and height of the governor,

$$h = BG / \tan\alpha = 164 / \tan 45° = 164 \text{ mm} = 0.164 \text{ m}$$

Let N_1 = Minimum speed of rotation, and

N_2 = Maximum speed of rotation.

We know that

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h}$$

$$= \frac{1.15 \times 9.81 + (20 \times 9.81 - 10)}{1.15 \times 9.81} \times \frac{895}{0.164} = 95\ 382$$

∴ $N_1 = 309$ r.p.m. **Ans.**

and
$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h}$$

$$= \frac{1.15 \times 9.81 + (20 \times 9.81 + 10)}{1.15 \times 9.81} \times \frac{895}{0.164} = 105\ 040$$

$N_2 = 324$ r.p.m. **Ans.**

18.7. Proell Governor

The Proell governor has the balls fixed at *B* and *C* to the extension of the links *DF* and *EG*, as shown in Fig. 18.12 (*a*). The arms *FP* and *GQ* are pivoted at *P* and *Q* respectively.

Consider the equilibrium of the forces on one-half of the governor as shown in Fig. 18.12 (*b*). The instantaneous centre (*I*) lies on the intersection of the line *PF* produced and the line from *D* drawn perpendicualr to the spindle axis. The prependicular *BM* is drawn on *ID*.

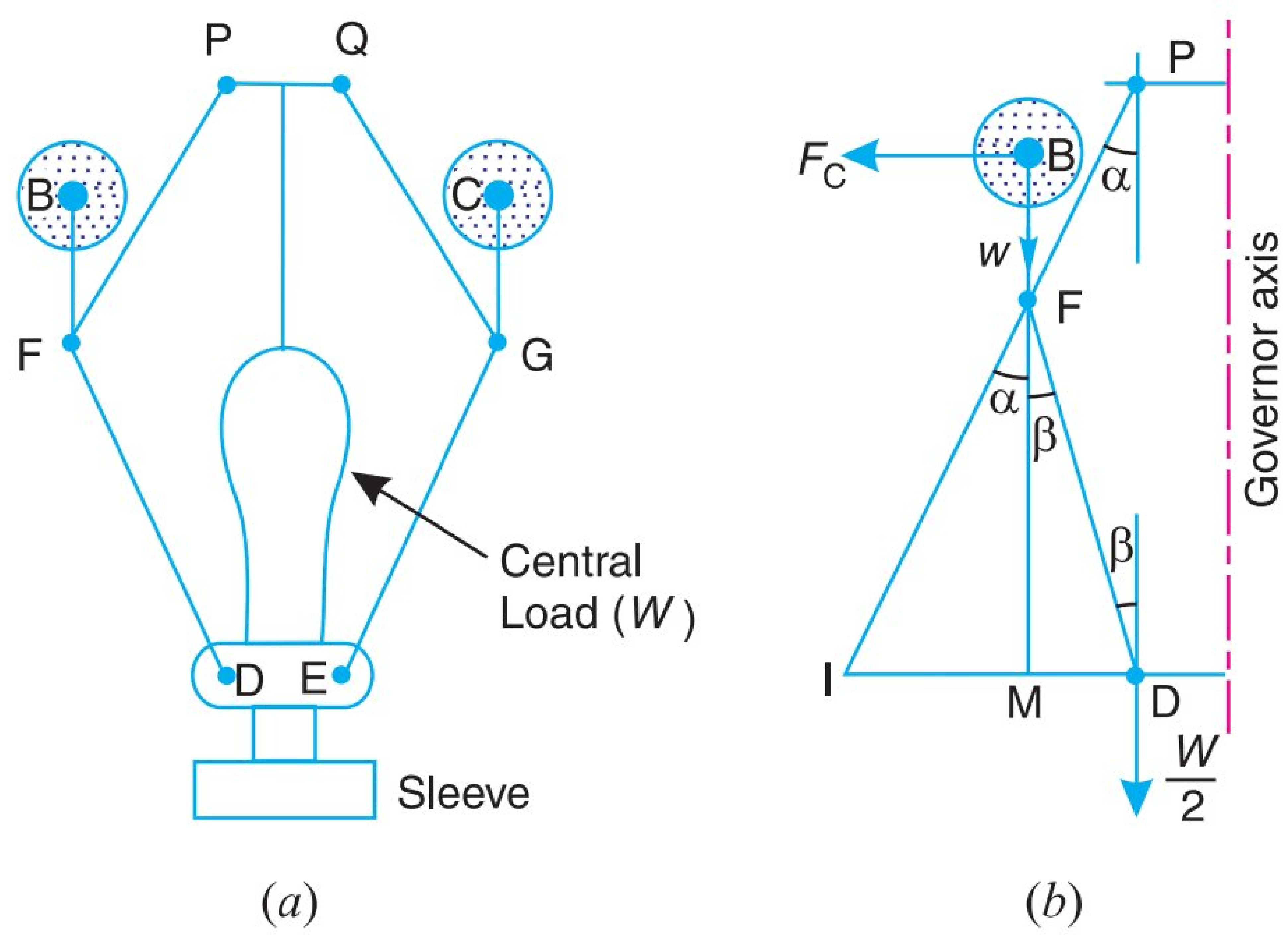

Fig. 18.12. Proell governor.

Taking moments about *I*, using the same notations as discussed in Art. 18.6 (Porter governor),

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \qquad \ldots (i)$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \qquad \ldots (\because\ ID = IM + MD)$$

Multiplying and dividing by *FM*, we have

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha \left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \dfrac{r}{h}$ and $q = \dfrac{\tan\beta}{\tan\alpha}$

$$\therefore \qquad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

and

$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{g}{h} \qquad \ldots (ii)$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81$ m/s^2, we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h} \qquad \ldots (iii)$$

Notes : 1. The equation ***(i)*** may be applied to any given configuration of the governor.

2. Comparing equation ***(iii)*** with the equation ***(v)*** of the Porter governor (Art. 18.6), we see that the equilibrium speed reduces for the given values of *m*, *M* and *h*. Hence in order to have the same equilibrium speed for the given values of *m*, *M* and *h*, balls of smaller masses are used in the Proell governor than in the Porter governor.

3. When $\alpha = \beta$, then $q = 1$. Therefore equation ***(iii)*** may be written as

$$N^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h} \qquad (h \text{ being in metres}) \ldots (iv)$$

Example 18.9. *A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.*

Solution. Given : $PF = DF = 300$ mm ; $BF = 80$ mm ; $m = 10$ kg ; $M = 100$ kg ; $r_1 = 150$ mm; $r_2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 18.13.

Let N_1 = Minimum speed when radius of rotation, $r_1 = FG = 150$ mm ; and

N_2 = Maximum speed when radius of rotation, $r_2 = FG = 200$ mm.

From Fig. 18.13 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (150)^2} = 260 \text{ mm} = 0.26 \text{ m}$$