...The Normal Zeeman Effect

...will now discuss the behavior of the H atom in ...ernal magnetic field. In the beginning we will ...semiclassical model (called the vector model), ...e the electron motion is described by a classical ...lar orbit, while the angular momentum is given by ...uantum mechanical expression

$|l| = \sqrt{l(l+1)}\hbar .$

...ectron with charge $-e$ moving with the velocity v ...he circular frequency $\nu = v/(2\pi r)$ on a circle with ...r represents an electric current

$$I = -e\nu = -\frac{ev}{2\pi r} , \qquad (5.24)$$

...h causes a magnetic moment

$$\mu = I A = I\pi r^2 \hat{n} , \qquad (5.25)$$

...e $A = \pi r^2 \hat{n}$ is the area vector perpendicular to the ... of the motion (Fig. 5.8).

...he angular momentum of the circulating electron

$$l = r \times p = m_e r v \hat{n} . \qquad (5.26)$$

...omparison of (5.25) and (5.26) gives the relation

$$\boxed{\mu = -\frac{e}{2m_e} l} \qquad (5.27)$$

...een magnetic moment μ and angular momentum l ...electron. Since μ is proportional to l, the orbital ...etic moment is often labeled μ_l.

...an external magnetic field the potential energy of ...netic dipole with magnetic moment μ is

$$E_{pot} = -\mu \cdot B . \qquad (5.28)$$

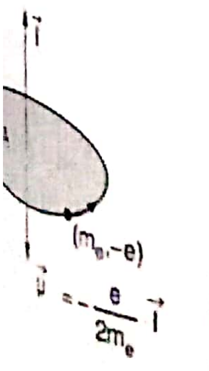


Fig. 5.8. Classical model of orbital angular momentum l and magnetic moment μ

Using the relation (5.27) this can be expressed by the angular momentum l as

$$E_{pot} = +\frac{e}{2m_e} l \cdot B . \qquad (5.29)$$

When the magnetic field points into the z-direction ($B = \{0, 0, B_z = B\}$), we obtain from (5.29), because of $l_z = m\hbar$,

$$E_{pot} = \frac{e\hbar}{2m_e} m B , \qquad (5.30)$$

where m (which had been introduced before as the projection of l onto the z-axis) is called the magnetic quantum number, that can take the values $-l \le m \le +l$.

The constant factor in (5.30)

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274015 \times 10^{-24}\,\mathrm{J/T} \qquad (5.31)$$

is called the ***Bohr magneton***.

We can now write the additional energy caused by the magnetic field as

$$\Delta E_m = \mu_B m B , \qquad (5.32)$$

which gives for the energies of the hydrogen atomic states in an external magnetic field:

$$\boxed{E_{n,l,m} = E_{Coul}(n) + \mu_B m B .} \qquad (5.33)$$

The $2l+1$ m states that are degenerate without magnetic field split into $2l+1$ equidistant Zeeman components with an energetic distance (Fig. 5.9)

$$\Delta E = E_{n,l,m} - E_{n,l,m-1} = \mu_B B , \qquad (5.34)$$

which is determined by the product of Bohr magneton μ_B and magnetic field strength B.

> The splitting of the $2l+1$ degenerate m components in an external magnetic field B due to the orbital magnetic moment related to the angular momentum $|l| = \sqrt{l(l+1)}\hbar$ is called the ***normal Zeeman effect***.

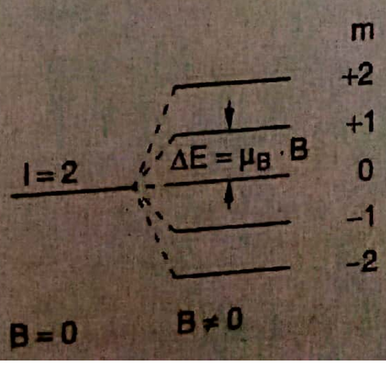


Fig. 5.9. Zeeman splitting of a level with $l = 2$ in a homogeneous magnetic field (normal Zeeman effect)

Using the Bohr magneton (5.11) we can write the orbital magnetic moment of the electron as

$$\boldsymbol{\mu}_l = -(\mu_B/\hbar)\,\boldsymbol{l} \tag{5.34x}$$

Since the external magnetic field with cylindrical symmetry breaks the spherical symmetry of the Coulomb potential the orbital angular momentum l of the electron is no longer constant, because the torque

$$\boldsymbol{D} = \boldsymbol{\mu}_l \times \boldsymbol{B} \tag{5.34y}$$

acts on the electron. In the case of a magnetic field $\boldsymbol{B} = \{0, 0, B_z = B\}$ in the z direction the z component of l stays constant. The vector $\boldsymbol{l}$ precesses around the z-axis on a cone with the apex angle 2α (Fig. 5.10), where

$$\cos\alpha = \frac{l_z}{|l|} = \frac{m}{\sqrt{l(l+1)}} \tag{5.34z}$$

The component l_z has the values

$$l_z = m\hbar \quad \text{with} \quad -l \le m \le +l \tag{5.35}$$

Also, the absolute value of l

$$|l| = \sqrt{l(l+1)}\,\hbar \tag{5.36}$$

is well defined, while the two other components l_x and l_y are not defined (see Sect. 4.4.2). Their quantum mechanical expectation value is zero, as is the classical time averaged value.

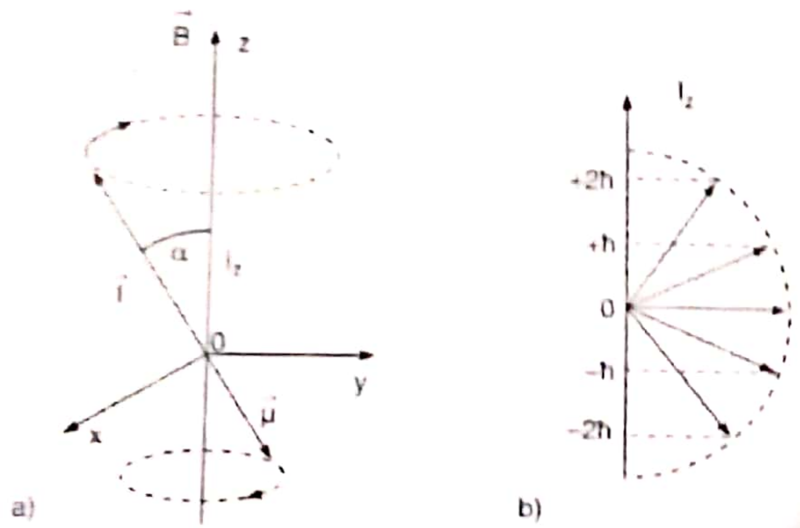


Fig. 5.10a,b. Vector model of the normal Zeeman effect. (**a**) Classical model of orbital angular momentum precessing around the field axis. (**b**) Possible orientations of l and projections $m\hbar$ in the quantum mechanical description

For the absorption or emission of light ... in a magnetic field our model makes the ... predictions.

When a circularly polarized σ^+ light ... gates into the z direction all photons have the ... If they are absorbed by atoms in the magnetic ... $B = \{0, 0, B_z\}$ they transfer their spin ($\hbar$) to the ... (because of conservation of angular momentum) ... therefore cause transitions with $\Delta l_z = +\hbar$ ... quantum number m changes by +1.

For σ^- polarization of the light wave ... with $\Delta m = -1$ are induced.

A similar consideration is valid for the ... of light by atoms in a magnetic field. For ... emitted into the direction of the magnetic field ... the z direction) the two circularly polarized ... components are observed, while for light emitted ... the direction perpendicular to the field three ... polarized components are observed. One component ... with the E vector parallel to B, which is not shifted against the field free transition, and two components with $E \perp B$, which are shifted to opposite sides of the unshifted line (Fig. 5.11).

According to (5.34), the Zeeman splitting $\Delta E = \mu_B B$ is independent of the quantum numbers ... and l. This implies that all atomic states should have the same separation of the Zeeman components. Therefore every spectral line corresponding to a transition $(n_1 l_1) \to (n_2, l_2)$ should always split in a magnetic field into three Zeeman components (Fig. 5.12) with σ^+, σ^- and π-polarization and a frequency separation of

$$\Delta\nu = \mu_B B/h .$$

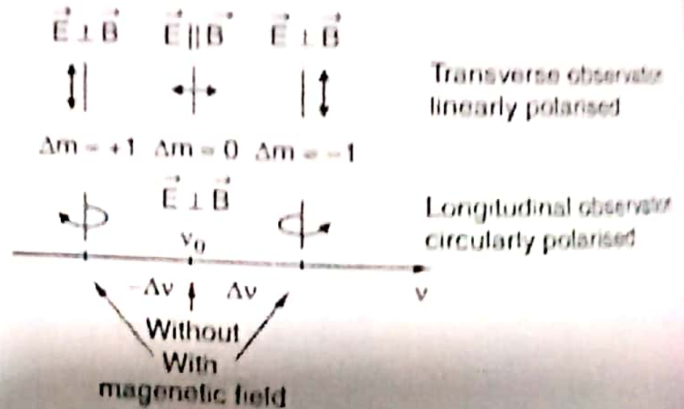
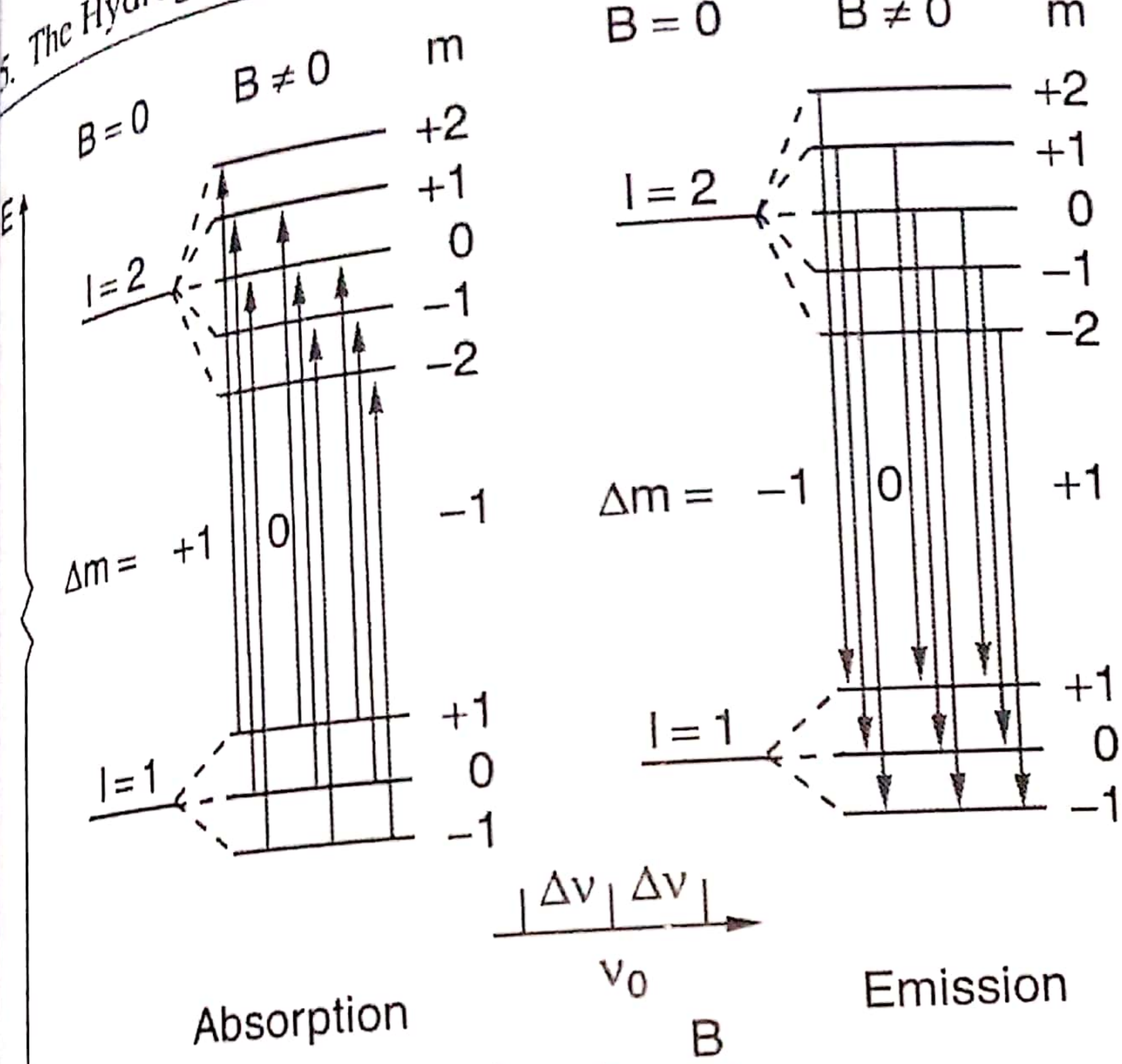


Fig. 5.11. Normal Zeeman effect. Zeeman splitting and polarizations of a spectral line with frequency ν_0 observed in emission. The splitting is $\Delta\nu = \mu_B B/h$



$$\Delta\nu = \mu_B \cdot \frac{B}{\hbar}$$

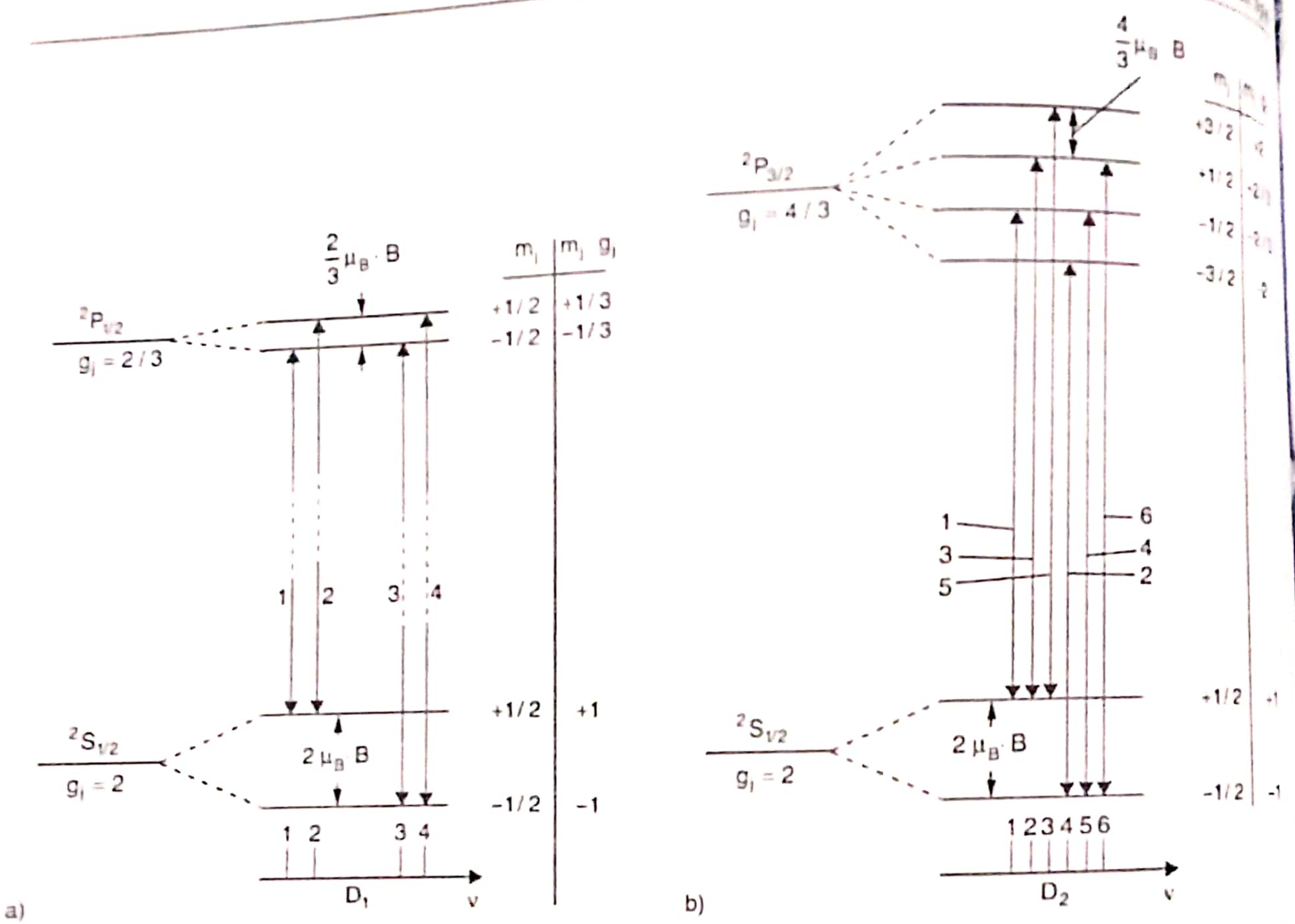


Fig. 5.24a,b. Anomalous Zeeman effect of the transitions (**a**) $^2P_{1/2} \leftarrow {}^2S_{1/2}$ and (**b**) $^2P_{3/2} \leftarrow {}^2S_{1/2}$ neglecting hyperfine structure

Paschen Buck Effect:-

strong External magnetic field

$$V_{mag} = -\vec{\mu}_J \cdot \vec{B}_z$$
$$= -\vec{\mu}_{L_z} \cdot \vec{B}_z - \vec{\mu}_{S_z} \cdot \vec{B}_z$$
$$= \frac{e}{2m_e} B_0 L_z + \frac{e}{2m_e} B_0 S_z$$
$$= \mu_B B_0 m_l + 2\mu_B B_0 m_s$$

$$\boxed{V_{mag} = \mu_B (m_l + 2m_s) B_0}$$

$$\mu_{L_z} = \frac{-\mu_B L_z}{\hbar}$$

$$\mu_{S_z} = \frac{-\mu_B 2 S_z}{\hbar}$$

$$\mu_B = \frac{e\hbar}{2m_e}$$