12

Toothed Gearing

Features

12.1. Introduction

We have discussed in the previous chapter, that the slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by means of ***gears*** or ***toothed wheels***. A gear drive is also provided, when the distance between the driver and the follower is very small.

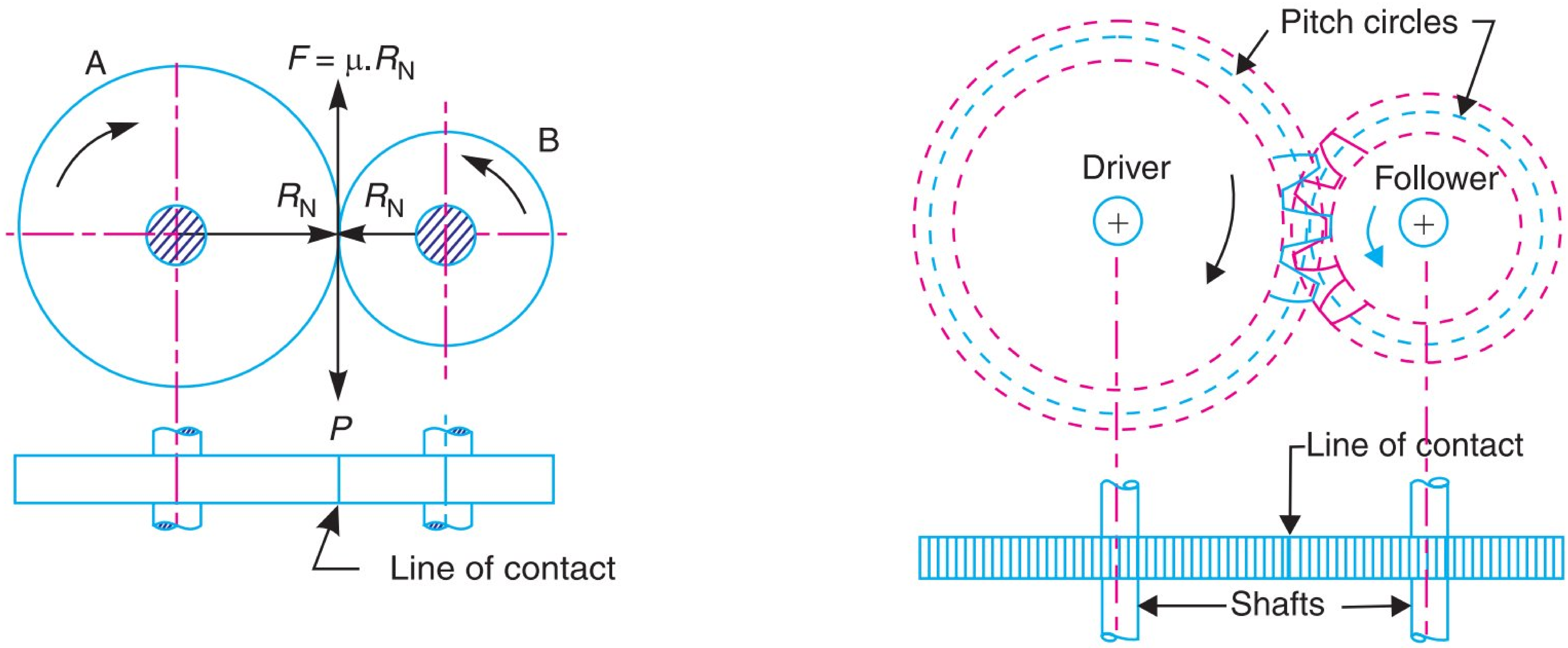
12.2. Friction Wheels

The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels or discs. In order to understand how the motion can be transmitted by two toothed wheels, consider two plain circular wheels *A* and *B* mounted on

shafts, having sufficient rough surfaces and pressing against each other as shown in Fig. 12.1 (*a*).

Let the wheel A be keyed to the rotating shaft and the wheel B to the shaft, to be rotated. A little consideration will show, that when the wheel A is rotated by a rotating shaft, it will rotate the wheel B in the opposite direction as shown in Fig. 12.1 (*a*).

The wheel B will be rotated (by the wheel A) so long as the tangential force exerted by the wheel A does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (P) exceeds the *frictional resistance (F), slipping will take place between the two wheels. Thus the friction drive is not a positive drive.



(*a*) Friction wheels. (*b*) Toothed wheels.

Fig. 12.1

In order to avoid the slipping, a number of projections (called teeth) as shown in Fig. 12.1 (*b*), are provided on the periphery of the wheel A, which will fit into the corresponding recesses on the periphery of the wheel B. A friction wheel with the teeth cut on it is known as ***toothed wheel* or *gear***. The usual connection to show the toothed wheels is by their **pitch circles.

Note : Kinematically, the friction wheels running without slip and toothed gearing are identical. But due to the possibility of slipping of wheels, the friction wheels can only be used for transmission of small powers.

12.3. Advantages and Disadvantages of Gear Drive

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives :

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

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* The frictional force F is equal to $\mu . R_N$, where μ = Coefficient of friction between the rubbing surface of two wheels, and R_N = Normal reaction between the two rubbing surfaces.

** For details, please refer to Art. 12.4.

12.4. Classification of Toothed Wheels

The gears or toothed wheels may be classified as follows :

1. *According to the position of axes of the shafts*. The axes of the two shafts between which the motion is to be transmitted, may be

(*a*) Parallel, (*b*) Intersecting, and (*c*) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. 12.1. These gears are called ***spur gears*** and the arrangement is known as ***spur gearing***. These gears have teeth parallel to the axis of the wheel as shown in Fig. 12.1. Another name given to the spur gearing is ***helical gearing***, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. 12.2 (*a*) and (*b*) respectively. The double helical gears are known as ***herringbone gears***. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to parallel shafts and having a line contact.

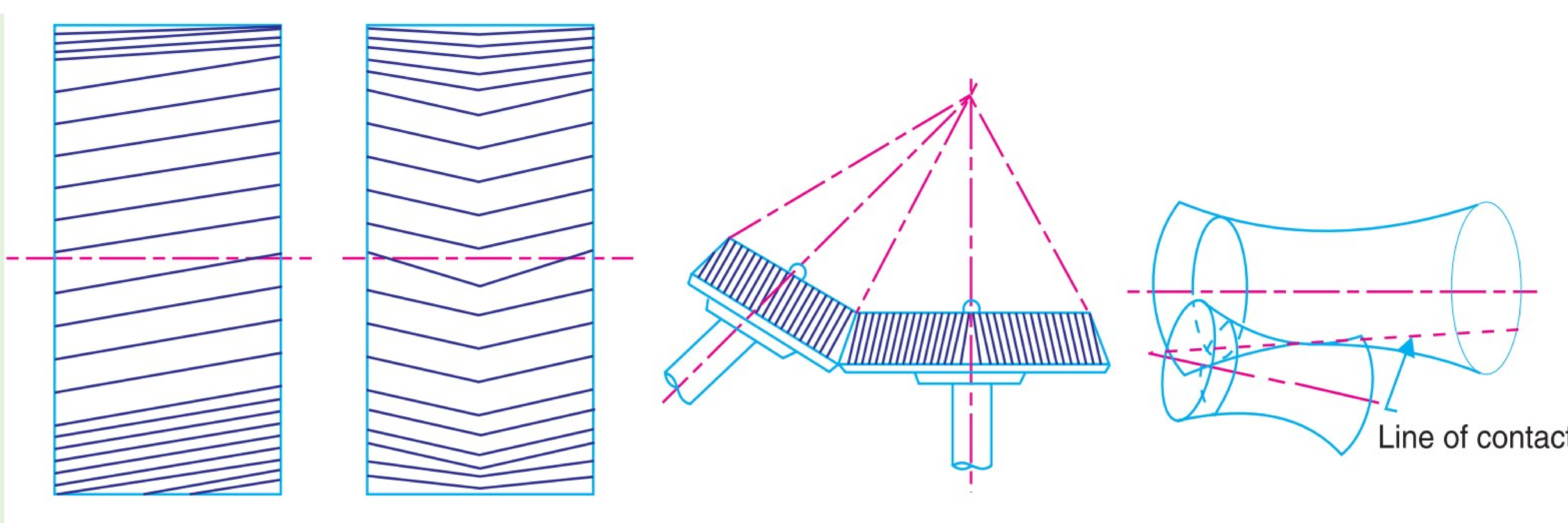
The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in Fig. 12.2 (*c*). These gears are called ***bevel gears*** and the arrangement is known as ***bevel gearing***. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as ***helical bevel gears***.

The two non-intersecting and non-parallel *i.e.* non-coplanar shaft connected by gears is shown in Fig. 12.2 (*d*). These gears are called ***skew bevel gears*** or ***spiral gears*** and the arrangement is known as ***skew bevel gearing*** or ***spiral gearing***. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as ***hyperboloids***.

Notes : (*a*) When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as ***mitres***.

(*b*) A hyperboloid is the solid formed by revolving a straight line about an axis (not in the same plane), such that every point on the line remains at a constant distance from the axis.

(*c*) The worm gearing is essentially a form of spiral gearing in which the shafts are usually at right angles.



(*a*) Single helical gear. (*b*) Double helical gear. (*c*) Bevel gear. (*d*) Spiral gear.

Fig. 12.2

2. *According to the peripheral velocity of the gears*. The gears, according to the peripheral velocity of the gears may be classified as :

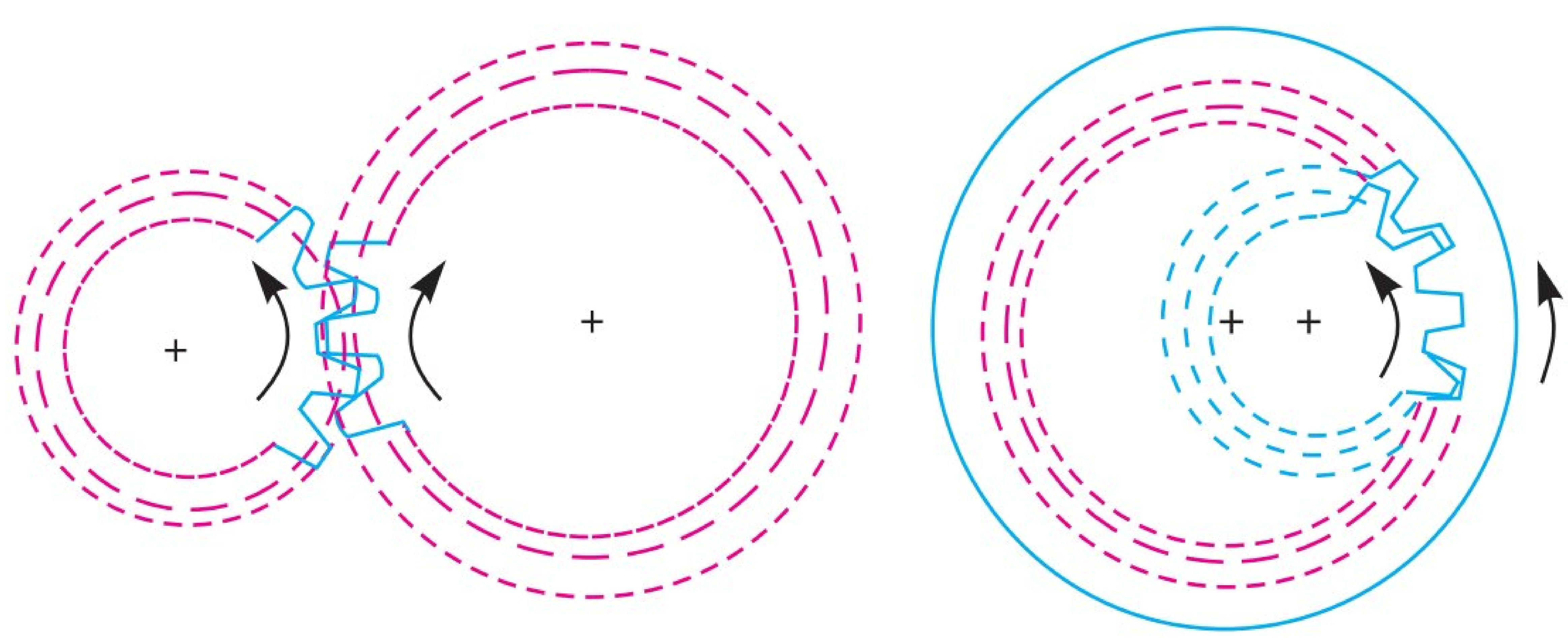
(*a*) Low velocity, (*b*) Medium velocity, and (*c*) High velocity.

The gears having velocity less than 3 m/s are termed as ***low velocity*** gears and gears having velocity between 3 and 15 m/s are known as ***medium velocity gears***. If the velocity of gears is more than 15 m/s, then these are called ***high speed gears***.

3. ***According to the type of gearing***. The gears, according to the type of gearing may be classified as :

(a) External gearing, ***(b)*** Internal gearing, and ***(c)*** Rack and pinion.

In ***external gearing***, the gears of the two shafts mesh externally with each other as shown in Fig. 12.3 (*a*). The larger of these two wheels is called ***spur wheel*** and the smaller wheel is called **pinion.** In an external gearing, the motion of the two wheels is always ***unlike***, as shown in Fig. 12.3 (*a*).



(*a*) External gearing. (*b*) Internal gearing.

Fig. 12.3

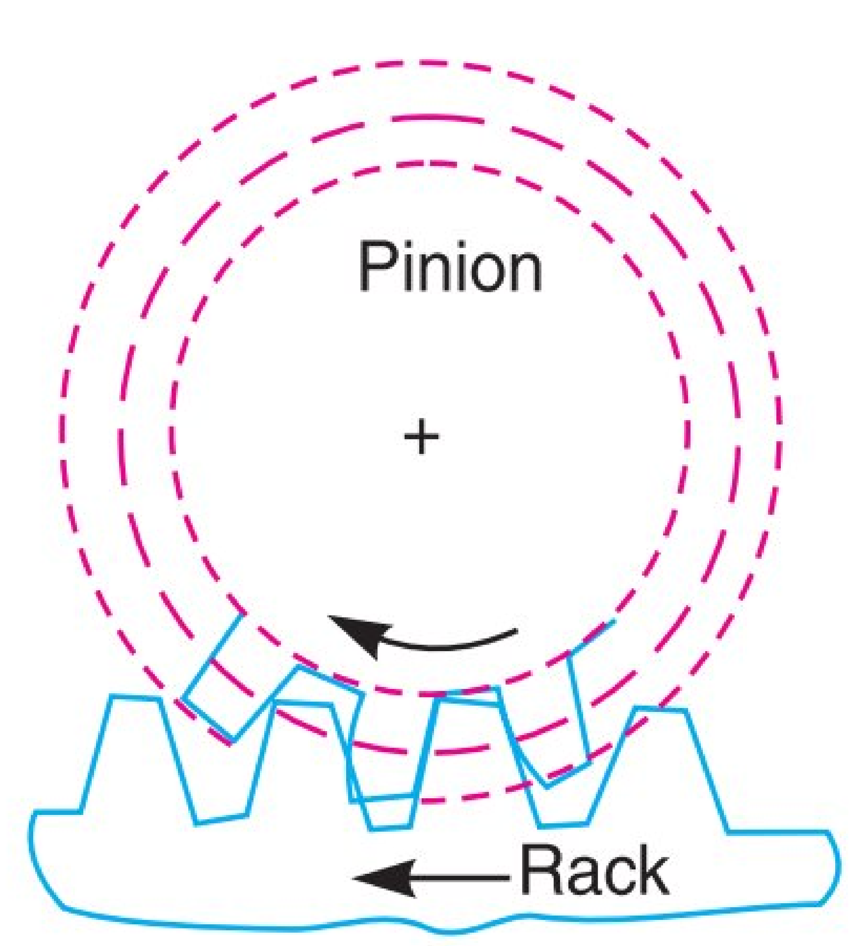


Fig. 12.4. Rack and pinion.

In ***internal gearing***, the gears of the two shafts mesh ***internally*** with each other as shown in Fig. 12.3 (*b*). The larger of these two wheels is called ***annular wheel*** and the smaller wheel is called ***pinion***. In an internal gearing, the motion of the two wheels is always ***like***, as shown in Fig. 12.3 (*b*).

Sometimes, the gear of a shaft meshes externally and internally with the gears in a *straight line, as shown in Fig. 12.4. Such type of gear is called ***rack and pinion***. The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and ***vice-versa*** as shown in Fig. 12.4.

4. ***According to position of teeth on the gear surface***. The teeth on the gear surface may be (*a*) straight, (*b*) inclined, and (*c*) curved.

We have discussed earlier that the spur gears have straight teeth where as helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.

Internal gears

Rack and pinion

12.5. Terms Used in Gears

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in Fig. 12.5.

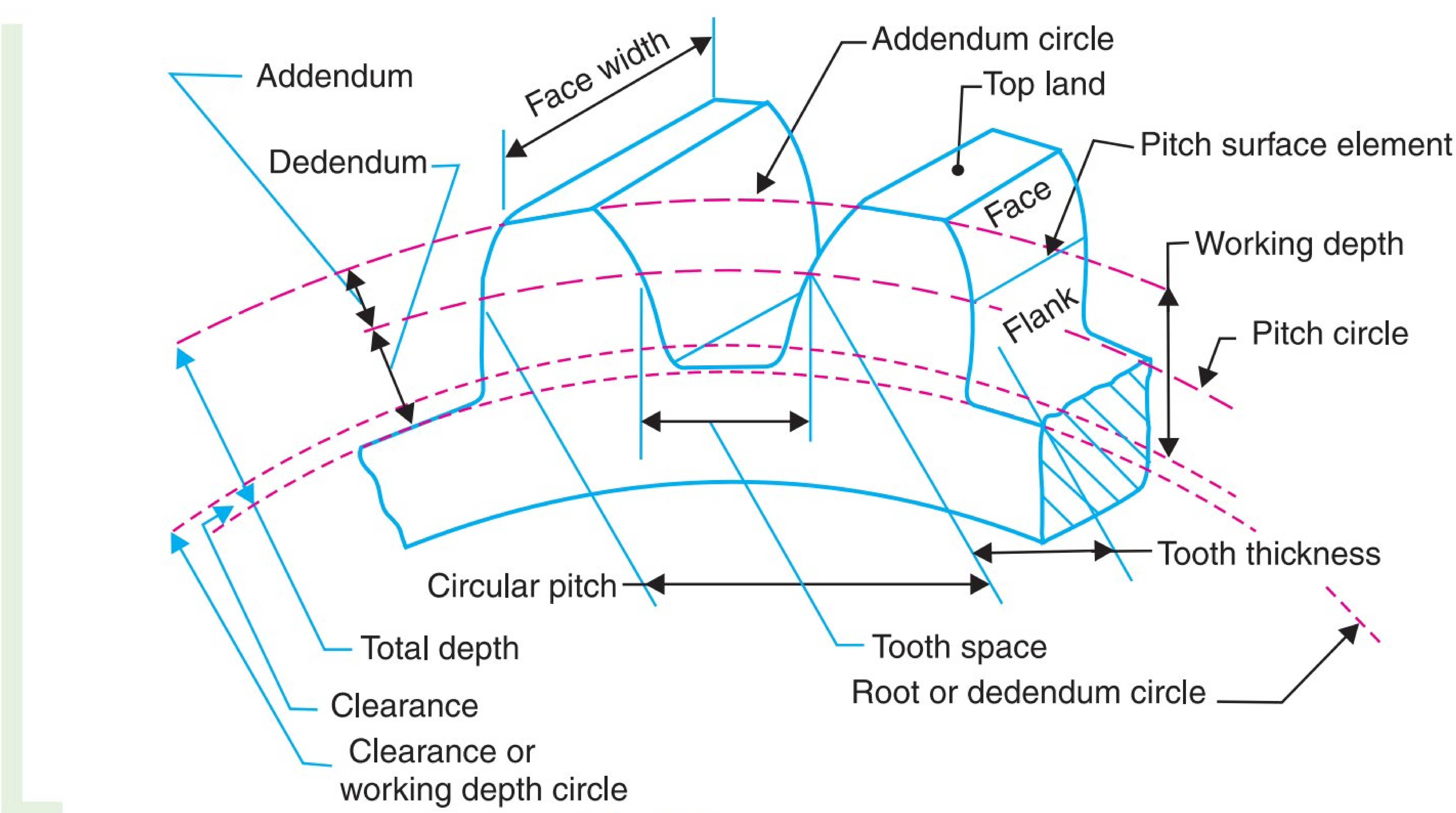


Fig. 12.5. Terms used in gears.

1. ***Pitch circle***. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

* A straight line may also be defined as a wheel of infinite radius.

2. *Pitch circle diameter*. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as ***pitch diameter***.

3. *Pitch point*. It is a common point of contact between two pitch circles.

4. *Pitch surface*. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. *Pressure angle or angle of obliquity*. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are $14\frac{1}{2}°$ and 20°.

6. *Addendum*. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. *Dedendum*. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. *Addendum circle*. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. *Dedendum circle*. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note : Root circle diameter = Pitch circle diameter × cos ϕ, where ϕ is the pressure angle.

10. *Circular pitch*. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically,

$$\text{Circular pitch,} \quad p_c = \pi D/T$$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

11. *Diametral pitch*. It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by p_d . Mathematically,

$$\text{Diametral pitch,} \quad p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots \left(\because\ p_c = \frac{\pi D}{T}\right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

12. *Module*. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically,

$$\text{Module,} \quad m = D/T$$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

13. *Clearance*. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle***.

14. *Total depth*. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

15. *Working depth*. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. *Tooth thickness*. It is the width of the tooth measured along the pitch circle.

17. *Tooth space* . It is the width of space between the two adjacent teeth measured along the pitch circle.

18. *Backlash*. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. *Face of tooth*. It is the surface of the gear tooth above the pitch surface.

20. *Flank of tooth*. It is the surface of the gear tooth below the pitch surface.

21. *Top land*. It is the surface of the top of the tooth.

22. *Face width*. It is the width of the gear tooth measured parallel to its axis.

23. *Profile*. It is the curve formed by the face and flank of the tooth.

24. *Fillet radius*. It is the radius that connects the root circle to the profile of the tooth.

25. *Path of contact*. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. **Length of the path of contact*. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. * Arc of contact*.** It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

(*a*) *Arc of approach*. It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(*b*) *Arc of recess*. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Note : The ratio of the length of arc of contact to the circular pitch is known as ***contact ratio*** *i.e*. number of pairs of teeth in contact.

12.6. Gear Materials

The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc. The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze. The non-metallic materials like wood, raw hide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise.

The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method. The cast iron gears with cut teeth may be employed, where smooth action is not important.

The steel is used for high strength gears and steel may be plain carbon steel or alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness.

The phosphor bronze is widely used for worm gears in order to reduce wear of the worms which will be excessive with cast iron or steel.

12.7. Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the

* For details, see Art. 12.16.

** For details, see Art. 12.17.