$OAGH$ is the acceleration diagram.

$$f_{ba}^{n} = \omega^2 \cdot AG$$
$$f_{ba}^{t} = \omega^2 \cdot GH$$
$$f_{ba} = \omega^2 \cdot AH$$
$$\omega^2 = \left(\frac{2\pi \times 300}{60}\right)^2 = 986.96\ (\text{rad/s})^2$$

Locate point X in AB, such that $AX = 400$ mm. Draw Xx parallel to BO to meet AH at x. Join Ox. Then

$$Ox \propto f_x$$
$$f_b = \omega^2 OH$$

(a) Acceleration of piston, $f_b = \omega^2 \times OH = \dfrac{986.96 \times 1.1 \times 100}{1000} = 108.56\ \text{m/s}^2$

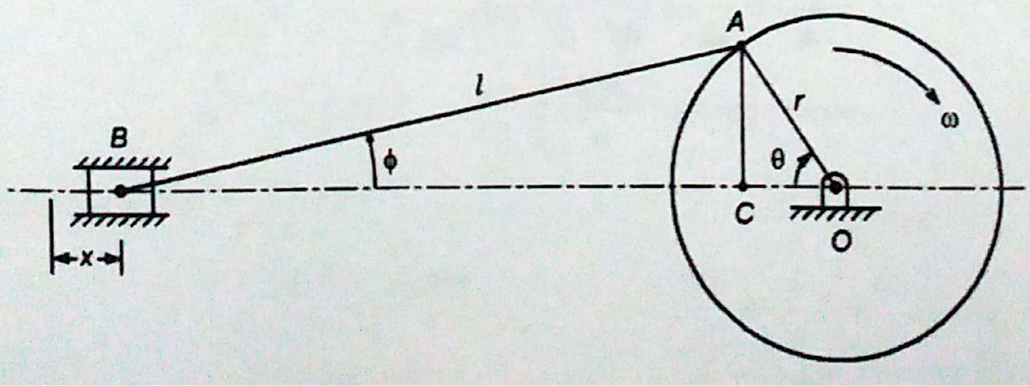
(b) Tangential acceleration of rod, $f_{ba}^{t} = \omega^2 \times GH = \dfrac{986.96 \times 2.6 \times 100}{1000} = 256.61\ \text{m/s}^2$

Angular acceleration of rod, $\alpha_{AB} = \dfrac{f_{ba}^{t}}{AB} = \dfrac{256.61}{1.2} = 213.84\ \text{rad/s}^2$

(c) Acceleration of point X on rod $= \omega^2 \times Ox = \dfrac{986.96 \times 2.2 \times 100}{1000} = 217.13\ \text{m/s}^2$

3.7 ANALYTICAL ANALYSIS OF SLIDER-CRANK MECHANISM

Consider the slider-crank mechanism shown in Fig.3.26. Let θ be the angle turned through by the crank $OA = r$ when the slider B has moved by an amount x to the right, and ϕ the angle, which the connecting rod $AB = l$ makes with the line of stroke.



$$x = r + l - (OC + BC)$$
$$= r + l - (r\cos\theta + l\cos\phi)$$
$$= r(1-\cos\theta) + l(1-\cos\phi)$$

Now $AC = r\sin\theta = l\sin\phi$

$$\sin\phi = \frac{r}{l}\sin\theta$$

Let $n = \frac{l}{r}$

Then $\cos\phi = (1-\sin^2\phi)^{0.5}$

$$= \left[1 - \frac{\sin^2\theta}{n^2}\right]^{0.5}$$

$$x = r(1-\cos\theta) + l\left\{1 - \left(1 - \frac{\sin^2\theta}{n^2}\right)^{0.5}\right\}$$

$$= r\left[(1-\cos\theta) + n\left\{1 - \left(1 - \frac{\sin^2\theta}{n^2}\right)^{0.5}\right\}\right]$$

$$= r\left[(1-\cos\theta) + n\left\{1 - \left(1 - \frac{\sin^2\theta}{2n^2} - \ldots\right)\right\}\right]$$

$$\approx r\left[(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right]$$

$$\approx r\left[(1-\cos\theta) + \frac{(1-\cos 2\theta)}{4n}\right] \quad (3.13)$$

Velocity of slider, $v_b = \frac{dx}{dt} = \frac{dx}{d\theta}\cdot\frac{d\theta}{dt} = \frac{dx}{d\theta}\cdot\omega$

$$= \omega r\left[\sin\theta + \frac{\sin 2\theta}{2n}\right] \quad (3.14)$$

Acceleration of slider,

$$f_b = \frac{d^2x}{dt^2} = \frac{dv_b}{dt} = \frac{dv_b}{d\theta}\cdot\frac{d\theta}{dt} = \omega\cdot\frac{dv_b}{d\theta}$$

$$= \omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

Now $\sin\phi = \frac{\sin\theta}{n}$

$$\cos\phi\cdot\frac{d\phi}{dt} = \frac{\cos\theta}{n}\times\frac{d\theta}{dt}$$

$$= \frac{\omega\cos\theta}{n}$$

Angular velocity of connecting rod,

$$\omega_{ba} = \frac{d\phi}{dt} = \frac{\omega}{n} \cdot \frac{\cos\theta}{\cos\phi}$$

$$= \frac{\omega \cos\theta}{(n^2 - \sin^2\theta)^{0.5}}$$

$$\approx \left(\frac{\omega}{n}\right) \cos\theta \quad (3.15)$$

Angular acceleration of connecting rod,

$$\alpha_{ba} = \frac{d\omega_{ba}}{dt} = -\omega^2 \sin\theta \frac{(n^2 - 1)}{(n^2 - \sin^2\theta)^{3/2}}$$

$$\approx -\frac{\omega^2}{n} \cdot \sin\theta \quad (3.16$$

Example 3.15