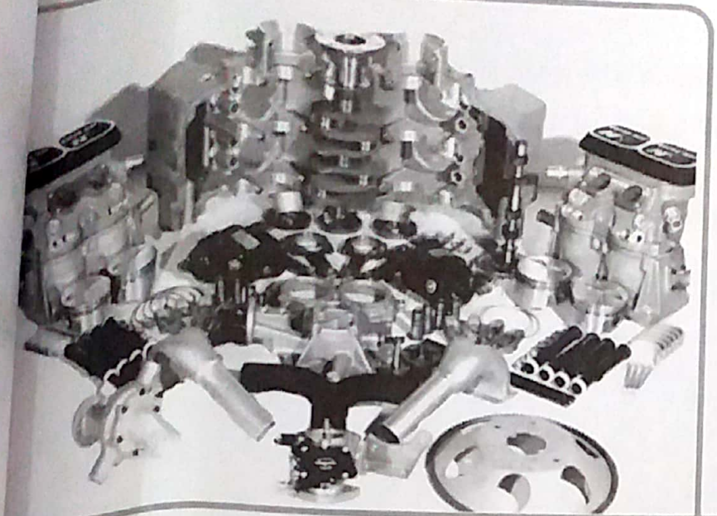

2 VELOCITY IN MECHANISMS

Chapter Outline

2.1 INTRODUCTION

Kinematics deals with the study of relative motion between the various links of a machine ignoring the forces involved in producing such motion. Thus, kinematics is the study to determine the displacement velocity and acceleration of the various links of the mechanism. A machine is a mechanism or a combination of mechanisms that not only imparts definite motion to the various links but also transmits and modifies the available mechanical energy into some kind of useful energy. In this chapter, we shall study the various methods of determination of velocity in mechanisms.

2.2 VELOCITY DIAGRAMS

Displacement: The displacement of a body is its change of position with reference to a certain fixed point.

Velocity: Velocity is the state of change of displacement of a body with respect to time. It is a vector quantity.

Linear velocity: It is the rate of change of velocity of a body along a straight line with respect to time. Its units are m/s.

Angular velocity: It is the rate of change of angular position of a body with respect to time. Its units are rad/s.

The relationship between velocity v and angular velocity ω is:

$$v = r\,\omega \tag{2.1}$$

where r = distance of point undergoing displacement from the centre of rotation.

Relative velocity: The relative velocity of a body A with respect to a body B is obtained by adding to the velocity of A the reversed velocity of B. If $v_a > v_b$, then

$$\vec{v}_{ab} = \vec{v}_a - \vec{v}_b$$

or

$$ba = oa - ob$$

Similarly,

$$\vec{v}_{ba} = \vec{v}_b - \vec{v}_a$$

or

$$ab = ob - oa$$

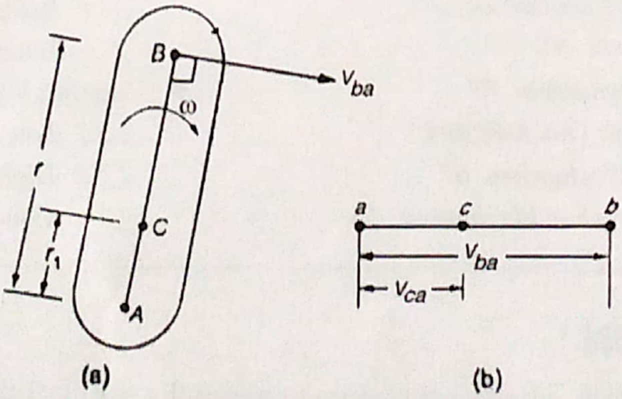

Fig.2.1 Relative velocity of a point

Consider two points A and B on a rigid link rotating clockwise about A as shown in Fig.2.1(a). There can be no relative motion between A and B as long as the distance between A and B remains the same. Therefore, the relative motion of B with respect to A must be perpendicular to AB. Hence, the direction of relative velocity of two points in a rigid link is always along the perpendicular to the straight line joining these points. Let relative velocity of B with respect to A be represented by $v_{ba} = \omega \cdot AB$, then ab is drawn perpendicular to AB to a convenient scale, as shown in Fig.2.1(b). Similarly, the linear velocity of any other point C on AB with respect to A is $v_{ca} = \omega \cdot AC$ and is represented by vector ac. Hence,

$$v_{ba}/v_{ca} = (\omega \cdot AB)/(\omega \cdot AC) = AB/AC$$

or

$$ab/ac = AB/AC = r/r_1 \tag{2.2}$$

Hence the point C divides the vector ab in the same ratio as the point C divides the link AB.

2.3 DETERMINATION OF LINK VELOCITIES

There are two methods to determine the velocities of links of mechanisms.

1. Relative velocity method
2. Instantaneous centre method

2.3.1 Relative Velocity Method

Consider a rigid link AB, as shown in Fig.2.2(a), such that the velocity of A (v_a) is vertical and velocity of B (v_b) is horizontal. To construct the velocity diagram, take a point o. Draw oa representing the magnitude and direction of velocity of A. Draw ob along the direction of v_b. From point 'a' draw a line ab perpendicular to AB, meeting ob in b. Then oab is the velocity triangle, as shown in Fig.2.2(b). $Ob = v_b$; $ab = v_{ba}$, i.e. the velocity of B with respect to A. Vector ab is called the *velocity image* of link AB. The velocity of any point C in AB with respect to A is given by,

$$v_{ca} = ac = v_{ba} \cdot \left(\frac{AC}{AB}\right)$$

$$v_c = v_a + \left(\frac{AC}{AB}\right) v_{ba}$$

$$= oa + \left(\frac{ac}{ab}\right) \cdot ab$$

$$= oa + ac$$

$$= oc$$

Hence vector oc represents the velocity of point C.

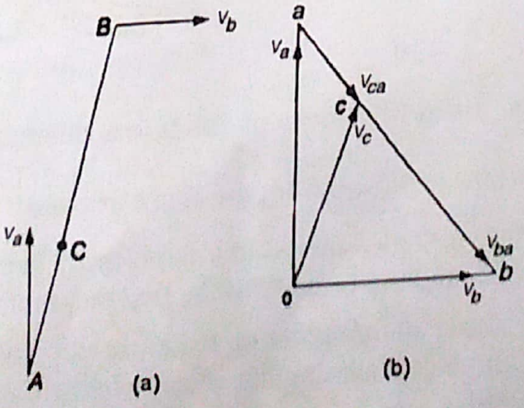

Fig.2.2 Relative velocity of points in a kinematic link

2.3.2 Relative Velocity of Points in a Kinematic Link

Consider a link A_1B_1 first undergoing rotation by an amount $\Delta\theta$ to A_1B_1' and then undergoing translation by an amount Δs_A to occupy the new position A_2B_2, as shown in Fig.2.3(a). Then

$$\Delta s_B = \Delta s_{BA} + \Delta s_A \tag{a}$$

Now let the link A_1B_1 first undergo linear translation Δs_A to A_2B_1'' and then angular rotation of $\Delta\theta$ to A_2B_2, as shown in Fig.2.3(b). Then

$$\Delta s_B = \Delta s_A + \Delta s_{BA} \tag{b}$$

Eqs. (a) and (b) are same. Dividing by Δt, we get

$$\frac{\Delta s_B}{\Delta t} = \frac{\Delta s_A}{\Delta t} + \frac{\Delta s_{BA}}{\Delta t}$$

or

$$v_b = v_a + v_{ba}$$

Therefore, the velocity of point B is obtained by adding vectorially the relative velocity of point B w.r.t. point A to the velocity of point A.

Now

$$\Delta s_{BA} = A_1B_1 \cdot \Delta\theta$$

or

$$\frac{\Delta s_{BA}}{\Delta t} = A_1B_1 \cdot \frac{\Delta\theta}{\Delta t}$$

$$v_{ba} = A_1B_1 \cdot \omega$$

Also

$$\angle\phi = 90^\circ$$

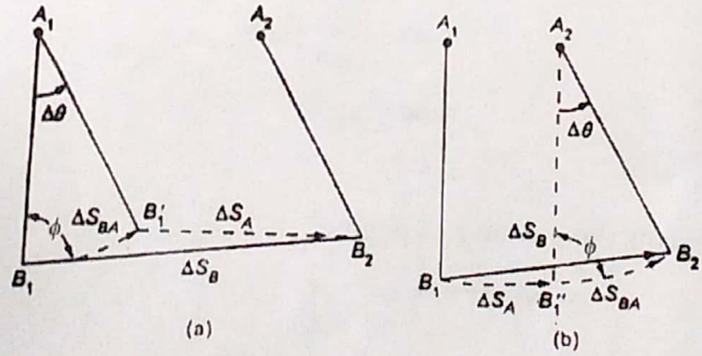

Fig.2.3 Relative velocity of points in a kinematic link

The following conclusions may be drawn from the above analysis:

- The velocity of any point on the kinematic link is given by the vector sum of the velocity of some other point in the link and the velocity of the first point relative to the other.
- The magnitude of the velocity of any point on the kinematic link relative to the other point in the kinematic link is the product of the angular velocity of the link and distance between the two points under consideration.
- The direction of the velocity of any point on a link relative to any other point on the link is perpendicular to the line joining the two points.

2.3.3 Relative Angular Velocities

Consider two links *OA* and *OB* connected by a pin joint at *O*, as shown in Fig.2.4. Let ω_1 and ω_2 be the angular velocities of the links *OA* and *OB*, respectively. Relative angular velocity of *OA* with respect to *OB* is,

$$\omega_{12} = \omega_1 - \omega_2$$

and relative angular velocity of *OB* with respect to *OA* is,

$$\omega_{21} = \omega_2 - \omega_1$$
$$= -\omega_{12}$$

If r = radius of the pin at joint *O*, then rubbing (or sliding) velocity at the pin joint *O*,

$$= (\omega_1 - \omega_2)r, \text{ when the links move in the same direction} \quad (2.3a)$$
$$= (\omega_1 + \omega_2)r, \text{ when the links move in the opposite direction} \quad (2.3b)$$

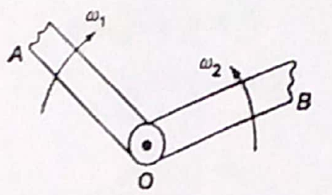

Fig.2.4 Relative angular velocities

2.3.4 Relative Velocity of Points on the Same Link

Consider a ternary link *ABC*, as shown in Fig.2.5(a), such that *C* is any point on the link. Let v_a and v_b be the velocities of points *A* and *B*, respectively. Then

$$v_b = v_a + v_{ba}$$

$$\omega_{ab} = \frac{v_{ba}}{AB} = \frac{ab}{AB}$$

$$v_c = v_a + v_{ca}$$

or
$$= v_b + v_{cb}$$

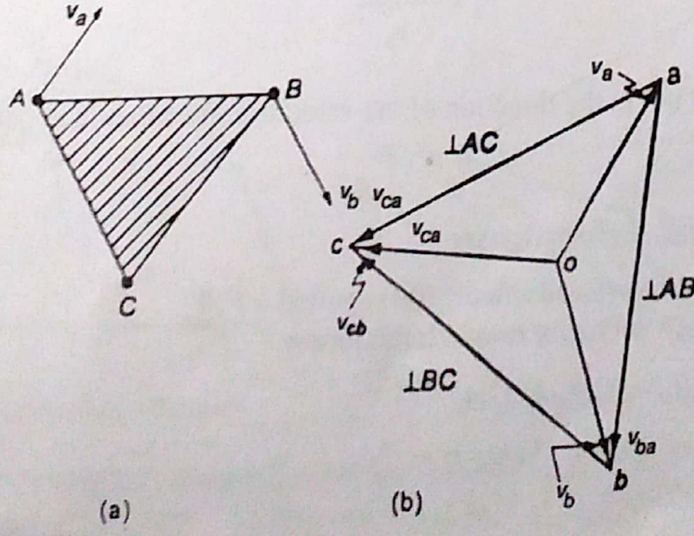

Fig.2.5 Relative velocity of points on the same link

The velocity diagram is shown in Fig.2.5(b).

Angular velocity of link ABC,

$$\omega_{abc} = \frac{v_{ba}}{AB} = \frac{v_{cb}}{BC} = \frac{v_{ca}}{AC}$$

$$= \frac{ab}{AB} = \frac{bc}{BC} = \frac{ac}{AC} \quad (2.4)$$

2.3.5 Forces in a Mechanism

Consider a link AB subjected to the action of forces and velocities, as shown in Fig.2.6. Let A be the driving end and B the driven end. When the direction of the forces and velocities is the same, then

Energy at A = Energy at B

$$F_a \times v_a = F_b \times v_b$$

or

$$F_b = \frac{F_a v_a}{v_b} \quad (2.5a)$$

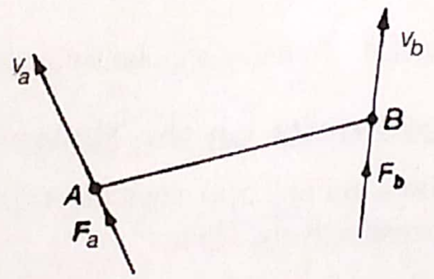

Fig.2.6 Force and velocity diagram

Considering the effect of friction, the efficiency of transmission,

$$\eta = \frac{\text{output}}{\text{input}} = \frac{(F_b v_b)}{(F_a v_a)}$$

or

$$F_b = \frac{\eta F_a v_a}{v_b} \quad (2.5b)$$

When the forces are not in the direction of the velocities, then their components along the velocities should be taken.

2.3.6 Mechanical Advantage

Mechanical advantage, MA = Load lifted/Effort applied = F_b/F_a

For a mecahnism, MA = Output torque/Input torque

$$= T_b/T_a = \omega_a/\omega_b \quad (2.6a)$$

Considering the effect of friction, MA = $\eta\ \omega_a / \omega_b$ (2.6b)

Scanned by CamScanner