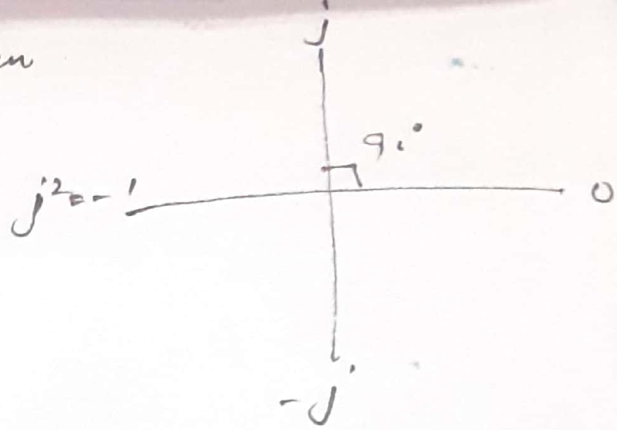
$V_L = j\omega IL$

$V_L = jI\omega L$

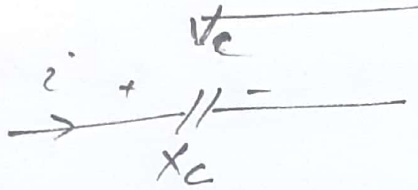
we see from Expression that Inductive voltage is lead 90°

$\therefore$ i : Phasor Diagram





3rd Concept Consider Capacitor



$X_c = \frac{1}{j 2\pi f c}$

$X_c = \frac{1}{j\omega c}$

Capactive ~~Impedance~~ Reactance = $X_c = \frac{1}{j\omega c}$

$\therefore$ Voltage drop across Capacitor $= V_c = X_c i$

$X_c = \frac{1}{j\omega c}$

$V_c = \frac{I}{j\omega c}$

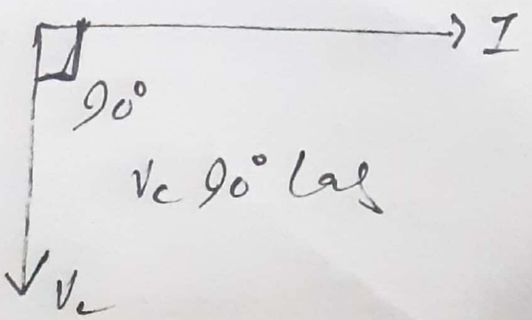
Rationalize

$V_c = -jI/\omega c$

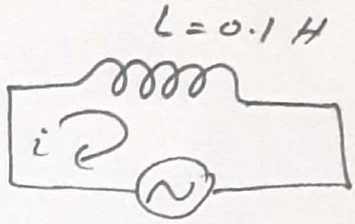
$V_c = \frac{-jI}{\omega c}$

$\Rightarrow$ Current lead & Voltage lag

V_c will Lag 90° of current



Example 9.8 :. The voltage $v = 12\cos(60t + 45^\circ)$ is applied to a 0.1 H Inductor. Find the Steady State current through the Inductor.



$v = 12\cos(60t + 45^\circ)$

Solution

for Inductor $V = j\omega L I$

where $\omega = 60$ rad/s & $V = 12\angle 45^\circ$

As Per formula

$$I = \frac{V}{j\omega L}$$

$$I = \frac{12\angle 45^\circ}{j \times 60 \times 0.1}$$

$$I = \frac{12\angle 45^\circ}{1\angle 90^\circ \times 6}$$

$$I = \frac{\cancel{12}^{\,2}\angle 45^\circ}{\cancel{6}\angle 90^\circ}$$

$$I = 2\angle 45^\circ - 90^\circ$$

$$I = 2\angle -45^\circ \text{ Amp}$$

Converting this to the time domain

$$\boxed{i(t) = 2\cos(60t - 45^\circ)\,A}$$

$V = L\dfrac{di}{dt}$

$V = L\,j\omega\,I$

$I = \dfrac{V}{j\omega L}$

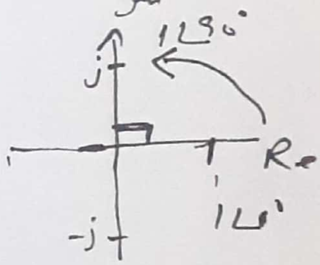
$j = 1\angle 90^\circ$

90° Lead

$j^0 = 1$

$j^1 = j = 1\angle$

$-j = 1\angle -90^\circ$



$i = \sqrt{-1}$

$j = \sqrt{-1}$

$j = (-1)^{1/2}$

$1\angle 90^\circ$

90° Shift

iota = $i = j$

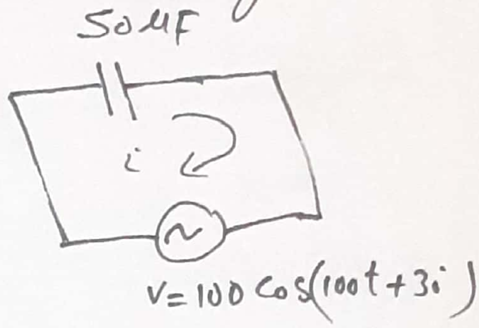
$\underbrace{j = \sqrt{-1} \quad \text{but in E.E we read as } 1\angle 90^\circ}$

$\therefore \int \;:\; 4\int dx = 4x \quad = \int \Rightarrow j4 \Rightarrow 4\angle 90^\circ$

$\frac{d}{dt} \;:\; \frac{d4}{dt} = 0 \qquad 90^\circ$ Shift

~~Example~~ Practice Problem 9.8

If voltage $v = 10\cos(100t + 30^\circ)$ is applied to a $50\ \mu F$ Capacitor, calculate the current through the Capacitor.?



Solution

$$v = 10\cos(100t + 30^\circ)$$

$$V = 10\angle 30^\circ$$

$$\omega = 100\ rad/s$$

$$C = 50\times10^{-6}\ F$$

$$I = ?$$

∴ As we know that

$$V = X_c I$$

$$V = \frac{1}{j\omega C}\cdot I$$

$$V\cdot j\omega C = I$$

∴ $j = 1\angle 90^\circ$

So Putting formula

$$I = j\omega C\cdot V$$

$$I = 1\angle 90^\circ\cdot 100\times 50\times10^{-6}\times 10\angle 30^\circ$$

$$I = 10\angle 90^\circ+30^\circ\cdot(0.005)$$

$$I = 0.05\angle 120^\circ\ A$$

$$I = 50\angle 120^\circ\ A$$

In time Domain

$$i(t) = 50\cos(100t + 120^\circ)\ mA$$

Impedance & Admittance

Ac Impedance

Voltage - current Relationship for the three Passive Element

Time Domain	Phasor domain
Generalized ohm's Law	
$v = iR$ [Resistance]	$V_R = IR$
$v_L = L\frac{di}{dt}$ [Inductor]	$V_L = j\omega L I$
$\frac{dv}{dt} = \frac{i}{c}$ [Capacitor]	$V_c = \frac{1}{c} \cdot \frac{I}{j\omega}$
$v = \frac{1}{c}\int i.dt$	$V_c = \frac{I}{j\omega c}$

$i = c\frac{dv}{dt}$

$V = X_L \cdot I$

$V = j\omega L \cdot I$

or $V_c = X_c \cdot I$

$V_c = \frac{I}{j\omega c}$

$V = IR$, $V = j\omega L I$, $V_c = \frac{I}{j\omega c}$

These Equation may be written in term of Ratio of the Phasor voltage to the Phasor current

$\frac{V}{I} = R$, $\frac{V}{I} = j\omega L$, $\frac{V}{I} = \frac{1}{j\omega c}$

$Z = R$ $\quad Z = j\omega L \quad Z = \frac{1}{j\omega c}$

$Z = -j/\omega c$

" Definition of Impedance

"The Impedance Z of a circuit is the "Ratio of the ~~circuit~~ Phasor voltage (V) to the Phasor current (I), measured in ohm (Ω)

Mathematically

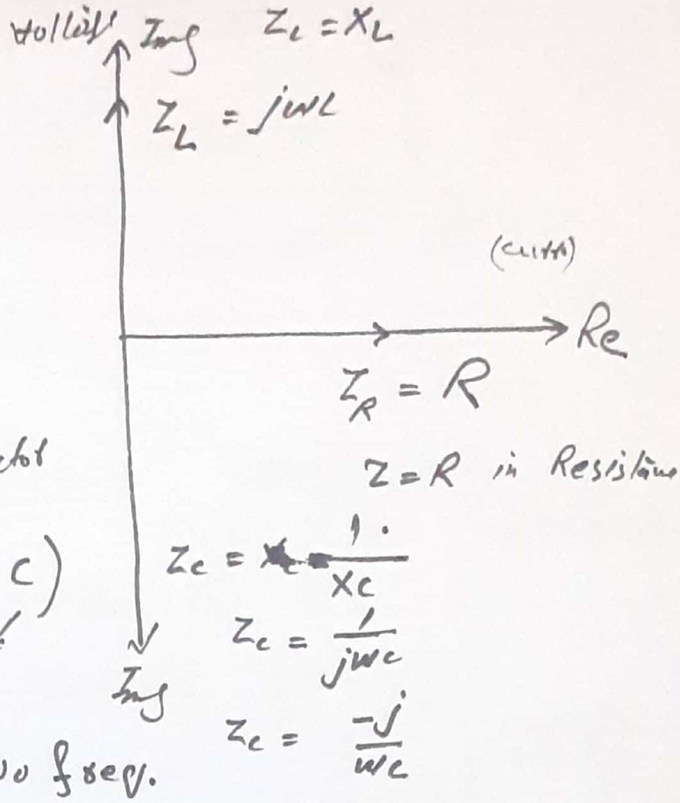
$$Z = \frac{V}{I} \Rightarrow V = IZ$$

" The impedance Represents the opposition that the circuit exhibits to the flow of Sinusoidal Current".

$Z_R = R$

$Z_L = jwL$

$Z_c = \frac{-j}{wc}$



Consider two Extreme cases of angular frequency.

Inductor

(i) <u>DC Source</u> (behavior of L & C)

capacitor

when $\omega = 0$

$Z_L = j\omega L$

$Z_L = j2\pi f L$ as DC has No freq.

$\boxed{Z_L = 0}$ So $X_L = 2\pi f L$, No Inductive Reactance

So The Inductor acts like a short circuit at dc.

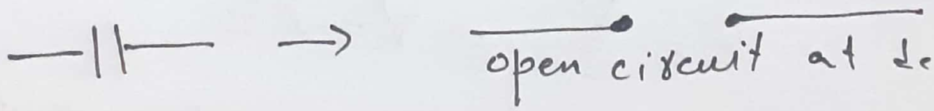
if f is high then X_L (Inductive Reactance) will be High, then Impedance will also very high & act as open circuit for AC

(ii) when $w_c \rightarrow \infty$ i.e $\boxed{W = 0}$ in capacitive Reactance

$$X_c = \frac{1}{2\pi f c} = x_c = \frac{1}{0} = \infty$$

So $$Z_c = \frac{1}{jwc} \Rightarrow Z_c = \frac{1}{0} = \infty$$

The capacitor acts like an open circuit at dc because of Infinite capacitive Impedance.



$\downarrow Z_c = \frac{1}{jwc \uparrow}$ / $\downarrow X_c = \frac{1}{2\pi f c \uparrow}$

Admittance (Y)

"The admittance (Y) is the reciprocal of impedance, measured in Semens (S) → mho

$\therefore \quad Y = 1/Z$

Element	Impedance	Admittance
R	$Z = R$	$Y = \frac{1}{Z}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

$\therefore \quad Z = R + jX$

Real ← → Imaginary

$Z = R + jX$ is Inductive /

$Z = R - jX$ is capactive

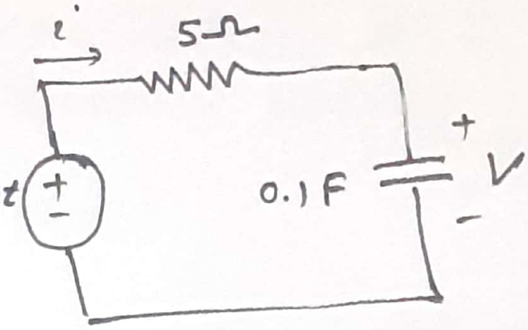
So Admittance $= Y = \frac{1}{Z} = \frac{1}{R+jX}$

$Y = G + jB$

Conductance $G = \frac{1}{R}$

susceptance ⇒ jB

Example 9.9 Find $v(t)$ and $i(t)$ in the circuit shown in Fig. below



Data

$V_s = 10\cos 4t$

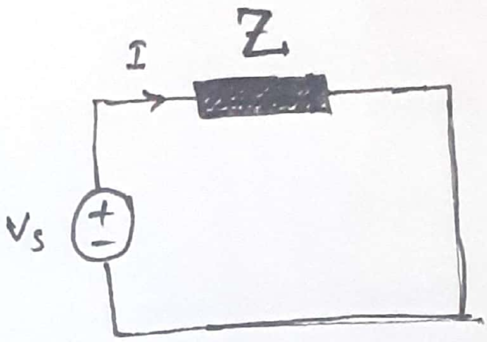
$\omega = 4$

$C = 0.1\ F$

$\therefore V_s = 10\cos(4t + 0^\circ)$

$\mathbf{V_s} = 10\angle 0^\circ$

As Required $I = \frac{V}{Z}$



First, we will find the "Z" Impedance

$$Z = R + X_c$$

$$Z = R + \frac{1}{j\omega c}$$

$$Z = 5 + \frac{j}{j^2 \times 4 \times 0.1}$$

$$Z = 5 - \frac{j}{0.4}$$

$$Z = 5 - j2.5\ \Omega$$

Hence the current $\Rightarrow I = \frac{V_s}{Z}$

$$\mathbf{I} = \frac{10\angle 0^\circ}{5 - j2.5}$$

$$I = \frac{10}{5 - j2.5} \times \frac{5 + j2.5}{5 + j2.5}$$

$$I = \frac{50 + j25}{25 - j^2 6.25}$$

$$I = \frac{50 + j25}{25 + 6.25} = \frac{50}{31.25} + \frac{j25}{31.25}$$

$$I = 1.6 + j0.8$$

Convert into Polar form

Convert into Polar directly

$$I = 1.789 \angle 26.57^\circ \ A$$

⇒ The voltage across the capacitor is

$$V = I Z_c \qquad \therefore Z_c = \frac{1}{j\omega c}$$

$$V = \frac{I}{j\omega c}$$

$\because i = c\frac{dv}{dt}$

$\int \frac{i}{c} = \int \frac{d}{dt}(v)$

$\frac{1}{c}\int I = V$

$V = \frac{I}{j\omega c}$

$$V = \frac{1.789 \angle 26.57}{1\angle 90^\circ \times 4 \times 0.1}$$

$$V = \frac{1.789 \angle 26.57^\circ}{0.4 \angle 90^\circ}$$

$$V = \frac{1.789}{0.4} \angle 26.57^\circ - 90^\circ$$

$$V = 4.47 \angle -63.43^\circ \ V$$

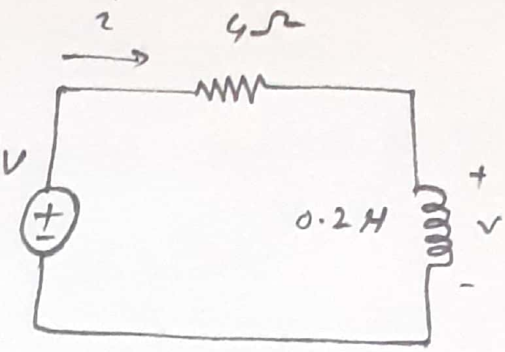
Converting I & V into the time domain

$$i(t) = 1.789 \cos(4t + 26.57^\circ) \ A$$

& $$v(t) = 4.47 \cos(4t - 63.43^\circ) \ V$$

Notice that $i(t)$ leads $v(t)$ by 90° as expected.

Practice Problem 9.7

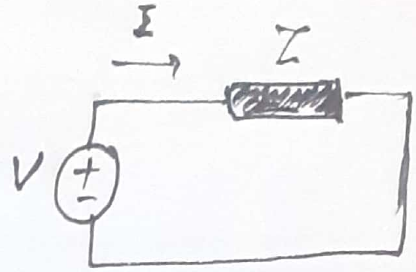


$V_s = 20\sin(10t + 30°)\,V$

Determine $v(t)$ & $i(t)$ and also write your analysis of comments.

Data $R = 4\Omega$
$L = 0.2\ H$
$V_s = 20\sin(10t + 30°)$

first of all convert sin into cosine



$$V_s = 20\cos(10t + 30° - 90°)$$

$$V_s = 20\cos(10t - 60°)$$

$\boxed{\cos(\omega t - 90°) = \sin\omega t}$

in Phasor Domain $\quad V_s = 20\angle -60° \quad \& \ \omega = 10$

The Required Formula is

$$I = \frac{V}{Z} \qquad \therefore \text{first we will find } Z$$

The Impedance $= Z = R + jX_L$

$$Z = R + j\omega L \qquad \therefore \sin(\omega t + 90°) = \cos\omega t$$

$$Z = 4 + j \times 10 \times 0.2$$

$$Z = 4 + j2 \ \Omega$$

Hence the Current is

$$I = \frac{V}{Z} = \frac{20\angle -60°}{4 + j2}$$

→ Convert from Rectangular into Polar

$$Z = 4 + j2$$
$$Z = \sqrt{(4)^2 + (2)^2}$$
$$Z = \sqrt{16 + 4}$$
$$Z = \sqrt{20}$$
$$Z = 4.47$$
$$\theta = \tan^{-1}\left(\frac{2}{4}\right)$$
$$\theta = 26.56$$

$$I = \frac{20\angle -60}{4.47\angle 26.56}$$

$$I = \frac{20}{4.47}\angle -60 - 26.56$$

$$I = 4.472\angle -86.56 \ A$$

No write in time domain as required in question

$$i(t) = 4.472\cos(10t - 86.56)\ A$$

$$i(t) = 4.472\sin(10t - 86.56 + 90°)\ A$$

$$i(t) = 4.472\sin(10t + 3.44)\ A$$

Now we can find the voltage across the Inductor.

$$Z_L = j\omega L$$

$$V = I Z_L$$

$$V = 4.472 \angle -86.56 \times j\omega L$$

$$V = 4.472 \angle -86.56 \times 1 \angle 90^\circ \times 10 \times 0.2$$

$$V = 4.472 \angle -86.56 \times 2 \angle 90^\circ$$

$$V = 8.944 \angle 3.44^\circ \quad V$$

in time domain

$$\cancel{V(t)} =$$

$$v(t) = 8.944 \cos(10t + 3.44)\ V$$

The Answer should be in given question format if in Sine then Answer will be in Sin
⇒ Convert into ~~sin~~. Sin.

i.e

$$v(t) = 8.99 \sin(10t + 3.44 + 90^\circ)$$

$$v(t) = 8.99 \sin(10t + 93.44^\circ)\ V \qquad \because \sin(\omega t + 90^\circ) = \cos\omega t$$

Extra

Analysis of this P.P. 9.9

$$i(t) = 4.72 \sin(10t + 3.44^\circ)\ \text{Amp}$$

$$v(t) = 8.944 \sin(10t + 93.44^\circ)\ \text{Volts}$$

Notice/Analyze that the voltage is leads 90° by current, as Expected.

9.6

Kirchhoff's Law in the frequency Domain

KCL (current Law) — KVL (Voltage Law)

KCL

"In any network the algebraic sum of the current meeting at a point (or Junction) is zero.

"OR"

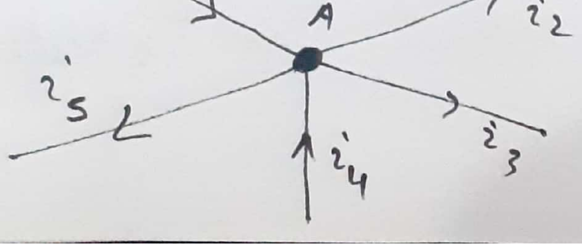
"The sum of the current flowing towards a point is equal to the sum of those flowing away from it"

"OR"

The algebraic sum of the currents entering to a Node equals to the sum of the current leaving it

$$i_1 + i_4 = i_2 + i_3 + i_5$$

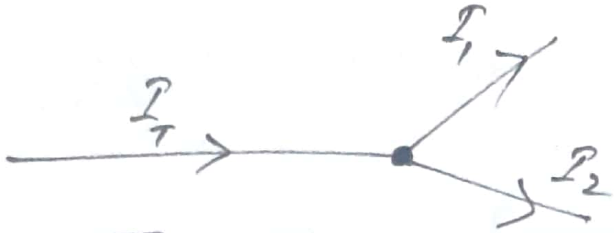
$$i_1 + i_4 - i_2 - i_3 - i_5 = 0$$



Incoming current = outgoing current

mathematically

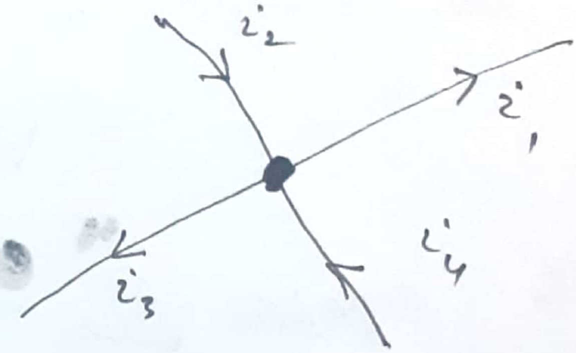
$$\sum I_{(entering)} = \sum I_{(Leaving)}$$



$$I_T = I_1 + I_2$$

∴ consider all current into a point as Positive & all current directed away from that Point as Negative

$$i_1 + i_3 = i_2 + i_4$$



Sum of $\theta°$ = Sum $\theta°$

in Phaso domain

$$I_1 \angle\theta° + I_3 \angle\theta = I_2 \angle\theta° + I_4 \angle\theta$$

$$\Rightarrow I_1(\cos\theta + j\sin\theta) + I_3(\cos\theta + j\sin\theta) = I_2(\cos\theta + j\sin\theta) + I_4(\cos\theta + j\sin\theta)$$

$$z = r(\cos\theta + j\sin\theta)$$

$$z = I(\cos\theta + j\sin\theta)$$

$$\Rightarrow I_1\cos\theta + jI_1\sin\theta + I_3\cos\theta + jI_3\sin\theta = I_2\cos\theta + jI_2\sin\theta + I_4\cos\theta + jI_4\sin\theta$$

seperate Real & Img. part

$$\underbrace{(I_1 + I_3)\cos\theta}_{\text{Real}} + j\underbrace{(I_1 + I_3)\sin\theta}_{\text{Imaginary}} = (I_2 + I_4)\cos\theta + j(I_2 + I_4)\sin\theta$$

Similarly KVL

$$V_1 \angle\theta + V_2 \angle\theta = V_3 \angle\theta$$

$$V_1(\cos\theta + j\sin\theta) + V_2(\cos\theta + j\sin\theta) = V_3(\cos\theta + j\sin\theta)$$

$$\underbrace{(V_1 + V_2)\cos\theta}_{\text{Real}} + j\underbrace{(V_1 + V_2)\sin\theta}_{\text{Imaginary}} = V_3\cos\theta + jV_3\sin\theta$$

Application of KCL & KVL in Impedence, Combination, Nodel & Mesh Analysis and Also in Superposition & Source Transformation

Definition of KVL

Kirchhoff's Voltage Law

" In any closed Path (or circuit) in a network, is the algebric Sum of all the Voltage is equal to Zero " /

$$\sum V = 0$$

" OR "

The Algebric Sum of the Voltages around any closed Path is Zero

" OR "

In any closed loop, the algebric Sum of the Voltage, applied is equal to the algebric Sum of the Voltage drop in the Element

$$V_1 + V_2 + \ldots\ldots\ldots V_n = 0$$

Each Voltage may be written in Cosine

$$V_{m_1} \cos(\omega t + \theta_1) + V_{m_2}(\omega t + \theta_2) + \ldots\ldots V_{mn}(\cos \omega t + \theta_n)$$