one person age 14
one person age 15
three persons age 16
two people age 22
two people age 24
five people age 25

if $N(j)$ represents the number of pepple of age j then

$N(14) = 1$
$N(15) = 1$
$N(16) = 3$
$N(22) = 2$
$N(24) = 2$
$N(25) = 5$

the total number of people in the room is $N = \sum_{j=0}^{\infty} N(j) = 14$

Q 1 what is the most probable age?

Ans 25

Q3 what is the propability that one selected person has age 15 ?

Ans $\frac{1}{14}$

$P(j)$ = probability of getting age j

So $P(14) = 1/14$ $P(15) = 1/14$

$P(16) = 3/14$

$$P(j) = \frac{N(j)}{N}$$

sum of the probabilites will be equal to 1

$$\sum_{j=1}^{\infty} P(j) = 1$$

total number of people = $N = \sum_{j=0}^{\infty} N(j) = 14$

probability of age j

$$P(j) = \frac{N(j)}{N}$$

total probability $\sum_{j=1}^{\infty} P(j) = 1$

Average

$$\langle j \rangle = \frac{\sum N(j) . j}{N} = \frac{\sum N(j) . j}{N}$$

$$\frac{14+15+3(16)+2(22)+2(24)+5(25)}{14} = -$$

$$\langle j \rangle = \sum P(j) . j$$

$$\langle j \rangle = \sum j P(j)$$

spread = ?

spread in a distribution with respect to the average $\Delta j = j - \langle j \rangle$

we get zero (trouble) (average of spread)

prove $\langle \Delta j \rangle = \sum [j - \langle j \rangle] P(j)$

$$= \sum j P(j) - \langle j \rangle [\sum P(j)]$$

$$= \langle j \rangle - \langle j \rangle 1 = 0$$

otherwhise ψ would not be ~~normalized~~ normalizable

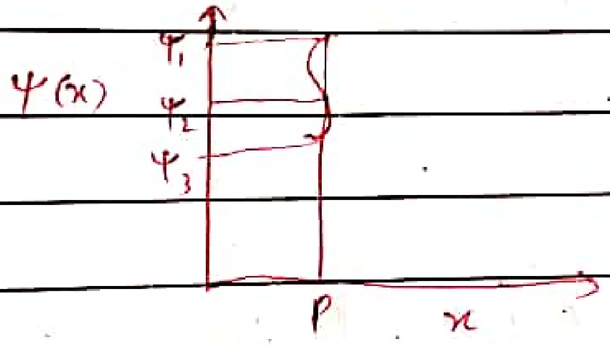
so

$$\frac{d}{dt}\int_{-\infty}^{+\infty} |\psi|^2 dx = 0$$

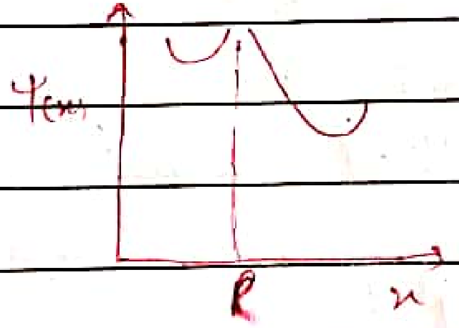
hence $\int_{-\infty}^{+\infty} |\psi|^2 dx$ = constant if ψ is normalized at $t=0$ it stay normalized for all future time page 2

② $\frac{\partial \psi}{\partial x}$, $\partial\psi/\partial y$. $\frac{\partial \psi}{\partial z}$ must be continous and single valued everywhere.

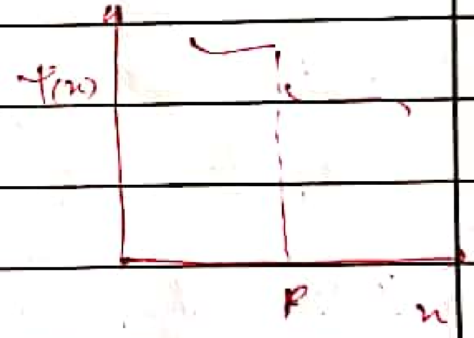
non acceptable wave functions



multi valued at 'p'



infinite at P



discontinous at p

③ ψ must be normalized

$$\frac{\partial \Psi}{\partial t} - \frac{i\hbar}{2m}\frac{\partial^2 \Psi}{\partial x^2} - \frac{i}{\hbar} V \Psi$$

complex conjugate

$$\frac{\partial \Psi^*}{\partial t} = -\frac{i\hbar}{2m}\frac{\partial^2 \Psi^*}{\partial x^2} + \frac{i}{\hbar} V \Psi^*$$

eq ② become

$$\frac{\partial}{\partial t}|\Psi|^2 = \frac{i\hbar}{2m}\left(\Psi^* \frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi^*}{\partial x^2}\Psi\right)$$

$$= \frac{\partial}{\partial x}\left[\frac{i\hbar}{2m}\left(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x}\Psi\right)\right].$$

equation ① become

$$\frac{d}{dt}\int_{-\infty}^{+\infty} |\Psi|^2 dx = \int \frac{\partial}{\cancel{\partial x}}\left[\frac{i\hbar}{2m}\left(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x}\Psi\right)\right] \cancel{dx}$$

$$\int_{-\infty} |\psi|^2 dx = 1$$

here ψ is wave function above condition is called normalization of wave function. we determine the wave function ψ by solving Schrodinger wave equation, for some solution to the Schrodinger wave equation ~~the~~ above integral ~~be~~ is infinite. The same goes for the trivial solution ($\psi = 0$) such non-normalizable solutions cannot represent particles and must me rejected

ψ must have the following properties

(i)- ψ must be continous and single valued everywhere.

(4) $\int_{-\infty}^{+\infty} |\Psi|^2 dx = 0, \infty, -ve$ are not accept

[Q] if Ψ is ~~not~~ normalized at time t =

How do i know that it will stay normaliz
as time goes on ?
proof of that point

(1) $\frac{d}{dt} \int_{-\infty}^{+\infty} |\Psi|^2 dx = \int_{-\infty}^{+\infty} \frac{\partial}{\partial t} |\Psi_{(x,t)}|^2 dx$

product rule

$\partial |\Psi|^2 = \partial (\Psi^* \Psi) = \Psi^* \partial \Psi + \partial \Psi^*$

$$= \frac{-i\hbar}{2m}\left[\int \psi^* \frac{\partial \psi}{\partial x}\,dx - \overset{0}{\cancel{\psi\psi^*\Big|_{-\infty}^{+\infty}}} + \int \psi^* \frac{\partial \psi}{\partial x}\,dx\right]$$

Page 4

$$= \frac{-i\hbar}{\cancel{2}m}\left[\cancel{2}\int \psi^* \frac{\partial \psi}{\partial x}\,dx\right]$$

$$\frac{d\langle x\rangle}{dt} = \frac{-i\hbar}{m}\int \psi^* \frac{\partial \psi}{\partial x}\,dx$$

multiply m on both sides

$$m\frac{d\langle x\rangle}{dt} = -i\hbar\int \psi^* \frac{\partial}{\partial x}\psi\,dx$$ to get momentum

$$\langle p\rangle = \int \psi^* \left(-i\hbar\frac{\partial}{\partial x}\right)\psi\,dx \qquad p = -i\hbar\frac{\partial}{\partial x}$$

Know calculate the average value of position of particle

$$\langle x \rangle = \int_{-\infty}^{+\infty} x|\Psi|^{*} dx$$

in more accurate form we can write

$$\langle x \rangle = \int_{-\infty}^{+\infty} \Psi x \Psi^* dx \qquad \text{as } |\Psi|^2 = \Psi$$

to calucculate momentum of partic

take derivate w.r.t time on both sides

$$\frac{d\langle x \rangle}{} = \frac{d}{dt}\int^{+\infty} \Psi x \Psi^* dx$$

as in previous lecture we derive ~~that~~ the relation for average value which is

$$\langle j \rangle = \sum j P(i)$$

if probelility functio is ~~contino~~ is continous

or we know that propability = $\int_{-\infty}^{+\infty} |\Psi|^2 dx$

$$P = \int_{-\infty}^{+\infty} |\Psi|^2 dx$$

put in above equation

$$\langle j \rangle = \int_{-\infty}^{+\infty} j\,|\Psi|^2 dx$$

$$\frac{d}{dt}\langle x\rangle = \int x \frac{\partial}{\partial t}|\psi|^2 dx$$

from previous lecture

$$\frac{d\langle x\rangle}{dt} = \frac{i\hbar}{2m}\int x \frac{\partial}{\partial x}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)dx$$

integration by parts

$$=$$

$$= \frac{i\hbar}{2m}\left[\underbrace{x\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)\Bigg|_{-\infty}^{+\infty}}_{0} - \int\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)\overset{\overset{1}{\downarrow}}{\frac{\partial x}{\partial x}}dx\right.$$

$$= \frac{-i\hbar}{2m}\int\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)dx$$

$$\frac{d\langle x\rangle}{dt} = \frac{-i\hbar}{2m}\int\psi^*\frac{\partial\psi}{\partial x}dx - \int\frac{\partial\psi^*}{\partial x}\psi\, dx$$

$$V_{quan} = \frac{\hbar K}{2m} \times \frac{2m}{2m} = \frac{\hbar K}{4m^2} \times 2m.$$

$$V_{quan} = \frac{\hbar K}{\sqrt{2m}} \times \frac{1}{\sqrt{2m}}$$

$$\boxed{V_{quan} = \sqrt{\frac{E}{2m}}}$$

$$V_{class} = \sqrt{\frac{2E}{m}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$V_{class} = \sqrt{\frac{4E}{2m}} = 2\sqrt{\frac{E}{2m}}$$

$$\cancel{\sqrt{E} = \sqrt{\frac{\hbar^2 K^2}{2m}}}$$

$$\cancel{\sqrt{E} = \frac{\hbar K}{\sqrt{2m}}}$$

$$\cancel{\sqrt{\frac{\hbar K^2}{4m \; 2m}}}$$

$$\underline{V_{class} = 2 V_{quantum}}$$

$$\frac{V_{class}}{2} = V_{quan}$$

because the pasticle unde considration is moving along $+x$

$$\Psi(x,t) = A e^{iK\left(x - \frac{\hbar K}{2m}t\right)}$$

coefficient of of 't' gives velocity

$$V_{quantum} = \frac{\hbar K}{2m} = \sqrt{\frac{E}{2m}} \quad \text{as} \quad K^2 = \frac{2mE}{\hbar^2}$$

classical speed of pasticle is

$$E = \frac{1}{2}mV^2 + 0 \quad \text{as} \quad V = 0$$

$$\frac{2E}{m} = V^2$$

$= V.$

find wave function of a free particle

for free particle $V(x) = 0$

time independen (SWE)

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{2mE}{\hbar^2}\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = -K^2\psi \quad \text{—— ①}$$

$$K^2 = \frac{2mE}{\hbar^2}$$

solution of eq ① is

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

include the time dependency

$$\Psi(x,t) = Ae^{ik\left(x - \frac{\hbar k}{2m}t\right)} + Be^{-ik\left(x + \frac{\hbar k}{2m}t\right)}$$

wave traviling at $+x$

wave traviling along $-x$

as wave is traviling only along $+x$

So second therm will be zero

$$V_{quantum} = \sqrt{\frac{E}{2m}} = \sqrt{\frac{2E}{4m}} = \frac{1}{2}\sqrt{\frac{2E}{m}} = \frac{1}{2} V_{class}$$

$$V_{quantum} = \frac{1}{2} V_{class}$$

$$2 V_{quantum} = V_{class}$$

this result represents that quantum mechinal wave function travels half the speed of particle which is not acceptible

also this wave function is not normalizable

$$\int_{-\infty}^{+\infty} \psi \psi^* dx = \int^{+\infty} A e^{ik(x - \frac{\hbar k}{2m} t)} A^* e^{-ik(x - \frac{\hbar k}{2m} t)} d$$

$$|A|^2\int_{-\infty}^{+\infty} 1\,dx = |A|^2(\infty)$$

$$= \infty$$

In the case of the free particle then the seprable solutions do not represents ~~p~~ physically realizable states, A free particle cannot exist in a stationary state

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi$$

$$\frac{d^2\psi}{dx^2} = -\boxed{\frac{2mE}{\hbar^2}}\psi \qquad K^2 = \frac{2mE}{\hbar^2}$$

$$\frac{d^2\psi}{\partial x^2} = -K^2\psi \qquad \frac{K^2\hbar^2}{2m} = E$$

$$D = \pm iK$$

$$\psi(x) = Ae^{iKx} + Be^{-iKx} \checkmark$$

⑪

$-h$ $E\Psi = i\hbar \frac{\partial \Psi}{\partial t}$:

$$\int \frac{E}{i\hbar}\, dt = \int \frac{d\Psi}{\Psi}$$

$$-\frac{iE}{\hbar} t = \ln \Psi$$

$$e^{-\frac{iE}{\hbar} t} = \Psi_{(t)}$$

$$\Psi_{(x,t)} = A e^{ikx} e^{-\frac{iEt}{\hbar}} + \cancel{Be^{-ikx}}\, e^{\frac{iEt}{\hbar}} \quad 11 \quad 0$$

wave traviling along $+x$ along $-x$

$$\Psi_{(x,t)} = A e^{ik\left(x - \frac{E}{\hbar K} t\right)}$$

$$\therefore \Psi_{(x,t)} = A e^{iK\left(x - \frac{\frac{\cancel{K^2\hbar^2}}{2m}}{\cancel{\hbar K}} t\right)}$$

$$\therefore \Psi_{(x,t)} = A e^{iK\left(x - \frac{\hbar K}{2m} t\right)}$$

no boundry condition

Checking normalizability.

$$\int_{-\infty}^{+\infty} |\Psi|^2 dx = 1 \checkmark$$

$$\int_{-\infty}^{+\infty} A e^{\cancel{ik(x-\frac{\hbar K}{2m}t)}} A^* e^{\cancel{-ik(x-\frac{\hbar K}{2m}t)}} dx$$

$$\int_{-\infty}^{+\infty} |A|^2 dx \Rightarrow |A|^2 \int_{-\infty}^{+\infty} dx$$

$$\alpha$$

$$|A|^2 (\infty) \neq 1$$

$$\Psi_{(x,t)} = A e^{ik(x-\frac{\hbar K}{2m}t)}$$

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$$V = \frac{\omega}{K} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = \underline{f\lambda} = V$$

$$\frac{\frac{\cancel{i}\hbar K^{\cancel{2}}}{2m}}{\cancel{i}\cancel{K}} = V_q = \frac{\hbar K}{2m} = V_{quant} = \frac{\hbar^2 K^2}{2m} \times \frac{\cancel{\hbar K}}{\hbar K} =$$

classicl. velocity

$$E = K.E + \cancelto{0}{V}$$

$$E = \frac{1}{2} m V_{cla}^2$$

$$\sqrt{\frac{2E}{m}} = V_{clas}$$

Similarly

$$(a_- a_+ - \frac{1}{2}\hbar\omega) a_- \psi = (E - \hbar\omega) a_- \psi$$

a_- lowering operator

a_+ Rising operator

a_-, a_+ ladder operator

$$= a_+\left(a_-a_+ + \frac{1}{2}\hbar\omega\right)\psi$$

$$= a_+\left[a_-a_+ - \frac{1}{2}\hbar\omega + \hbar\omega\right]\psi$$

$$= a_+\left[\left(a_-a_+ - \frac{1}{2}\hbar\omega\right)\psi + \hbar\omega\psi\right] \quad \text{from (i)}$$

$$= a_+\left[E\psi + \hbar\omega\psi\right]$$

$$= (E + \hbar\omega)\, a_+\psi$$

$$\underbrace{\left(a_+a_- + \frac{1}{2}\hbar\omega\right)}_{\text{Eigen operator}} \underbrace{a_+\psi}_{\text{Eigen function}} = \underbrace{(E + \hbar\omega)}_{\text{Eigen Value}}\, a_+\psi$$

give a ~~t~~test function $f(x) = f$

$$(a_- a_+) f(x) = \frac{1}{\sqrt{2m}}\left(\frac{\hbar}{i}\frac{d}{dx} - im\omega x\right)\frac{1}{\sqrt{2m}}\left(\frac{\hbar}{i}\frac{d}{dx} + im\omega x\right.$$

$$= \frac{1}{2m}\left(\frac{\hbar}{i}\frac{d}{dx} - im\omega x\right)\left(\frac{\hbar}{i}\frac{df}{dx} + im\omega x f\right)$$

$$= \frac{1}{2m}\left[-\hbar^2\frac{d^2 f}{dx^2} + \hbar m\omega\frac{d}{dx}(xf) - \hbar m\omega x\frac{df}{dx} - i^2 m^2\right.$$

$$= \frac{1}{2m}\left[-\hbar^2\frac{d^2 f}{dx^2} + \hbar m\omega\left(\frac{df}{dx}.x + f\,(1)\right) - \hbar m\omega x\frac{df}{dx} + (\right.$$

$$= \frac{1}{2m}\left[-\hbar^2\frac{d^2 f}{dx^2} + \cancel{\hbar m\omega x\frac{df}{dx}} + \hbar m\omega f - \cancel{\hbar m\omega x\frac{df}{dx}} + (\right.$$

$$= \frac{1}{2m}\left[-\hbar^2\frac{d^2 f}{dx^2} + (m\omega x)^2 f + \hbar m\omega f\right]$$

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give a ~~test~~ test function $f(x) = f$

$$(a_- a_+) f(x) = \frac{1}{\sqrt{2m}}\left(\frac{\hbar}{i}\frac{d}{dx} - im\omega x\right)\frac{1}{\sqrt{2m}}\left(\frac{\hbar}{i}\frac{d}{dx} + im\omega x\right) f(x)$$

$$= \frac{1}{2m}\left(\frac{\hbar}{i}\frac{d}{dx} - im\omega x\right)\left(\frac{\hbar}{i}\frac{df}{dx} + im\omega x f\right)$$

$$= \frac{1}{2m}\left[-\hbar^2 \frac{d^2 f}{dx^2} + \hbar m\omega \frac{d}{dx}(xf) - \hbar m\omega x \frac{df}{dx} - i^2 m^2 \omega^2 x^2 f\right.$$

$$= \frac{1}{2m}\left[-\hbar^2 \frac{d^2 f}{dx^2} + \hbar m\omega\left(\frac{df}{dx}\cdot x + f(1)\right) - \hbar m\omega x \frac{df}{dx} + (m\omega x)^2 f\right.$$

$$= \frac{1}{2m}\left[-\hbar^2 \frac{d^2 f}{dx^2} + \cancel{\hbar m\omega x \frac{df}{dx}} + \hbar m\omega f - \cancel{\hbar m\omega x \frac{df}{dx}} + (m\omega x)^2 f\right.$$

$$= \frac{1}{2m}\left[-\hbar^2 \frac{d^2 f}{dx^2} + (m\omega x)^2 f + \hbar m\omega f\right]$$

Harmonic oscillator in Quatum mechaniss

$$E = K.E + V$$

$$= \quad + \frac{1}{2}Kx^2 \qquad \omega = \sqrt{\frac{K}{m}}$$

Schrodinger wave equation (TIDSWE)

$$\frac{1}{2m}\left[\left(\frac{\hbar}{i}\frac{d}{dx}\right)^2 + (m\omega x)^2\right]\psi = E\psi$$

$$a^2 + b^2 = (a - ib)(a + ib)$$

$$a_{\pm} = \frac{1}{\sqrt{2m}}\left[\left(\frac{\hbar}{i}\frac{d}{dx} \pm im\omega x\right)\right]$$

STIWE

$$a_+a_- + \frac{1}{2}\hbar\omega = E\psi$$

$$a_-a_+ - a_+a_- = \frac{1}{2}\hbar\omega - \left(-\frac{1}{2}\hbar\omega\right)$$

$$= \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega$$

$$a_-a_+ - a_+a_- = \hbar\omega$$

Now

$$\left(a_+a_- + \frac{1}{2}\hbar\omega\right)a_+\psi$$

$$= \left(a_+a_-a_+\psi + \frac{1}{2}\hbar\omega a_+\right)\psi$$

$$(a_-a_+)\cancel{f(x)} = \frac{1}{2m}\left[\left(\frac{\hbar}{i}\frac{d}{dx}\right)^2 + (m\omega x)^2 + \hbar m\omega\right]\cancel{f(x)}$$

$$a_-a_+ = \frac{1}{2m}\left[\left(\frac{\hbar}{i}\frac{d}{dx}\right)^2 + (m\omega x)^2\right] + \frac{1}{2\cancel{m}}\hbar\cancel{m}\omega$$

$$+\frac{1}{2}\hbar\omega$$

TIDSWE

$$\frac{1}{2m}\left[\left(\frac{\hbar}{i}\frac{d}{dx}\right)^2 + (m\omega x)^2\right]\psi = E\psi$$

$$a_-a_+ - \frac{1}{2}\hbar\omega = E\psi \quad \text{— (i)}$$

Similarly (do it your self)

$$a_+a_- = \frac{1}{2m}\left[\left(\frac{\hbar}{i}\frac{d}{dx}\right)^2 + (m\omega x)^2\right] - \frac{1}{2}\hbar\omega$$

take antilog on both sides

$$\frac{\psi_0}{A} = e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\boxed{\psi_0 = A e^{-\frac{m\omega}{2\hbar}x^2}}$$

put in Schrodinger wave equation

$$\left(a_+a_- + \frac{1}{2}\hbar\omega\right)\psi_0 = E\psi_0$$

$$a_+\underbrace{\boxed{a_-\psi_0}}_{=0} + \frac{1}{2}\hbar\omega\psi_0 = E\psi_0 \qquad a_-\psi_0 = 0$$

$$\frac{1}{2}\hbar\omega\cancel{\psi_0} = E_0\cancel{\psi_0}$$

$$E_1 = \frac{1}{2}\hbar\omega + \hbar\omega$$

$$E_1 = \left(1 + \frac{1}{2}\right)\hbar\omega$$

Similarly $E_2 = \left(2+\frac{1}{2}\right)\hbar\omega$

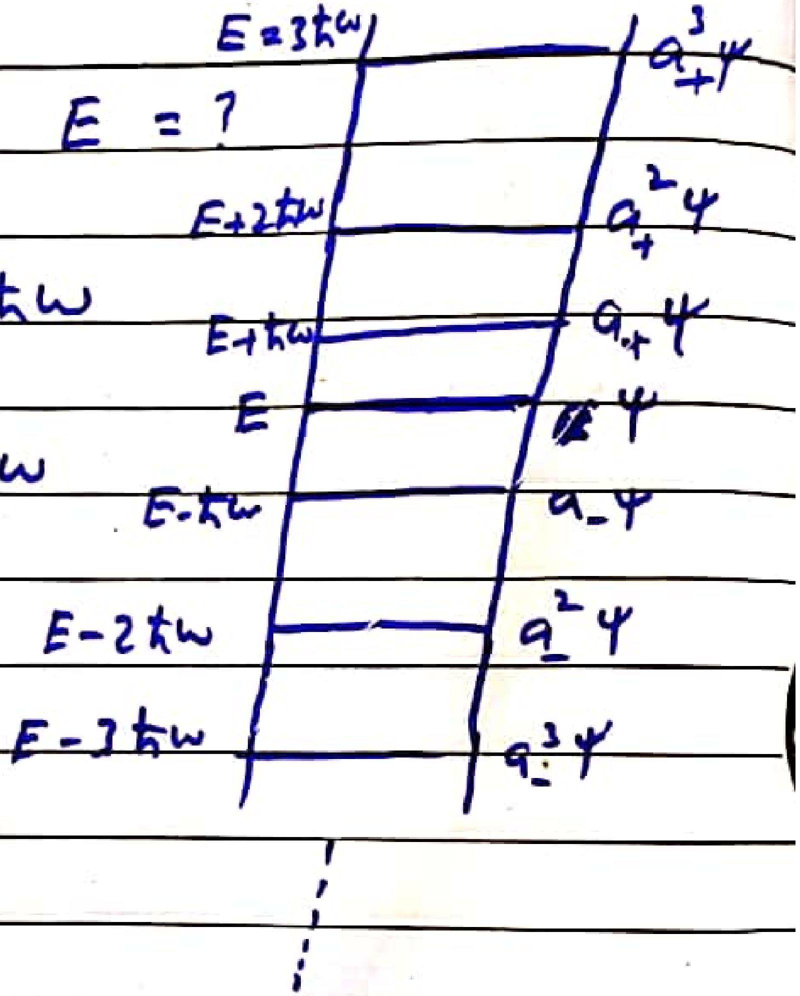
$\vdots$

$$\psi_n = A_n (a_+)^n e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\boxed{E_n = \left(n+\frac{1}{2}\right)\hbar\omega}$$

$n = 0, 1, 2, 3, \ldots$

harmonic Osaillator $E = ?$



$a_+\Psi$ $E + \hbar\omega$

$a_-\psi$ $E - \hbar\omega$

State ψ_0 has minum energy E_0

$$a_-\Psi_0 = 0$$

$$\frac{1}{\sqrt{2m}}\left(\frac{\hbar}{i}\frac{d\psi_0}{dx} - im\omega x\psi_0\right) = 0$$

$$\frac{\hbar}{i}\frac{d\psi_0}{dx} - im\omega x\psi_0 = 0$$

$$\boxed{E_0 = \frac{1}{2}\hbar\omega}$$

This is the minimum energy of Quahtum harmonic oscillator

to find the nent or heigher energy

$$\Psi = A e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\Psi_1 = A_1 a_+ e^{-\frac{m\omega}{2\hbar}x^2}$$

put this in Schrodinger wave equation to find E → do yourself

$$E_1 = E_0 + \hbar\omega$$

$$\frac{d\psi_0}{dx} = \frac{i \times i m\omega x \psi_0}{\hbar} \qquad i^2 = -1$$

$$\frac{d\psi_0}{dx} = -\frac{m\omega}{\hbar} x \psi_0$$

$$\int \frac{d\psi_0}{\psi_0} = -\frac{m\omega}{\hbar} \int x \, dx$$

$$\ln \psi_0 = \frac{-m\omega}{\hbar} \frac{x^2}{2} + \ln A$$

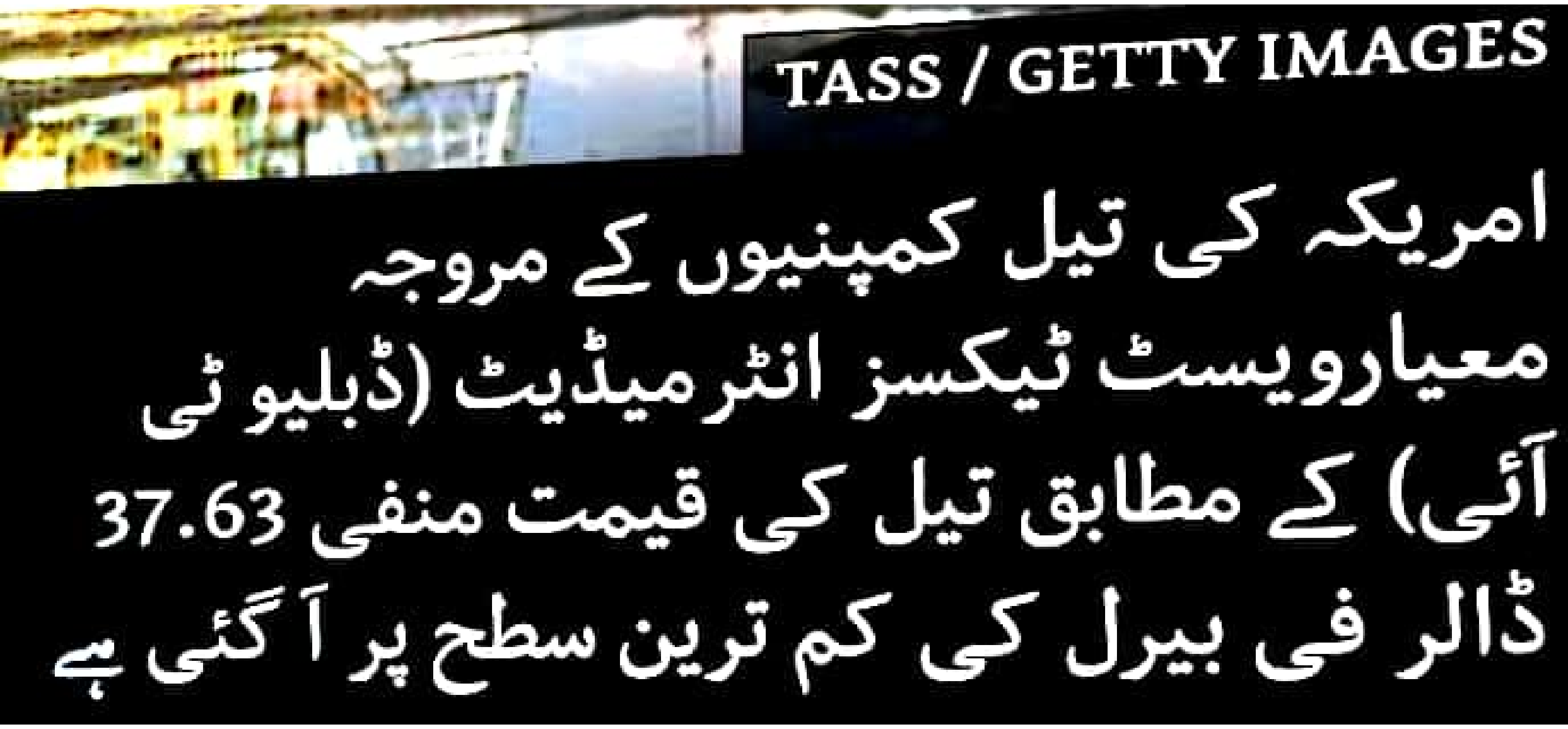
→ constant of integration

~~antilog~~

~~$\psi_0 = A e^{-\frac{m\omega x^2}{2\hbar}}$~~

$$\ln \psi_0 - \ln A = \frac{-m\omega x^2}{2\hbar}$$

$$\ln\left(\frac{\psi_0}{A}\right) = \frac{-m\omega x^2}{2\hbar}$$



TASS / GETTY IMAGES

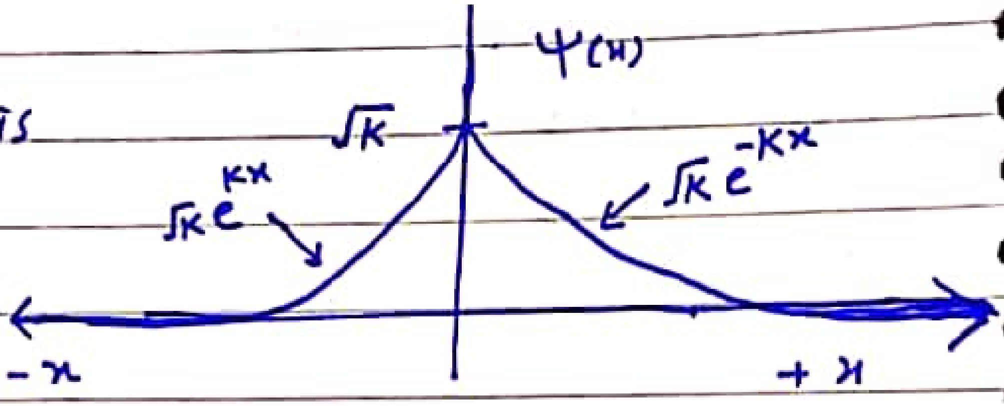
امریکہ کی تیل کمپنیوں کے مروجہ
معیارویسٹ ٹیکسز انٹرمیڈیٹ (ڈبلیو ٹی
آئی) کے مطابق تیل کی قیمت منفی 37.63
ڈالر فی بیرل کی کم ترین سطح پر آ گئی ہے

So

$$B = F$$

$$\Psi_{(x)} = \begin{cases} Be^{kx} & x \leq 0 \\ Be^{-kx} & x \geq 0 \end{cases}$$

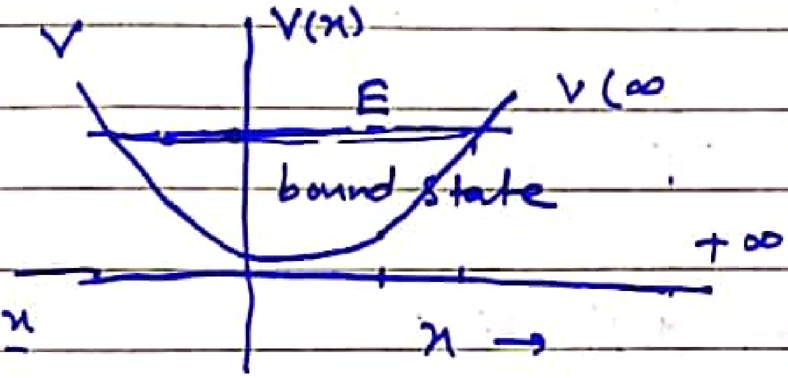
its graph is

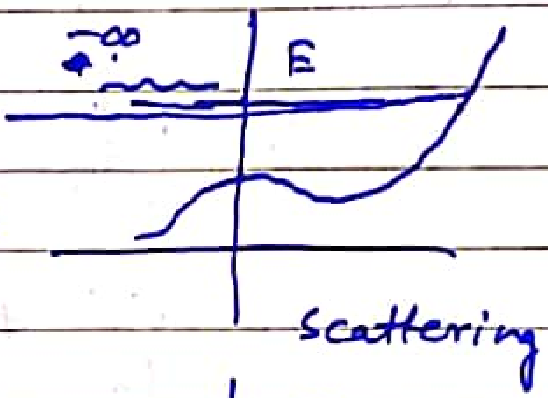


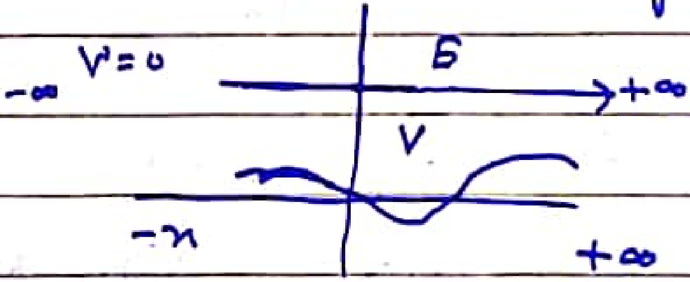
$$\frac{}{(T_{32} - T_{21})}$$

required minimum pumping power

Quantum Mechanics







E < V bound state

E > V scattering state

delta function

$$\delta(x) = \begin{cases} 0 & x \neq 0 \\ \infty & x = 0 \end{cases}$$

$$\int_{-\infty}^{+\infty} f(x)\,\delta(x-a)\,dx = f(a)$$

$$\int_{-\infty}^{+\infty} \delta(x)\,dx = 1$$

most potentials goes to zero at Infinity so we can write above

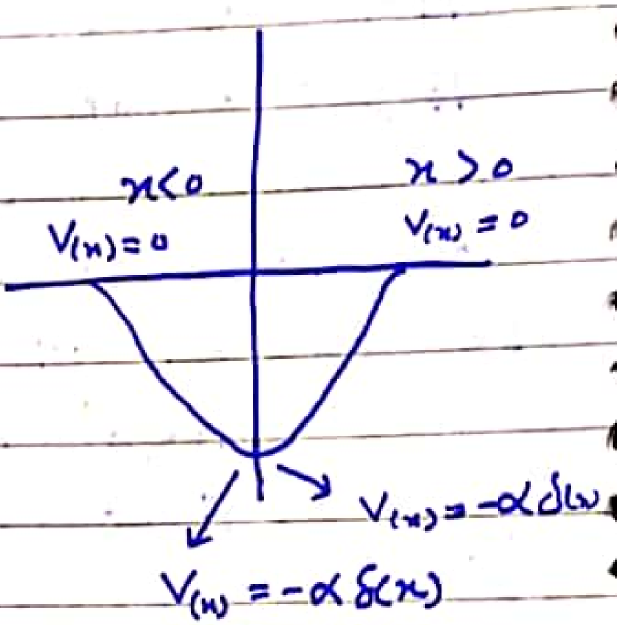
$$\begin{cases} E<0 & \text{bound state} \\ E>0 & \text{Scattering state} \end{cases}$$

in the case of free particle potential is zero every where so it allows scattering states, But in case of harmonic Oscillator $V \to \infty$ as $x \to \pm\infty$ so It allows only bound state

$$\frac{d^2\psi}{dx^2} = \frac{-2mE}{\hbar^2}\psi$$

$$\frac{d^2\psi}{dx^2} = K^2\psi \quad \text{— (i)}$$

here $K = \frac{\sqrt{-2mE}}{\hbar}$



Solution of eq (i) is

$$\psi_{(x)} = Ae^{-kx} + Be^{kx}$$

at $x=-\infty$ goes to ∞

at $x=-\infty$ remain finite

at x
$x=-\infty$

$$\left[e^{-(-\infty)} = e^{\infty} \right.$$
$$= \infty$$

$$\left[e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} \right.$$
$$= 0$$

so take $A = 0$

$$\psi_{(x)} = Be^{kx} \quad \text{— (ii)}$$

in the region $x>0$ $V_{(x)}$ is again zero so the solution of eq (i) is of the form

1- ψ is always continous

2- $\frac{d\psi}{dx}$ is continous except at points where potential is infinite

in case of our delta potential

$$V(x) = -\alpha\,\delta(x)$$

this potential is infinite only at $x=0$ because at $x=0$ $\delta(x) = \infty$

Scanned with CamScanner

$$\frac{d\psi}{dx}\bigg|_{x=0^+} = iK(F-G) \quad x>0$$

$$e^0 = 1$$

$$e^{-0} = 1$$

$$\frac{d\psi}{dx}\bigg|_{x=0^-} = iK(A-B) \quad x<0$$

$$\Delta\left({d\psi}/{dx}\right) = \frac{d\psi}{dx}\bigg|_{x=0^+} - \frac{d\psi}{dx}\bigg|_{x=0^-}$$

$$= iK(F-G) - iK(A-B)$$

$$K = \frac{m\alpha}{\hbar^2}$$

$$E = -\frac{\hbar^2 K^2}{2m}$$

$$E = -\frac{\cancel{\hbar^2}\, m^{\cancel{2}} \alpha^2}{2\cancel{m}\, \hbar^{\cancel{4}}}$$

$$\boxed{E = -\frac{m\alpha^2}{2\hbar^2}}$$

Now to find value of 'B' we normalize ψ

$$\int_{-\infty}^{+\infty} |\psi|^2 \, dx = 1$$

$-\infty$ 0

$$2\int_{0}^{+\infty} |B|^2 e^{-2Kx} dx = 1$$

$$|B|^2 2\int_{0}^{+\infty} e^{-2Kx} dx = 1$$

$$2|B|^2 \int_{0}^{\infty} \frac{1}{-2K} e^{-2Kx} (-2K) dx = 1$$

$$-\frac{|B|^2}{K} e^{-2Kx} \Big|_{0}^{\infty} = 1$$

$$-|B|^2 \left(e^{-\infty} - e^{0} \right) = K$$

$$-|B|^2 (0 - 1) = K$$

For Scattering state $E > 0$
for $x < 0$
the SWE reads

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$$

$$\frac{d^2\psi}{dx^2} = -K^2\psi \qquad K^2 = \frac{2mE}{\hbar^2}$$

the solution is $\qquad K = \frac{\sqrt{2mE}}{\hbar}$

$$\psi_{(x)} = Ae^{ikx} + Be^{-ikx}$$

non of the terms blows up (tends to ∞)

Similarly for $x > 0$

$$\psi_{(x)} = Fe^{ikx} + Ge^{-ikx}$$

as $\psi_{(x)}$ is continous at $x = 0$
So

$$Fe^{0} + Ge^{-(0)} = Ae^{0} + Be^{-(0)}$$

$$F + G = A + B$$

$\longrightarrow$ (i)