

Finding the Null Space, Row Space, and Column Space of a Matrix

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 3 & 0 & 1 & 8 \\ 2 & -2 & -2 & 6 \end{bmatrix} \rightarrow RREF \rightarrow \begin{bmatrix} 1 & 0 & 1/3 & 5/3 \\ 0 & 1 & 4/3 & -1/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\{R_1 = \left(1, 0, \frac{1}{3}, \frac{5}{3}\right), R_2 = \left(0, 1, \frac{4}{3}, -\frac{1}{3}\right)\}$$

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$$x_1 + \frac{1}{3}x_3 + \frac{5}{3}x_4 = 0$$

$$x_1 + \frac{1}{3}t + \frac{5}{3}s = 0$$

$$x_1 = -\frac{1}{3}t - \frac{5}{3}s$$

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$$x_3 = t$$

$$x_4 = s$$

$$x_1 + \frac{1}{3}x_3 + \frac{5}{3}x_4 = 0$$

$$x_2 + \frac{4}{3}x_3 - \frac{1}{3}x_4 = 0$$

$$x_1 = -\frac{1}{3}t - \frac{5}{3}s$$

$$x_1 + \frac{1}{3}t + \frac{5}{3}s = 0$$

$$x_2 + \frac{4}{3}t - \frac{1}{3}s = 0$$

$$x_2 = -\frac{4}{3}t + \frac{1}{3}s$$

$$x_3 = t$$

$$x_1 = -\frac{1}{3}t - \frac{5}{3}s$$

$$x_2 = -\frac{4}{3}t + \frac{1}{3}s$$

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$$x_3 = t$$

$$x_1 = -\frac{1}{3}t - \frac{5}{3}s$$

$$x_2 = -\frac{4}{3}t + \frac{1}{3}s$$

$$x_4 = s$$

$$\text{Null Space} = \left\{ \left(-\frac{1}{3}, -\frac{4}{3}, 1, 0 \right), \left(-\frac{5}{3}, \frac{1}{3}, 0, 1 \right) \right\}$$

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$$\text{Null Space} = \left\{ \left(-\frac{1}{3}, -\frac{4}{3}, 1, 0 \right), \left(-\frac{5}{3}, \frac{1}{3}, 0, 1 \right) \right\}$$

$$\text{Column Space} = \{ (1, 3, 2), (2, 0, -2) \}$$

$$\text{Row Space} = \left\{ \left(1, 0, \frac{1}{3}, \frac{5}{3} \right), \left(0, 1, \frac{4}{3}, -\frac{1}{3} \right) \right\}$$