2.19 ORTHOGONALITY OF WAVE FUNCTION

There may be many acceptable solutions to SWE $\hat{H}\Psi = E\Psi$ for a particular system. Each wave function has a corresponding energy value E. For any two wave functions Ψ_n and Ψ_m corresponding to the energy values E_n and E_m, the following condition must be fulfilled.

$$\int_{-\infty}^{+\infty} \Psi_n \Psi_m \, d\tau = 0 \qquad (2.61)$$

Such a condition is called condition of orthogonality of wave functions; the two functions Ψ_n and Ψ_m are said to be orthogonal to each other. Equation (2.61) is a general property of the wave functions. Wave functions that are solution of a given SWE are usually orthonormal to one another. They are independent of one another and the integral of their product over the whole space is zero. They are not degenerate wave functions (having same energy) and are exact solution of the wave equation.

The normalization and orthogonality conditions may be combined as follows

$$\int_{-\infty}^{+\infty} \Psi_n^* \Psi_m \, d\tau = 1, \qquad \text{if } n = m$$

$$\int_{-\infty}^{+\infty} \Psi_n^* \Psi_m \, d\tau = 0, \qquad \text{if } n \neq m$$

These relations can be combined by writing

$$\int_{-\infty}^{+\infty} \Psi_n^* \Psi_n \, d\tau = \delta_{nm}$$

or

$$\int_{-\infty}^{+\infty} \Psi_n \Psi_m^* \, d\tau = \delta_{nm} \qquad (2.62)$$

where δ_{nm} is called *Kronecker delta*, which is defined by

$$\delta_{nm} \begin{Bmatrix} 0 & \text{for} & n \neq m \\ 1 & \text{for} & n = m \end{Bmatrix} \qquad (2.63)$$

wave functions that satisfy equation (2.63) are said to be *orthonormal*.

An exception to the orthogonality rule occurs when two or more wave functions correspond to the same energy level. Such levels are said to be degenerate levels. Wave functions for degenerate levels are not always orthogonal to one another. However, they are orthogonal to all other wave functions, that are solutions of the same wave equation.