20.4. Terms Used in Radial Cams

Fig. 20.3 shows a radial cam with reciprocating roller follower. The following terms are important in order to draw the cam profile.

1. *Base circle.* It is the smallest circle that can be drawn to the cam profile.

2. *Trace point.* It is a reference point on the follower and is used to generate the *pitch curve*. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centre of the roller represents the trace point.

3. *Pressure angle.* It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

4. *Pitch point.* It is a point on the pitch curve having the maximum pressure angle.

5. *Pitch circle.* It is a circle drawn from the centre of the cam through the pitch points.

6. *Pitch curve.* It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.

7. *Prime circle.* It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

8. *Lift or stroke.* It is the maximum travel of the follower from its lowest position to the topmost position.

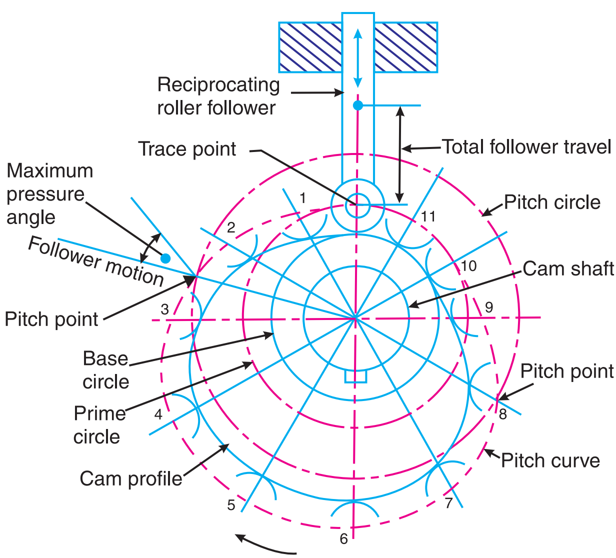

Fig. 20.3. Terms used in radial cams.

20.5. Motion of the Follower

The follower, during its travel, may have one of the following motions.

1. Uniform velocity, **2.** Simple harmonic motion, **3.** Uniform acceleration and retardation, and **4.** Cycloidal motion.

We shall now discuss the displacement, velocity and acceleration diagrams for the cam when the follower moves with the above mentioned motions.

20.6. Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity

The displacement, velocity and acceleration diagrams when a knife-edged follower moves with uniform velocity are shown in Fig. 20.4 (*a*), (*b*) and (*c*) respectively. The abscissa (base) represents the time (*i.e.* the number of seconds required for the cam to complete one revolution) or it may represent the angular displacement of the cam in degrees. The ordinate represents the displacement, or velocity or acceleration of the follower.

Since the follower moves with uniform velocity during its rise and return stroke, therefore the slope of the displacement curves must be constant. In other words, AB_1 and C_1D must be straight lines. A little consideration will show that the follower remains at rest during part of the cam rotation. The periods during which the follower remains at rest are known as ***dwell periods***, as shown by lines B_1C_1 and DE in Fig. 20.4 (*a*). From Fig. 20.4 (*c*), we see that the acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This is due to the fact that the follower is required to start from rest and has to gain a velocity within no time. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite. These conditions are however, impracticable.

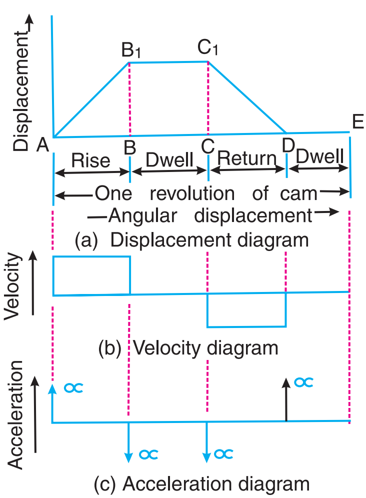

Fig. 20.4. Displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

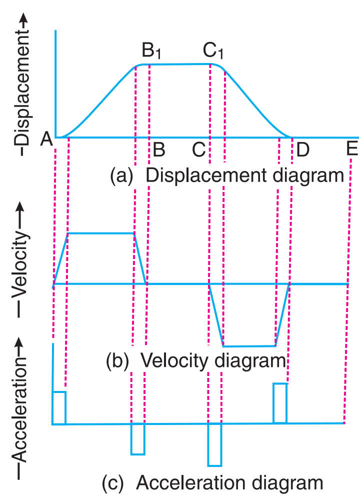

Fig. 20.5. Modified displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

In order to have the acceleration and retardation within the finite limits, it is necessary to modify the conditions which govern the motion of the follower. This may be done by rounding off the sharp corners of the displacement diagram at the beginning and at the end of each stroke, as shown in Fig. 20.5 (*a*). By doing so, the velocity of the follower increases gradually to its maximum value at the beginning of each stroke and decreases gradually to zero at the end of each stroke as shown in Fig. 20.5 (*b*). The modified

Camshaft of an IC engine.

displacement, velocity and acceleration diagrams are shown in Fig. 20.5. The round corners of the displacement diagram are usually parabolic curves because the parabolic motion results in a very low acceleration of the follower for a given stroke and cam speed.

20.7. Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion

The displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion are shown in Fig. 20.6 (*a*), (*b*) and (*c*) respectively. The displacement diagram is drawn as follows :

1. Draw a semi-circle on the follower stroke as diameter.
2. Divide the semi-circle into any number of even equal parts (say eight).
3. Divide the angular displacements of the cam during out stroke and return stroke into the same number of equal parts.
4. The displacement diagram is obtained by projecting the points as shown in Fig. 20.6 (*a*).

The velocity and acceleration diagrams are shown in Fig. 20.6 (*b*) and (*c*) respectively. Since the follower moves with a simple harmonic motion, therefore velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve. We see from Fig. 20.6 (*b*) that the velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke. On the other hand, the acceleration of the follower is maximum at the beginning and at the ends of the stroke and diminishes to zero at mid-stroke.

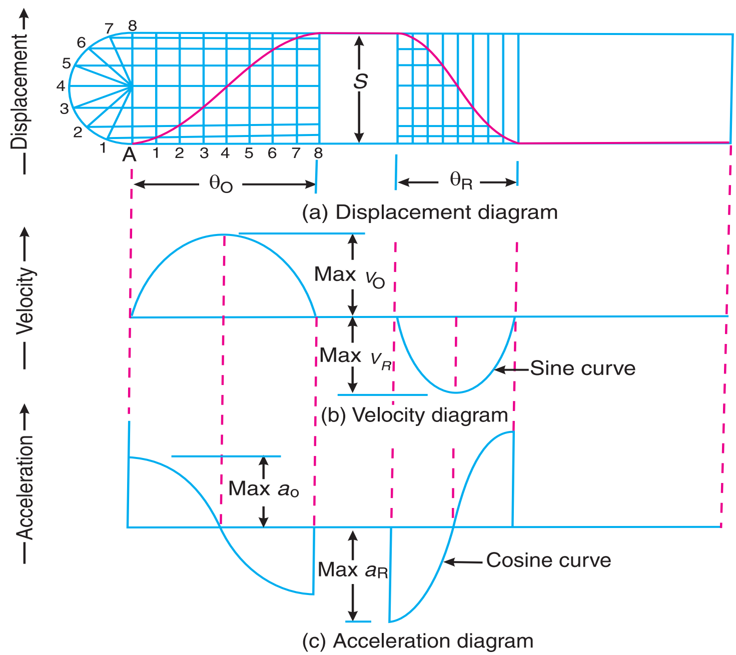

Fig. 20.6. Displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion.

Let S = Stroke of the follower,

θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians, and

ω = Angular velocity of the cam in rad/s.

∴ Time required for the out stroke of the follower in seconds,

$$t_O = \theta_O / \omega$$

Consider a point P moving at a uniform speed ω_P radians per sec round the circumference of a circle with the stroke S as diameter, as shown in Fig. 20.7. The point P' (which is the projection of a point P on the diameter) executes a simple harmonic motion as the point P rotates. The motion of the follower is similar to that of point P'.

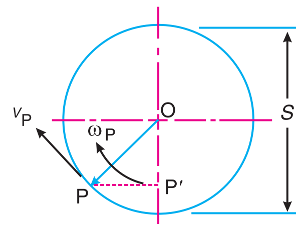

Fig. 20.7. Motion of a point.

∴ Peripheral speed of the point P',

$$v_P = \frac{\pi S}{2} \times \frac{1}{t_O} = \frac{\pi S}{2} \times \frac{\omega}{\theta_O}$$

and maximum velocity of the follower on the outstroke,

$$v_O = v_P = \frac{\pi S}{2} \times \frac{\omega}{\theta_O} = \frac{\pi \omega . S}{2\theta_O}$$

We know that the centripetal acceleration of the point P,

$$a_P = \frac{(v_P)^2}{OP} = \left(\frac{\pi \omega . S}{2\theta_O}\right)^2 \times \frac{2}{S} = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

∴ Maximum acceleration of the follower on the outstroke,

$$a_O = a_P = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi \omega . S}{2\theta_R}$$

and maximum acceleration of the follower on the return stroke,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2}$$

20.8. Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation

The displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation are shown in Fig. 20.8 (*a*), (*b*) and (*c*) respectively. We see that the displacement diagram consists of a parabolic curve and may be drawn as discussed below :

1. Divide the angular displacement of the cam during outstroke (θ_O) into any even number of equal parts (say eight) and draw vertical lines through these points as shown in Fig. 20.8 (*a*).
2. Divide the stroke of the follower (S) into the same number of equal even parts.
3. Join Aa to intersect the vertical line through point 1 at B. Similarly, obtain the other points C, D etc. as shown in Fig. 20.8 (*a*). Now join these points to obtain the parabolic curve for the out stroke of the follower.
4. In the similar way as discussed above, the displacement diagram for the follower during return stroke may be drawn.

Since the acceleration and retardation are uniform, therefore the velocity varies directly with the time. The velocity diagram is shown in Fig. 20.8 (*b*).

Let S = Stroke of the follower,

θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, and

ω = Angular velocity of the cam.

We know that time required for the follower during outstroke,

$$t_O = \theta_O / \omega$$

and time required for the follower during return stroke,

$$t_R = \theta_R / \omega$$

Mean velocity of the follower during outstroke

$$= S/t_O$$

and mean velocity of the follower during return stroke

$$= S/t_R$$

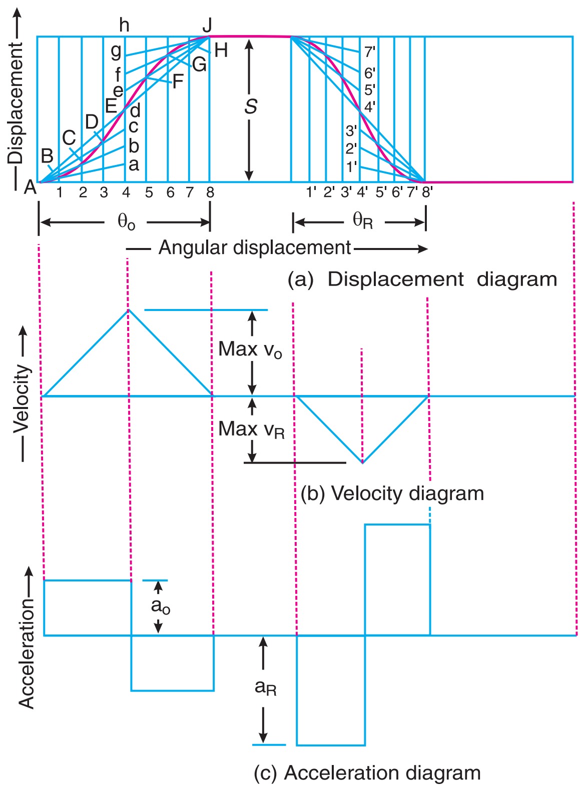

Fig. 20.8. Displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation.

Since the maximum velocity of follower is equal to twice the mean velocity, therefore maximum velocity of the follower during outstroke,

$$v_O = \frac{2S}{t_O} = \frac{2\omega.S}{\theta_O}$$

Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega.S}{\theta_R}$$

We see from the acceleration diagram, as shown in Fig. 20.8 (*c*), that during first half of the outstroke there is uniform acceleration and during the second half of the out stroke there is uniform retardation. Thus, the maximum velocity of the follower is reached after the time $t_O / 2$ (during out stroke) and $t_R /2$ (during return stroke).

∴ Maximum acceleration of the follower during outstroke,

$$a_O = \frac{v_O}{t_O / 2} = \frac{2 \times 2\,\omega.S}{t_O.\theta_O} = \frac{4\,\omega^2.S}{(\theta_O)^2} \qquad \ldots (\because \; t_O = \theta_O / \omega)$$

Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\,\omega^2.S}{(\theta_R)^2}$$

20.9. Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Cycloidal Motion

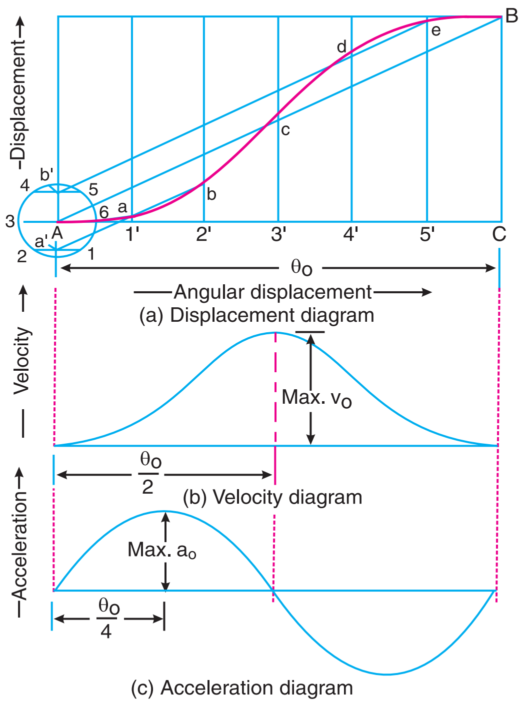

Fig. 20.9. Displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion.

The displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion are shown in Fig. 20.9 (*a*), (*b*) and (*c*) respectively. We know that cycloid is a curve traced by a point on a circle when the circle rolls without slipping on a straight line.

In case of cams, this straight line is a stroke of the follower which is translating and the circumference of the rolling circle is equal to the stroke (*S*) of the follower. Therefore the radius of the rolling circle is $S/2\pi$. The displacement diagram is drawn as discussed below :

1. Draw a circle of radius $S/2\pi$ with *A* as centre.

2. Divide the circle into any number of equal even parts (say six). Project these points horizontally on the vertical centre line of the circle. These points are shown by a' and b' in Fig. 20.9 (*a*).

3. Divide the angular displacement of the cam during outstroke into the same number of equal even parts as the circle is divided. Draw vertical lines through these points.

4. Join *AB* which intersects the vertical line through $3'$ at *c*. From a' draw a line parallel to *AB* intersecting the vertical lines through $1'$ and $2'$ at *a* and *b* respectively.

5. Similarly, from b' draw a line parallel to *AB* intersecting the vertical lines through $4'$ and $5'$ at *d* and *e* respectively.

6. Join the points *A a b c d e B* by a smooth curve. This is the required cycloidal curve for the follower during outstroke.

Cams are used in Jet and aircraft engines. The above picture shows an aircraft engine.

Let θ = Angle through which the cam rotates in time *t* seconds, and

ω = Angular velocity of the cam.

We know that displacement of the follower after time *t* seconds,

$$x = S\left[\frac{\theta}{\theta_O} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\theta_O}\right)\right] \quad \ldots (i)$$

$\therefore$ Velocity of the follower after time *t* seconds,

$$\frac{dx}{dt} = S\left[\frac{1}{\theta_O}\times\frac{d\theta}{dt} - \frac{2\pi}{2\pi\theta_O}\cos\left(\frac{2\pi\theta}{\theta_O}\right)\frac{d\theta}{dt}\right]$$

. . . [Differentiating equation (*i*)]

$$= \frac{S}{\theta_O}\times\frac{d\theta}{dt}\left[1-\cos\left(\frac{2\pi\theta}{\theta_O}\right)\right] = \frac{\omega.S}{\theta_O}\left[1-\cos\left(\frac{2\pi\theta}{\theta_O}\right)\right] \quad \ldots (ii)$$

The velocity is maximum, when

$$\cos\left(\frac{2\pi\theta}{\theta_O}\right) = -1 \quad \text{or} \quad \frac{2\pi\theta}{\theta_O} = \pi \quad \text{or} \quad \theta = \theta_O / 2$$

Substituting $\theta = \theta_O / 2$ in equation ***(ii)***, we have maximum velocity of the follower during outstroke,

$$v_O = \frac{\omega.S}{\theta_O}(1+1) = \frac{2\,\omega.S}{\theta_O}$$

Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\,\omega.S}{\theta_R}$$

Now, acceleration of the follower after time t sec,

$$\frac{d^2x}{dt^2} = \frac{\omega.S}{\theta_O}\left[\frac{2\pi}{\theta_O}\sin\left(\frac{2\pi\theta}{\theta_O}\right)\frac{d\theta}{dt}\right] \quad \text{. . . [Differentiating equation } (ii)]$$

$$= \frac{2\pi\omega^2.S}{(\theta_O)^2}\sin\left(\frac{2\pi\theta}{\theta_O}\right) \quad \text{. . . } \left(\because \frac{d\theta}{dt} = \omega\right) \quad \text{. . . } (iii)$$

The acceleration is maximum, when

$$\sin\left(\frac{2\pi\theta}{\theta_O}\right) = 1 \quad \text{or} \quad \frac{2\pi\theta}{\theta_O} = \frac{\pi}{2} \quad \text{or} \quad \theta = \theta_O / 4$$

Substituting $\theta = \theta_O / 4$ in equation (*iii*), we have maximum acceleration of the follower during outstroke,

$$a_O = \frac{2\pi\omega^2.S}{(\theta_O)^2}$$

Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{2\pi\omega^2.S}{(\theta_R)^2}$$

The velocity and acceleration diagrams are shown in Fig. 20.9 (*b*) and (*c*) respectively.

20.10. Construction of Cam Profile for a Radial Cam

In order to draw the cam profile for a radial cam, first of all the displacement diagram for the given motion of the follower is drawn. Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam is drawn.

In constructing the cam profile, the principle of kinematic inversion is used, *i.e.* the cam is imagined to be stationary and the follower is allowed to rotate in the ***opposite direction*** to the ***cam rotation.***

The construction of cam profiles for different types of follower with different types of motions are discussed in the following examples.

Example 20.1. *A cam is to give the following motion to a knife-edged follower :*

1. *Outstroke during 60° of cam rotation ;* ***2.*** *Dwell for the next 30° of cam rotation ;* ***3.*** *Return stroke during next 60° of cam rotation, and* ***4.*** *Dwell for the remaining 210° of cam rotation.*

The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes. Draw the profile of the cam when ***(a)*** *the axis of the follower passes through the axis of the cam shaft, and* ***(b)*** *the axis of the follower is offset by 20 mm from the axis of the cam shaft.*

Construction

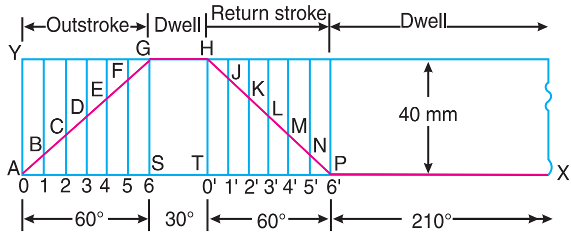

Fig. 20.10

First of all, the displacement diagram, as shown in Fig. 20.10, is drawn as discussed in the following steps :

1. Draw a horizontal line *AX* = 360° to some suitable scale. On this line, mark *AS* = 60° to represent outstroke of the follower, *ST* = 30° to represent dwell, *TP* = 60° to represent return stroke and *PX* = 210° to represent dwell.
2. Draw vertical line *AY* equal to the stroke of the follower (*i.e.* 40 mm) and complete the rectangle as shown in Fig. 20.10.
3. Divide the angular displacement during outstroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
4. Since the follower moves with uniform velocity during outstroke and return stroke, therefore the displacement diagram consists of straight lines. Join *AG* and *HP*.
5. The complete displacement diagram is shown by *AGHPX* in Fig. 20.10.

(a) Profile of the cam when the axis of follower passes through the axis of cam shaft

The profile of the cam when the axis of the follower passes through the axis of the cam shaft, as shown in Fig. 20.11, is drawn as discussed in the following steps :

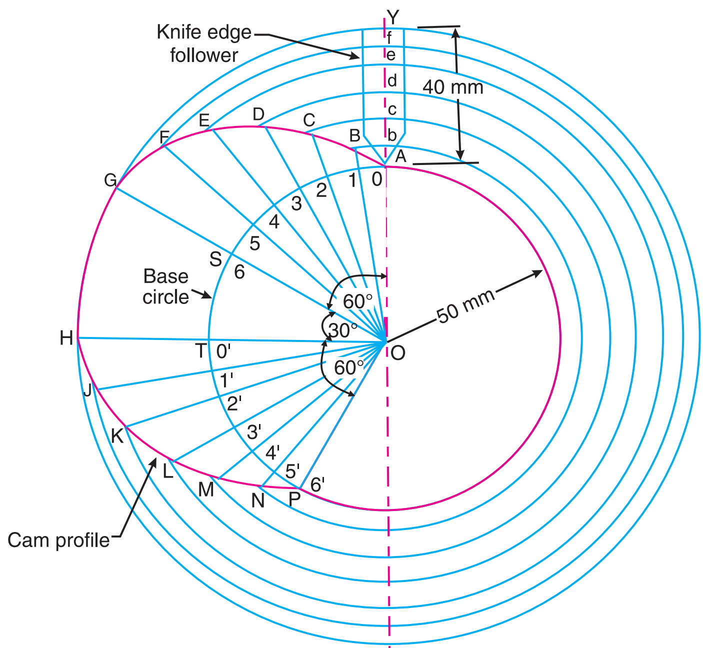

Fig. 20.11

1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with *O* as centre.
2. Since the axis of the follower passes through the axis of the cam shaft, therefore mark trace point *A*, as shown in Fig. 20.11.
3. From *OA*, mark angle *AOS* = 60° to represent outstroke, angle *SOT* = 30° to represent dwell and angle *TOP* = 60° to represent return stroke.
4. Divide the angular displacements during outstroke and return stroke (*i.e.* angle *AOS* and angle *TOP*) into the same number of equal even parts as in displacement diagram.
5. Join the points 1, 2, 3 ...etc. and $0', 1', 2', 3'$, ... etc. with centre *O* and produce beyond the base circle as shown in Fig. 20.11.
6. Now set off 1*B*, 2*C*, 3*D* ... etc. and $0'H, 1'J$ ... etc. from the displacement diagram.
7. Join the points *A*, *B*, *C*,... *M*, *N*, *P* with a smooth curve. The curve *AGHPA* is the complete profile of the cam.

Notes : The points *B*, *C*, *D* .... *L*, *M*, *N* may also be obtained as follows :

1. Mark *AY* = 40 mm on the axis of the follower, and set of *Ab*, *Ac*, *Ad*... etc. equal to the distances 1*B*, 2*C*, 3*D*... etc. as in displacement diagram.

2. From the centre of the cam *O*, draw arcs with radii *Ob*, *Oc*, *Od* etc. The arcs intersect the produced lines *O*1, *O*2... etc. at *B*, *C*, *D* ... *L*, *M*, *N*.

(b) Profile of the cam when the axis of the follower is offset by 20 mm from the axis of the cam shaft

The profile of the cam when the axis of the follower is offset from the axis of the cam shaft, as shown in Fig. 20.12, is drawn as discussed in the following steps :

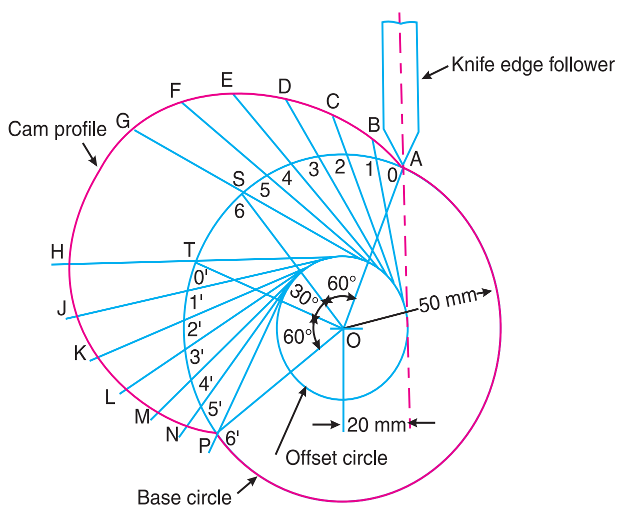

Fig. 20.12

1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with *O* as centre.
2. Draw the axis of the follower at a distance of 20 mm from the axis of the cam, which intersects the base circle at *A*.
3. Join *AO* and draw an offset circle of radius 20 mm with centre *O*.
4. From *OA*, mark angle *AOS* = 60° to represent outstroke, angle *SOT* = 30° to represent dwell and angle *TOP* = 60° to represent return stroke.

5. Divide the angular displacement during outstroke and return stroke (*i.e.* angle *AOS* and angle *TOP*) into the same number of equal even parts as in displacement diagram.
6. Now from the points 1, 2, 3 ... etc. and $0', 1', 2', 3'$ ... etc. on the base circle, draw tangents to the offset circle and produce these tangents beyond the base circle as shown in Fig. 20.12.
7. Now set off 1*B*, 2*C*, 3*D* ... etc. and $0'H, 1'J$ ... etc. from the displacement diagram.
8. Join the points *A*, *B*, *C* ...*M*, *N*, *P* with a smooth curve. The curve *AGHPA* is the complete profile of the cam.

Example 20.2. *A cam is to be designed for a knife edge follower with the following data :*

1. *Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion.*
2. *Dwell for the next 30°.*
3. *During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion.*
4. *Dwell during the remaining 180°.*

Draw the profile of the cam when

(a) the line of stroke of the follower passes through the axis of the cam shaft, and

(b) the line of stroke is offset 20 mm from the axis of the cam shaft.

The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

Solution. Given : S = 40 mm = 0.04 m; θ_O = 90° = $\pi/2$ rad = 1.571 rad ; θ_R = 60° = $\pi/3$ rad = 1.047 rad ; N = 240 r.p.m.

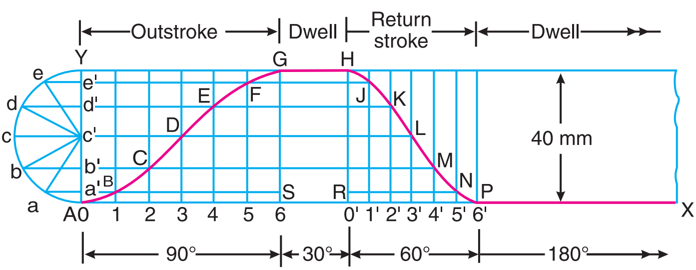

Fig. 20.13

First of all, the displacement diagram, as shown in Fig 20.13, is drawn as discussed in the following steps :

1. Draw horizontal line *AX* = 360° to some suitable scale. On this line, mark *AS* = 90° to represent out stroke ; *SR* = 30° to represent dwell ; *RP* = 60° to represent return stroke and *PX* = 180° to represent dwell.
2. Draw vertical line *AY* = 40 mm to represent the cam lift or stroke of the follower and complete the rectangle as shown in Fig. 20.13.
3. Divide the angular displacement during out stroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
4. Since the follower moves with simple harmonic motion, therefore draw a semicircle with *AY* as diameter and divide into six equal parts.
5. From points *a*, *b*, *c* ... etc. draw horizontal lines intersecting the vertical lines drawn through 1, 2, 3 ... etc. and $0', 1', 2'$ ...etc. at *B*, *C*, *D* ... *M*, *N*, *P*.
6. Join the points *A*, *B*, *C* ... etc. with a smooth curve as shown in Fig. 20.13. This is the required displacement diagram.