Outline of the Chapter. Section 4–1 has just presented introductory material for the chapter. Section 4–2 discusses liquid-level systems. Section 4–3 treats pneumatic systems—in particular, the basic principles of pneumatic controllers. Section 4–4 first discusses hydraulic servo systems and then presents hydraulic controllers. Finally, Section 4–5 analyzes thermal systems and obtains mathematical models of such systems.

4–2 LIQUID-LEVEL SYSTEMS

In analyzing systems involving fluid flow, we find it necessary to divide flow regimes into laminar flow and turbulent flow, according to the magnitude of the Reynolds number. If the Reynolds number is greater than about 3000 to 4000, then the flow is turbulent. The flow is laminar if the Reynolds number is less than about 2000. In the laminar case, fluid flow occurs in streamlines with no turbulence. Systems involving laminar flow may be represented by linear differential equations.

Industrial processes often involve flow of liquids through connecting pipes and tanks. The flow in such processes is often turbulent and not laminar. Systems involving turbulent flow often have to be represented by nonlinear differential equations. If the region of operation is limited, however, such nonlinear differential equations can be linearized. We shall discuss such linearized mathematical models of liquid-level systems in this section. Note that the introduction of concepts of resistance and capacitance for such liquid-level systems enables us to describe their dynamic characteristics in simple forms.

Resistance and Capacitance of Liquid-Level Systems. Consider the flow through a short pipe connecting two tanks. The resistance R for liquid flow in such a pipe or restriction is defined as the change in the level difference (the difference of the liquid levels of the two tanks) necessary to cause a unit change in flow rate; that is,

$$R = \frac{\text{change in level difference, m}}{\text{change in flow rate, m}^3/\text{sec}}$$

Since the relationship between the flow rate and level difference differs for the laminar flow and turbulent flow, we shall consider both cases in the following.

Consider the liquid-level system shown in Figure 4–1(a). In this system the liquid spouts through the load valve in the side of the tank. If the flow through this restriction is laminar, the relationship between the steady-state flow rate and steady-state head at the level of the restriction is given by

$$Q = KH$$

where Q = steady-state liquid flow rate, m^3/sec
K = coefficient, m^2/sec
H = steady-state head, m

For laminar flow, the resistance R_l is obtained as

$$R_l = \frac{dH}{dQ} = \frac{H}{Q}$$

The laminar-flow resistance is constant and is analogous to the electrical resistance.

If the flow through the restriction is turbulent, the steady-state flow rate is given by

$$Q = K\sqrt{H} \tag{4–1}$$

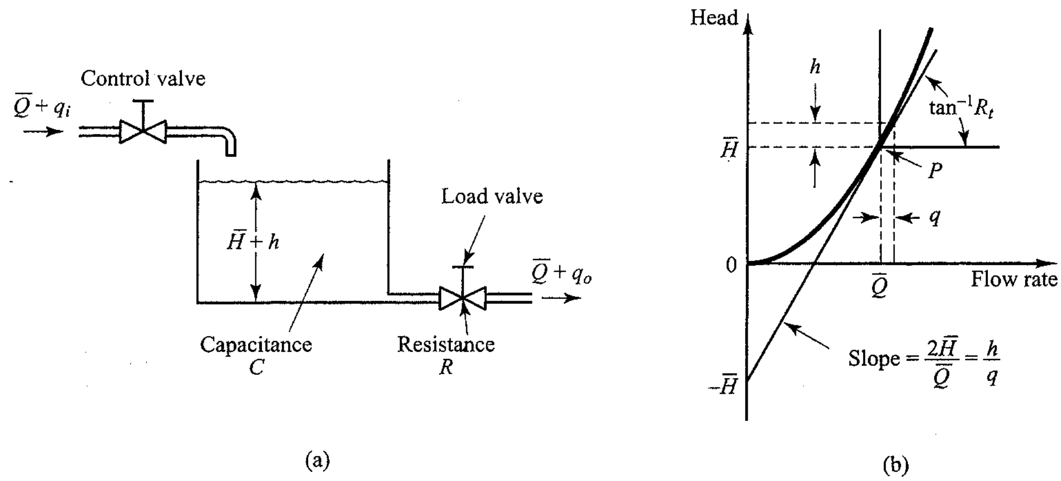

Figure 4–1 (a) Liquid-level system; (b) head versus flow rate curve.

where Q = steady-state liquid flow rate, m^3/sec
K = coefficient, $m^{2.5}/sec$
H = steady-state head, m

The resistance R_t for turbulent flow is obtained from

$$R_t = \frac{dH}{dQ}$$

Since from Equation (4–1) we obtain

$$dQ = \frac{K}{2\sqrt{H}}\,dH$$

we have

$$\frac{dH}{dQ} = \frac{2\sqrt{H}}{K} = \frac{2\sqrt{H}\sqrt{H}}{Q} = \frac{2H}{Q}$$

Thus,

$$R_t = \frac{2H}{Q}$$

The value of the turbulent-flow resistance R_t depends on the flow rate and the head. The value of R_t, however, may be considered constant if the changes in head and flow rate are small.

By use of the turbulent-flow resistance, the relationship between Q and H can be given by

$$Q = \frac{2H}{R_t}$$

Such linearization is valid, provided that changes in the head and flow rate from their respective steady-state values are small.

In many practical cases, the value of the coefficient K in Equation (4–1), which depends on the flow coefficient and the area of restriction, is not known. Then the resistance may be determined by plotting the head versus flow rate curve based on experimental data and measuring the slope of the curve at the operating condition. An example of such a plot is shown in Figure 4–1(b). In the figure, point P is the steady-state operating point. The tangent line to the curve at point P intersects the ordinate at point $(0, -\bar{H})$. Thus, the slope of this tangent line is $2\bar{H}/\bar{Q}$. Since the resistance R_t at the operating point P is given by $2\bar{H}/\bar{Q}$, the resistance R_t is the slope of the curve at the operating point.

Consider the operating condition in the neighborhood of point P. Define a small deviation of the head from the steady-state value as h and the corresponding small change of the flow rate as q. Then the slope of the curve at point P can be given by

$$\text{Slope of curve at point } P = \frac{h}{q} = \frac{2\bar{H}}{\bar{Q}} = R_t$$

The linear approximation is based on the fact that the actual curve does not differ much from its tangent line if the operating condition does not vary too much.

The capacitance C of a tank is defined to be the change in quantity of stored liquid necessary to cause a unit change in the potential (head). (The potential is the quantity that indicates the energy level of the system.)

$$C = \frac{\text{change in liquid stored, m}^3}{\text{change in head, m}}$$

It should be noted that the capacity (m^3) and the capacitance (m^2) are different. The capacitance of the tank is equal to its cross-sectional area. If this is constant, the capacitance is constant for any head.

Liquid-Level Systems. Consider the system shown in Figure 4–1(a). The variables are defined as follows:

$\bar{Q}$ = steady-state flow rate (before any change has occurred), m^3/sec
q_i = small deviation of inflow rate from its steady-state value, m^3/sec
q_o = small deviation of outflow rate from its steady-state value, m^3/sec
$\bar{H}$ = steady-state head (before any change has occurred), m
h = small deviation of head from its steady-state value, m

As stated previously, a system can be considered linear if the flow is laminar. Even if the flow is turbulent, the system can be linearized if changes in the variables are kept small. Based on the assumption that the system is either linear or linearized, the differential equation of this system can be obtained as follows: Since the inflow minus outflow during the small time interval dt is equal to the additional amount stored in the tank, we see that

$$C\,dh = (q_i - q_o)\,dt$$

From the definition of resistance, the relationship between q_o and h is given by

$$q_o = \frac{h}{R}$$

The differential equation for this system for a constant value of R becomes

$$RC\frac{dh}{dt} + h = Rq_i \tag{4–2}$$

Note that RC is the time constant of the system. Taking the Laplace transforms of both sides of Equation (4–2), assuming the zero initial condition, we obtain

$$(RCs + 1)H(s) = RQ_i(s)$$

where

$$H(s) = \mathscr{L}[h] \quad \text{and} \quad Q_i(s) = \mathscr{L}[q_i]$$

If q_i is considered the input and h the output, the transfer function of the system is

$$\frac{H(s)}{Q_i(s)} = \frac{R}{RCs + 1}$$

If, however, q_o is taken as the output, the input being the same, then the transfer function is

$$\frac{Q_o(s)}{Q_i(s)} = \frac{1}{RCs + 1}$$

where we have used the relationship

$$Q_o(s) = \frac{1}{R}H(s)$$

Liquid-Level Systems with Interaction. Consider the system shown in Figure 4–2. In this system, the two tanks interact. Thus the transfer function of the system is not the product of two first-order transfer functions.

In the following, we shall assume only small variations of the variables from the steady-state values. Using the symbols as defined in Figure 4–2 , we can obtain the following equations for this system:

$$\frac{h_1 - h_2}{R_1} = q_1 \tag{4–3}$$

$$C_1 \frac{dh_1}{dt} = q - q_1 \tag{4–4}$$

$$\frac{h_2}{R_2} = q_2 \tag{4–5}$$

$$C_2 \frac{dh_2}{dt} = q_1 - q_2 \tag{4–6}$$

If q is considered the input and q_2 the output, the transfer function of the system is

$$\frac{Q_2(s)}{Q(s)} = \frac{1}{R_1C_1R_2C_2s^2 + (R_1C_1 + R_2C_2 + R_2C_1)s + 1} \tag{4–7}$$

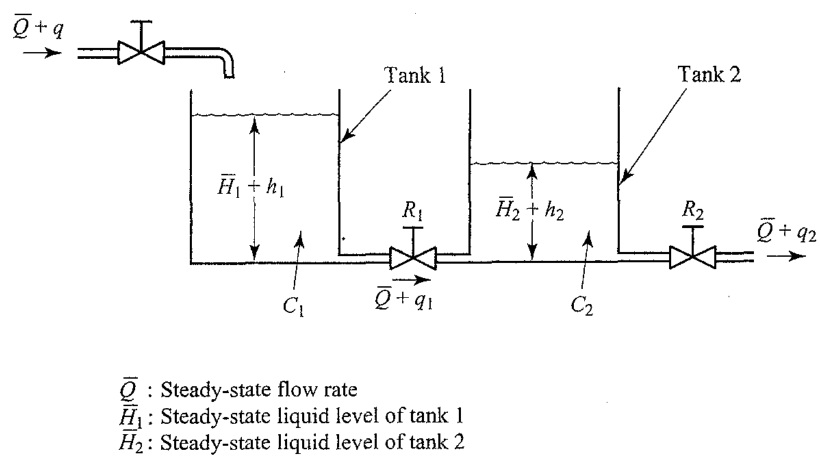

Figure 4–2
Liquid-level system with interaction.

It is instructive to obtain Equation (4–7), the transfer function of the interacted system, by block diagram reduction. From Equations (4–3) through (4–6), we obtain the elements of the block diagram, as shown in Figure 4–3(a). By connecting signals properly, we can construct a block diagram, as shown in Figure 4–3(b). This block diagram can be simplified, as shown in Figure 4–3(c). Further simplifications result in Figures 4–3(d) and (e). Figure 4–3(e) is equivalent to Equation (4–7).

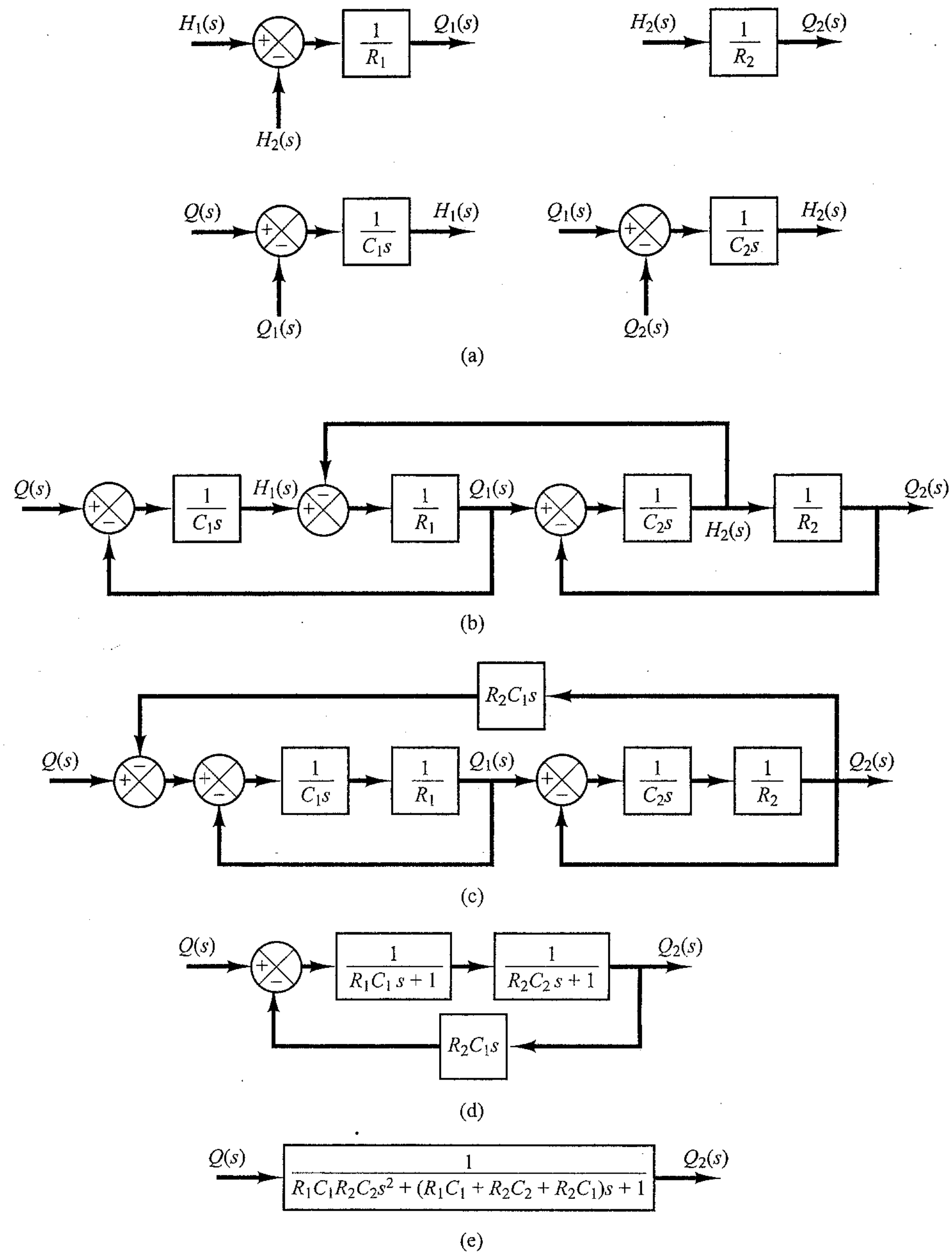

Figure 4–3
(a) Elements of the block diagram of the system shown in Figure 4–2; (b) block diagram of the system; (c)-(e) successive reductions of the block diagram.

Notice the similarity and difference between the transfer function given by Equation (4–7) and that given by Equation(3–72). The term R_2C_1s that appears in the denominator of Equation (4–7) exemplifies the interaction between the two tanks. Similarly, the term R_1C_2s in the denominator of Equation (3–72) represents the interaction between the two RC circuits shown in Figure 3–23.

4–3 PNEUMATIC SYSTEMS

In industrial applications pneumatic systems and hydraulic systems are frequently compared. Therefore, before we discuss pneumatic systems in detail, we shall give a brief comparison of these two kinds of systems.

Comparison Between Pneumatic Systems and Hydraulic Systems. The fluid generally found in pneumatic systems is air; in hydraulic systems it is oil. And it is primarily the different properties of the fluids involved that characterize the differences between the two systems. These differences can be listed as follows:

1. Air and gases are compressible, whereas oil is incompressible, (except at high pressure).
2. Air lacks lubricating property and always contains water vapor. Oil functions as a hydraulic fluid as well as a lubricator.
3. The normal operating pressure of pneumatic systems is very much lower than that of hydraulic systems.
4. Output powers of pneumatic systems are considerably less than those of hydraulic systems.
5. Accuracy of pneumatic actuators is poor at low velocities, whereas accuracy of hydraulic actuators may be made satisfactory at all velocities.
6. In pneumatic systems, external leakage is permissible to a certain extent, but internal leakage must be avoided because the effective pressure difference is rather small. In hydraulic systems internal leakage is permissible to a certain extent, but external leakage must be avoided.
7. No return pipes are required in pneumatic systems when air is used, whereas they are always needed in hydraulic systems.
8. Normal operating temperature for pneumatic systems is 5° to 60°C (41° to 140°F). The pneumatic system, however, can be operated in the 0° to 200°C (32° to 392°F) range. Pneumatic systems are insensitive to temperature changes, in contrast to hydraulic systems, in which fluid friction due to viscosity depends greatly on temperature. Normal operating temperature for hydraulic systems is 20° to 70°C (68° to 158°F).
9. Pneumatic systems are fire- and explosion-proof, whereas hydraulic systems are not, unless nonflammable liquid is used.

In what follows we begin with a mathematical modeling of pneumatic systems. Then we shall present pneumatic proportional controllers.

We shall first give detailed discussions of the principle by which proportional controllers operate. Then we shall treat methods for obtaining derivative and integral control actions. Throughout the discussions, we shall place emphasis on the

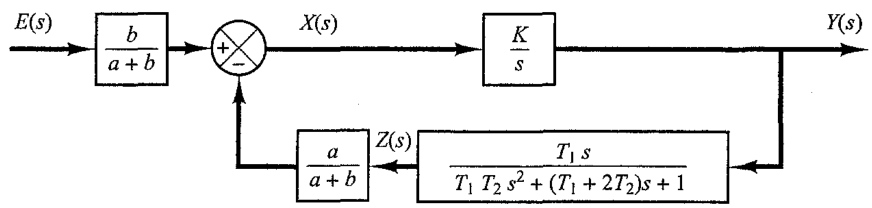

Figure 4–25
Block diagram for the system shown in Figure 4–24.

A block diagram for this system is shown in Figure 4–25. The transfer function $Y(s)/E(s)$ can be obtained as

$$\frac{Y(s)}{E(s)} = \frac{b}{a+b} \frac{\frac{K}{s}}{1 + \frac{a}{a+b} \frac{K}{s} \frac{T_1 s}{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1}}$$

Under normal operation of the system we have

$$\left| \frac{a}{a+b} \frac{K}{s} \frac{T_1 s}{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1} \right| \gg 1$$

Hence

$$\begin{aligned} \frac{Y(s)}{E(s)} &= \frac{b}{a} \frac{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1}{T_1 s} \\ &= K_p + \frac{K_i}{s} + K_d s \end{aligned}$$

where

$$K_p = \frac{b}{a} \frac{T_1 + 2T_2}{T_1}, \qquad K_i = \frac{b}{a} \frac{1}{T_1}, \qquad K_d = \frac{b}{a} T_2$$

Thus, the controller shown in Figure 4–24 is a proportional-plus-integral-plus-derivative controller (PID controller).

4–5 THERMAL SYSTEMS

Thermal systems are those that involve the transfer of heat from one substance to another. Thermal systems may be analyzed in terms of resistance and capacitance, although the thermal capacitance and thermal resistance may not be represented accurately as lumped parameters since they are usually distributed throughout the substance. For precise analysis, distributed-parameter models must be used. Here, however, to simplify the analysis we shall assume that a thermal system can be represented by a lumped-parameter model, that substances that are characterized by resistance to heat flow have negligible heat capacitance, and that substances that are characterized by heat capacitance have negligible resistance to heat flow.

There are three different ways heat can flow from one substance to another: conduction, convection, and radiation. Here we consider only conduction and convection. (Radiation heat transfer is appreciable only if the temperature of the emitter is very high compared to that of the receiver. Most thermal processes in process control systems do not involve radiation heat transfer.)

For conduction or convection heat transfer,

$$q = K\,\Delta\theta$$

where q = heat flow rate, kcal/sec
$\Delta\theta$ = temperature difference, °C
K = coefficient, kcal/sec °C

The coefficient K is given by

$$K = \frac{kA}{\Delta X}, \qquad \text{for conduction}$$

$$= HA, \qquad \text{for convection}$$

where k = thermal conductivity, kcal/m sec °C
A = area normal to heat flow, m^2
ΔX = thickness of conductor, m
H = convection coefficient, kcal/m^2 sec °C

Thermal Resistance and Thermal Capacitance. The thermal resistance R for heat transfer between two substances may be defined as follows:

$$R = \frac{\text{change in temperature difference, °C}}{\text{change in heat flow rate, kcal/sec}}$$

The thermal resistance for conduction or convection heat transfer is given by

$$R = \frac{d(\Delta\theta)}{dq} = \frac{1}{K}$$

Since the thermal conductivity and convection coefficients are almost constant, the thermal resistance for either conduction or convection is constant.

The thermal capacitance C is defined by

$$C = \frac{\text{change in heat stored, kcal}}{\text{change in temperature, °C}}$$

or

$$C = mc$$

where m = mass of substance considered, kg
c = specific heat of substance, kcal/kg °C

Thermal System. Consider the system shown in Figure 4–26(a). It is assumed that the tank is insulated to eliminate heat loss to the surrounding air. It is also assumed that there is no heat storage in the insulation and that the liquid in the tank is perfectly mixed so that it is at a uniform temperature. Thus, a single temperature is used to describe the temperature of the liquid in the tank and of the outflowing liquid.

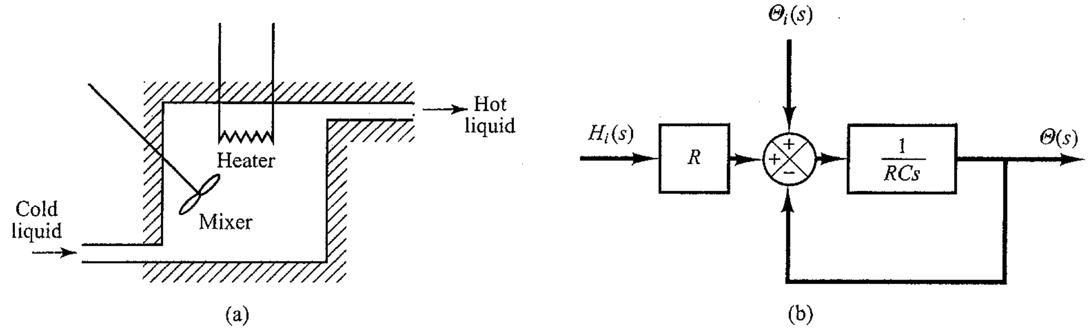

Figure 4–26
(a) Thermal system: (b) block diagram of the system.

Let us define

$\bar{\Theta}_i$ = steady-state temperature of inflowing liquid, °C

$\bar{\Theta}_o$ = steady-state temperature of outflowing liquid, °C

G = steady-state liquid flow rate, kg/sec

M = mass of liquid in tank, kg

c = specific heat of liquid, kcal/kg °C

R = thermal resistance, °C sec/kcal

C = thermal capacitance, kcal/°C

$\bar{H}$ = steady-state heat input rate, kcal/sec

Assume that the temperature of the inflowing liquid is kept constant and that the heat input rate to the system (heat supplied by the heater) is suddenly changed from $\bar{H}$ to $\bar{H} + h_i$, where h_i represents a small change in the heat input rate. The heat outflow rate will then change gradually from $\bar{H}$ to $\bar{H} + h_o$. The temperature of the outflowing liquid will also be changed from $\bar{\Theta}_o$ to $\bar{\Theta}_o + \theta$ For this case, h_o, C, and R are obtained, respectively, as

$$h_o = Gc\theta$$

$$C = Mc$$

$$R = \frac{\theta}{h_o} = \frac{1}{Gc}$$

The heat balance equation for this system is

$$C d\theta = (h_i - h_o)\,dt$$

or

$$C\frac{d\theta}{dt} = h_i - h_o$$

which may be rewritten as

$$RC\frac{d\theta}{dt} + \theta = Rh_i$$

Note that the time constant of the system is equal to RC or M/G seconds. The transfer function relating θ and h_i is given by

$$\frac{\Theta(s)}{H_i(s)} = \frac{R}{RCs + 1}$$

where $\Theta(s) = \mathscr{L}[\theta(t)]$ and $H_i(s) = \mathscr{L}[h_i(t)]$.

In practice, the temperature of the inflowing liquid may fluctuate and may act as a load disturbance. (If a constant outflow temperature is desired, an automatic controller may be installed to adjust the heat inflow rate to compensate for the fluctuations in the temperature of the inflowing liquid.) If the temperature of the inflowing liquid is suddenly changed from $\bar{\Theta}_i$ to $\bar{\Theta}_i + \theta_i$ while the heat input rate H and the liquid flow rate G are kept constant, then the heat outflow rate will be changed from $\bar{H}$ to $\bar{H} + h_o$, and the temperature of the outflowing liquid will be changed from $\bar{\Theta}_o$ to $\bar{\Theta}_o + \theta$. The heat balance equation for this case is

$$C\,d\theta = (Gc\theta_i - h_o)\,dt$$

or

$$C\frac{d\theta}{dt} = Gc\theta_i - h_o$$

which may be rewritten

$$RC\frac{d\theta}{dt} + \theta = \theta_i$$

The transfer function relating θ and θ_i is given by

$$\frac{\Theta(s)}{\Theta_i(s)} = \frac{1}{RCs + 1}$$

where $\Theta(s) = \mathscr{L}[\theta(t)]$ and $\Theta_i(s) = \mathscr{L}[\theta_i(t)]$.

If the present thermal system is subjected to changes in both the temperature of the inflowing liquid and the heat input rate, while the liquid flow rate is kept constant, the change θ in the temperature of the outflowing liquid can be given by the following equation:

$$RC\frac{d\theta}{dt} + \theta = \theta_i + Rh_i$$

A block diagram corresponding to this case is shown in Figure 4–26(b). Notice that the system involves two inputs.