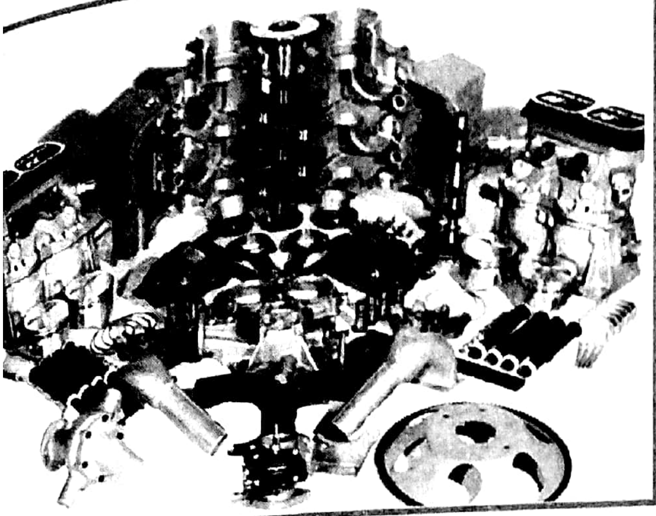


3

ACCELERATION IN MECHANISMS

Chapter Outline

3.1 INTRODUCTION

In the Chapter 2, we have defined the concept of velocity. In this chapter, we shall study the concept of acceleration. The acceleration may be defined as the rate of change of velocity of a body with respect to time. The acceleration can be linear or angular. Linear acceleration is the rate of change of linear velocity of a body with respect to time. Angular acceleration is the rate of change of angular velocity of a body with respect to time.

Acceleration diagram. It is the graphical representation of the accelerations of the various links of a mechanism drawn on a suitable scale. It helps us to determine the acceleration of various links of the mechanism.

The determination of acceleration of various links is important from the point of view of calculating the forces and torques in the various links to carry out the dynamic analysis.

3.2 ACCELERATION OF A BODY MOVING IN A CIRCULAR PATH

Consider a body moving in a circular path of radius r with angular speed ω, as shown in Fig.3.1(a). The body is initially at point A and in time δt moves to point B. Let the velocity change from v to $v + \delta v$ at B and the angle covered be $\delta\theta$.

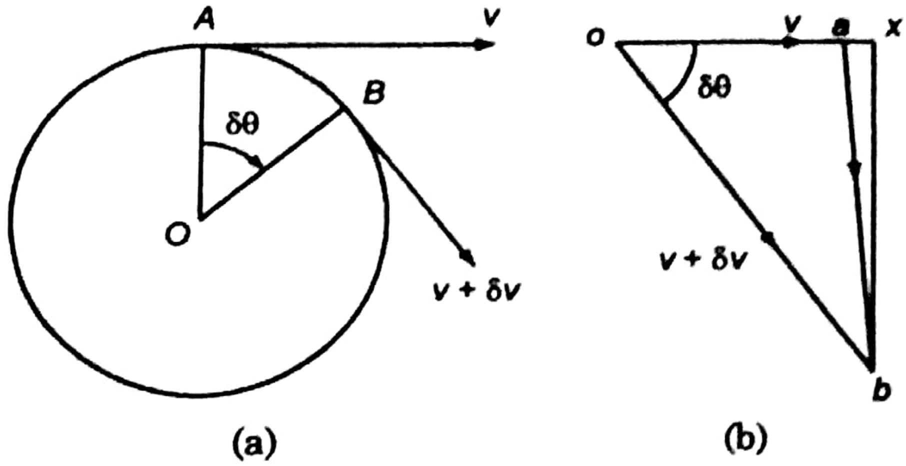


Fig.3.1 Acceleration for a link

The change in velocity can be determined by drawing the velocity diagram as shown in Fig.3.1(b). In this diagram, $oa = v$, $ob = v + \delta v$, and ab = change in velocity during time δt. The vector ab is resolved into two components ax and xb, parallel and perpendicular to oa, respectively.

Now $$ax = ox - oa = ob\cos\delta\theta - oa = (v+\delta v)\cos\theta - v$$

$$xb = ob\sin\delta\theta = (v+\delta v)\sin\delta\theta$$

The rate of change of velocity is defined as the acceleration. It has two components: tangential and normal. The tangential component of acceleration f^t is the acceleration in the tangential direction. It is defined by,

f^t = change of velocity in the tangential direction per unit time

$$= \frac{ax}{\delta t}$$

$$= \frac{(v+\delta v)\cos\theta - v}{\delta t}$$

$$\approx \frac{(v+\delta v) - v}{\delta t}$$ [Because for small angle $\cos\delta\theta \approx 1$]

$$= \frac{\delta v}{\delta t} = \frac{dv}{dt}$$

$$= \frac{d(\omega r)}{dt} = r\left(\frac{d\omega}{dt}\right) = ra \quad (3.1)$$

where α = angular acceleration.

The normal component of acceleration f^n is the acceleration in the direction normal to the tangent at that instant. This component is directed towards the centre of the circular path. It is also called the radial or centripetal acceleration. It is defined by,

f^n = change of velocity component in a direction normal to tangent per unit time

$$= \frac{xb}{\delta t}$$

$$= \frac{(v+\delta v)\ \sin\theta}{\delta t}$$

$$\approx \frac{(v+\delta v)\,\delta\theta}{\delta t} \qquad \text{[Because for small angle } \sin\delta\theta \approx \delta\theta\text{]}$$

$$= \frac{v\delta\theta}{\delta t} + \frac{\delta v\,\delta\theta}{\delta t} \approx \frac{v\delta\theta}{\delta t} \qquad \text{[Neglecting second term, being small]}$$

$$= \frac{v\,d\theta}{dt} = v\omega = r\omega^2 = \frac{v^2}{r} \tag{3.2}$$

Two cases arise regarding the motion of the body.

(i) When the body is rotating with uniform angular velocity, then $d\omega/dt = 0$, so that $f^t = 0$. The body will have only normal acceleration, $f^n = r\,\omega^2$.

(ii) When the body is moving on a straight path, r will be infinitely large and $\frac{v^2}{r}$ will tend to zero, so that $f^n = 0$. The body will have only tangential acceleration, $f^t = r\alpha$.

3.3 ACCELERATION DIAGRAMS

3.3.1 Total Acceleration of a Link

Consider two points A and B on a rigid link, as shown in Fig.3.2(a), such that the point B moves relative to point A with an angular velocity ω and angular acceleration α.

Centripetal (or normal or radial) acceleration of point B with respect to point A is,

$$f_{ba}^n = \omega^2 \cdot AB = \frac{v_{ba}^2}{AB} \tag{3.3}$$

Tangential acceleration of point B with respect to point A,

$$f_{ba}^t = \alpha \cdot AB \tag{3.4}$$

Total acceleration of B w.r.t. A,

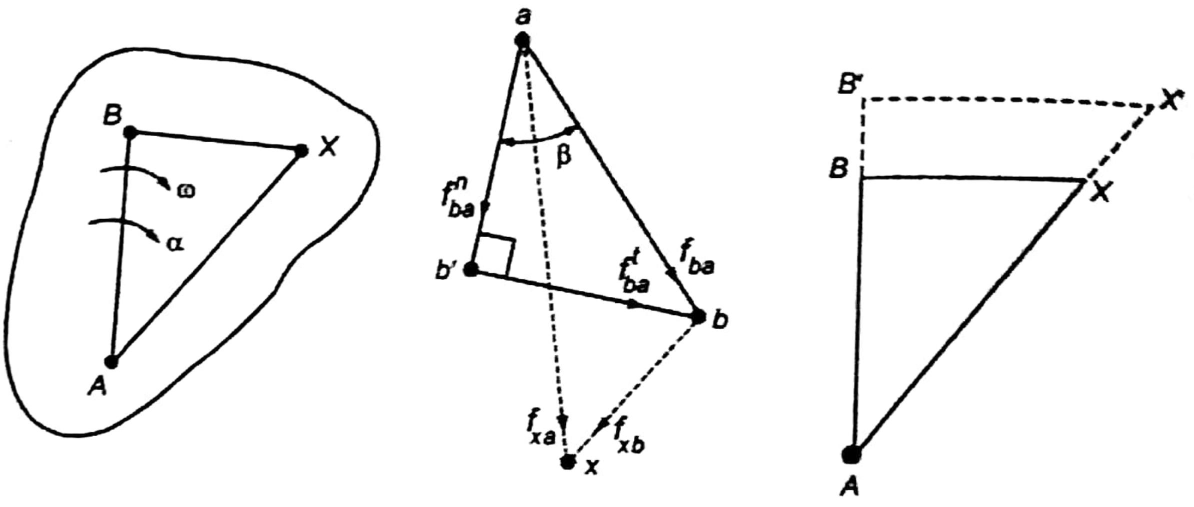
$$f_{ba} = f_{ba}^n + f_{ba}^t \quad \text{(vector sum)}$$

$$= \omega^2 \times AB + \alpha \times AB$$

$$= (\omega^2 + \alpha) \cdot AB \tag{3.5}$$

$$\tan\beta = \frac{f_{ba}^t}{f_{ba}^n} = \frac{\alpha}{\omega^2} \tag{3.6}$$

The acceleration diagram has been represented in Fig.3.2(b).



(a) Link subjected to acceleration (b) Acceleration diagram (c) Acceleration image

Fig.3.2 Total acceleration of a link

3.3.2 Acceleration of a Point on a Link

The accelerations of any point X on the rigid link w.r.t. A as shown in Fig.3.2(a) are:

$$f^n_{xa} = \omega^2 \cdot AX \tag{3.7}$$

$$f^t_{xa} = \alpha \cdot AX \tag{3.8}$$

Total acceleration, $$f_{xa} = f^n_{xa} + f^t_{xa} \quad \text{(vector sum)} \tag{3.9}$$

Therefore, f_{xa} Denoted by ax in the acceleration diagram shown in Fig.3.2(b) is inclined to XA at the same angle β. Triangles abx and ABX are similar. Thus, point x can be fixed on the *acceleration image*, corresponding to point X on the link. Total acceleration of X relative to A,

$$f_{xa} = ax$$

Total acceleration of X relative to B,

$$f_{xb} = bx$$

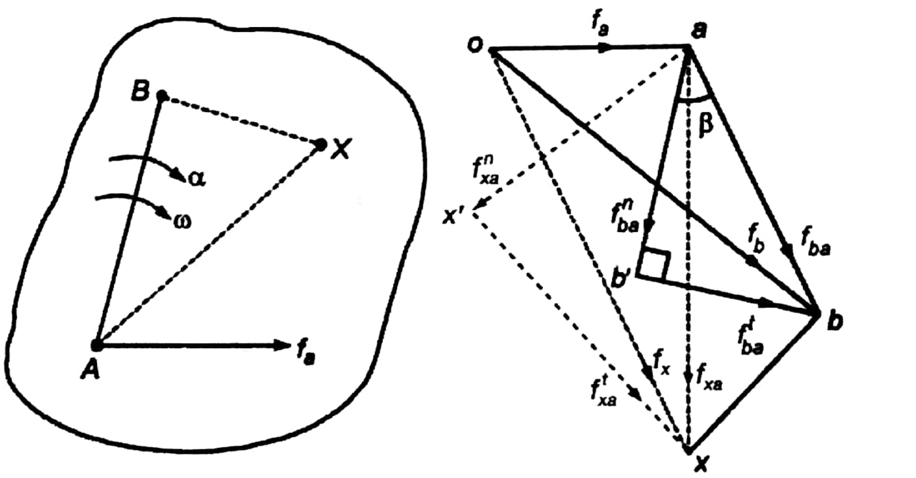
Acceleration Image The concept of velocity image was explained in Section 2.3.7(b) in Chapter 2. It was stated that the velocity images are useful in finding velocities of offset points of links. In the same way, acceleration images are also helpful to find the accelerations of offset points of the links. The acceleration image is obtained in the same manner as a velocity image.

An easier method of making Δabx similar to ΔABX is by making AB' on AB equal to ab and drawing a line parallel to BX, meeting AX in X'. $AB'X'$ is the exact size of the triangle to be made on ab.

Take $ax = AX'$ and $bx = B'X'$
Thus the point x is located.
The method is illustrated in Fig.3.2(c).

3.3.3 Absolute Acceleration for a Link

Consider the rigid link AB such that point B is rotating about A with angular velocity ω and angular acceleration α, as shown in Fig.3.3(a). The point A itself has acceleration f_a. The acceleration diagram is shown in Fig.3.3(b). Absolute acceleration of B,



(a) Rigid link subjected to acceleration **(b) Acceleration diagram**

Fig.3.3 Absolute acceleration of a rigid link

$$f_b = f_a + f_{xa}^n + f_{xa}^{t'} \quad \text{(vector sum)}$$

$$= f_a + \omega^2 \cdot AB + \alpha \cdot AB \tag{3.10}$$

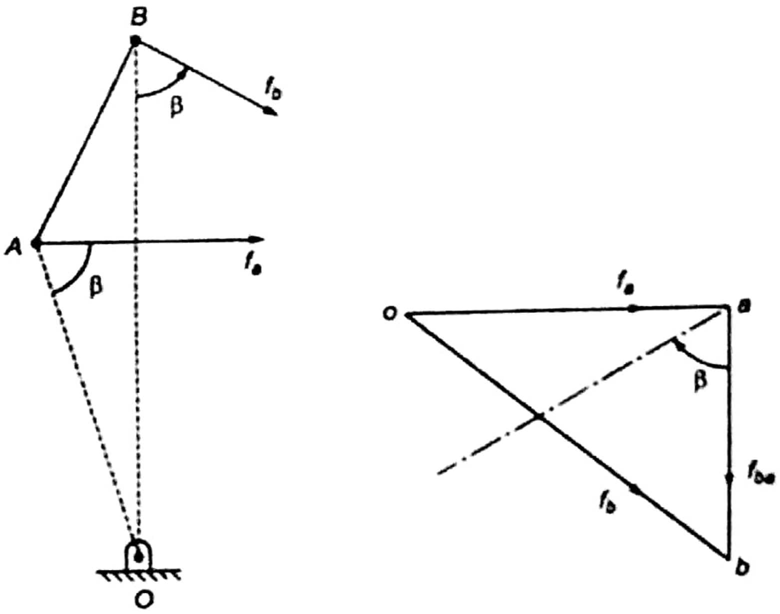
Similarly for any other point X,

$$f_x = f_a + f_{xa}^n + f'xa$$

$$= f_a + \omega^2 \cdot AX + \alpha \cdot AX \tag{3.11}$$

3.3.4 Acceleration Centre

Consider a rigid link AB whose ends A and B have accelerations f_a and f_b respectively, as shown in Fig.3.4(a). The acceleration diagram is shown in Fig.3.4(b). If we select a point O on the link such that triangles aob and AOB are similar, then the acceleration of point O relative to a fixed link or fixed point O is zero. The point O is called the instantaneous centre of acceleration of link AB or *acceleration centre.*

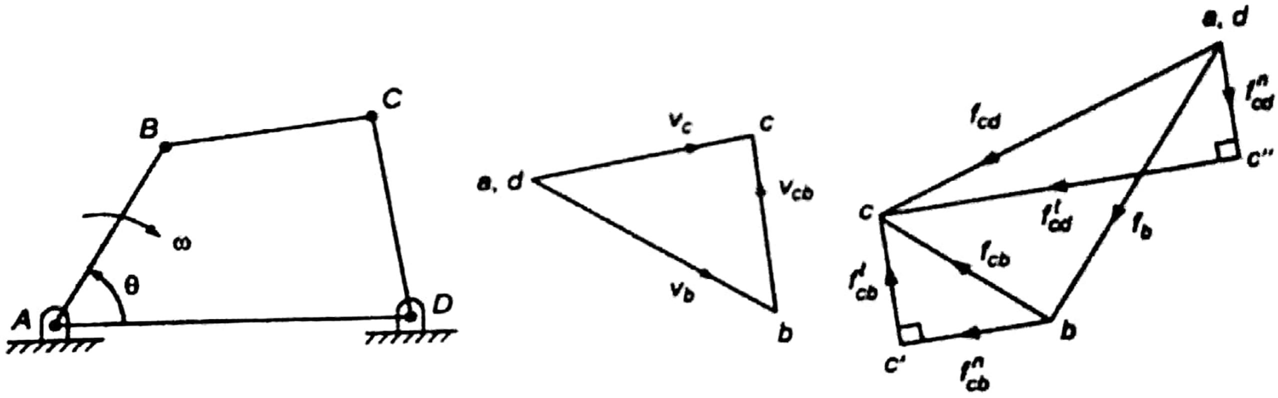


(a) Link having acceleration (b) Acceleration diagram

Fig.3.4 Acceleration centre

3.3.5 Acceleration Diagram for Four-Bar Mechanism

The four-bar mechanism is shown in Fig.3.5(a). The velocity of point B, $v_b = \omega \cdot AB$. The velocity diagram is shown in Fig.3.5(b), and has been drawn as explained in Section 2.3.7(b). In this diagram,



(a) Configuration diagram (b) Velocity diagram (c) Acceleration diagram

Fig.3.5 Acceleration diagram for four-bar mechanism

$$v_b = ab \perp AB$$

$$dc \perp DC$$

and

$$bc \perp BC$$

Then Velocity of C relative to D, $v_{cd} = v_c = dc$

Velocity of C relative to B, $v_{cb} = bc$

Now we calculate the accelerations of various points and links.

$$f_{ba}^n = \frac{v_b^2}{AB} = ab$$

$$f_{cb}^{n} = \frac{v_{cb}^{2}}{CB} = bc'$$

$$f_{cd}^{n} = \frac{v_{c}^{2}}{CD} = dc''$$

$$f_{cb} = f_{cb}^{n} + f_{cb}^{t}$$

$$bc = bc' + c'c$$

$$f_{cd} = f_{cd}^{n} + f_{cd}^{t}$$

$$dc = dc'' + c''c$$

The acceleration diagram is shown in Fig.3.5(c) to a suitable scale. To construct the acceleration diagram, proceed as follows:

1. Draw $ab = f_b$ parallel to AB, which is known in magnitude and direction.
2. Draw $bc' = f_{cb}^{n}$ parallel to BC, which is known in magnitude and direction.
3. Draw cc', representing f_{cb}^{t} perpendicular to bc', which is known in direction only.
4. Now draw $dc'' = f_{cd}^{n}$ parallel to CD, which is known in magnitude and direction.
5. Draw $c''c$, representing f_{cd}^{t}, perpendicular to dc'' to intersect $c'c$ at point c.
6. Join dc and bc. Then

Acceleration of C relative to B, $f_{cb} = bc$

Acceleration of C relative to D, $f_{cd} = f_c = dc$

3.3.6 Four-Bar Mechanism with Ternary Link

The four-bar mechanism with a ternary link is shown in Fig.3.6(a), in which the driving crank has angular velocity ω and angular acceleration $\alpha \cdot v_a = \omega \times O_1A$. The velocity diagram is shown in Fig.3.6(b), and has been drawn as explained in Section 2.3.7 (a). In this diagram,

(i) Velocity diagram

$$v_a = o_1a \perp O_1A$$

$$ab \perp AB$$

$$o_2b \perp O_2B$$

Then velocity of B relative to A, $v_{ba} = ab$

Velocity of B relative to O_2, $v_{bo_2} = v_b = o_2b$

Now $\quad bc \perp BC$

and $\quad ac \perp AC$

which locates point c.

Velocity of C relative to O_1, $v_c = o_1c$

Then Δabc is the velocity image of ternary link ABC.

Now $\quad \dfrac{ad}{ab} = \dfrac{AD}{AB}$