6 Controllers

6.1 Background

The concept of a control system is to sense deviation of the output from the desired value and correct it, till the desired output is achieved. The deviation of the actual output form its desired value is called an error. The measurement of error is possible because of feedback. The feedback allows us to compare the actual output with its desired value to generate the error. The error is denoted as e(t). The desired value of the output is also called **reference input** or a **set point**. The error obtained is required to be analysed to take the proper corrective action.

The **controller** is an element which accepts the error in some form and decides the proper corrective action. The output of the controller is then applied to the process or final control element. This brings the output back to its desired set point value. The controller is the heart of a control system. The accuracy of the entire system depends on how sensitive is the controller to the error detected and how it is manipulating such an error. The controller has its own logic to handle the error. Now a days for better accuracy, the digital controllers such as microprocessors, microcontrollers, computers are used. Such controllers execute certain algorithm to calculate the manipulating signal.

This chapter explains the basic discontinuous controllers such as on-off controller, continuous controllers such as proportional, integral etc. and composite controllers such as proportional plus integral, proportional plus derivative etc. Let us study first the general properties of the controller.

6.2 Properties of Controller

Consider a control system shown in the Fig. 6.1 which includes a controller.

The actual output is sensed by a sensor and converted to a proper feedback signal b(t) using a feedback element. The set point value is the reference input r(t). For example the actual output variable may be temperature but using the thermocouple as the feedback element, the feedback signal b(t) is an electrical voltage. This is then compared with reference input which is also an electrical voltage. The thermocouple senses the output temperature and produces the corresponding electrical e.m.f as the feedback signal. Hence actual output variable sensed and the feedback signal may be having different forms.

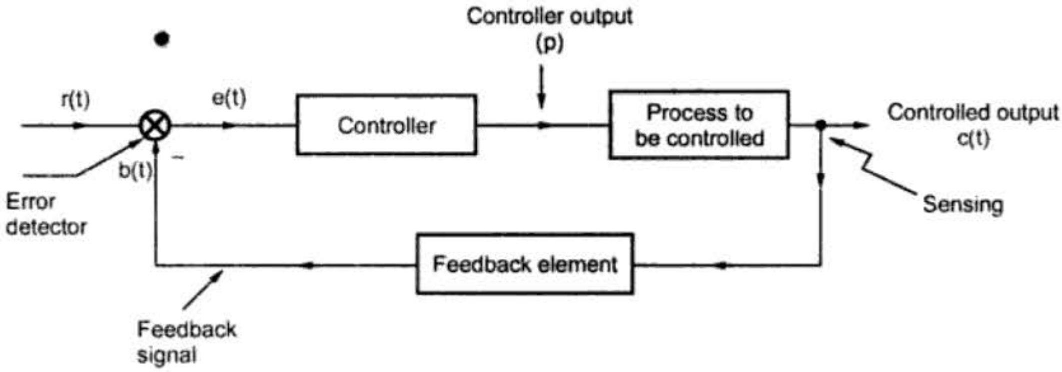


Fig. 6.1 Basic control system

6.2.1 Error

The error detector compares the feedback signal b(t) with the reference input r(t) to generate an error.

$$\therefore \quad \boxed{e(t) = r(t) - b(t)}$$

This gives an absolute indication of an error.

For example if the set point for a range of 5 mV to 20 mV is 12 mV and the feedback signal is 11.8 mV then error is 0.2 mV. But actual vairable to be controlled may be different such as temperature, pressure etc. Hence to obtain correct information from the error, it is expressed in percentage form related to the controller operation. It is expressed as the percentage of the measured variable range. The range of the measured variable b(t) is also called **span**.

Thus $\quad \text{span} = b_{max} - b_{min}$

Hence error can be expressed as percent of span as,

$$\therefore \quad \boxed{e_p = \frac{r-b}{b_{max}-b_{min}} \times 100}$$

Where $\quad e_p = $ error as % of span

Example 6.1 : *The range of measured variable for a certain control system is 2 mV to 12 mV and a setpoint of 7 mV. Find the error as percent of span when the measured variable is 6.5 mV.*

Solution : $b_{max} = 12$ mV, $\quad b_{min} = 2$ mV, $b = 6.5$ mV, $\quad r = 7$ mV

$$\therefore \quad e_p = \frac{r-b}{b_{max}-b_{min}} \times 100 = \frac{7-6.5}{12-2} \times 100$$

$$= \mathbf{5\%}$$

6.2.2 Variable Range

In practical systems, the controlled variable has a range of values within which the control is required to be maintained. This range is specified as the maximum and minimum values allowed for the controlled variable. It can be specified as some nominal values and plus-minus tolerance allowed about this value. Such range is important for the design of contollers.

6.2.3 Controller Output Range

Similar to the controlled variable, a range is associated with a controller output variable. It is also specified interms of the maximum and minimum values.

But often the controller output is expressed as a percentage where minimum controller output is 0% and maximum controller output is 100 %. But 0 % controller output does not mean, zero output. For example it is necessary requirement of the system that a steam flow corresponding to $\frac{1}{4}^{th}$ opening of the valves should be mininum. Thus 0% controller output in such case corresponds to the $\frac{1}{4}^{th}$ opening of the valve.

The controller output as a percent of full scale when the output changes within the specified range is expressed as,

$$p = \frac{u - u_{min}}{u_{max} - u_{min}} \times 100$$

Where
p = controller output as a percent of full scale
u = value of the output
u_{max} = maximum value of controlling variable
u_{min} = minimum value of controlling variable

6.2.4 Control Lag

The control system can have a lag associated with it. The control lag is the time required by the process and controller loop to make the necessary changes to obtain the output at its setpoint. The control lag must be compared with the process lag while designing the controllers. For example in a process a valve is required to be open or closed for controlling the output variable. Physically the action of opening or closing of the valve is very slow and is the part of the process lag. In such a case there is no point in designing a fast controller than the process lag.

6.2.5 Dead Zone

Many a times a dead zone is associated with a process control loop. The time corresponding to dead zone is called dead time. The time elapsed between the instant when error occurs and the instant when first corrective action occurs is called dead time.

Nothing happens in the system, during this time though the error occurs. This part is also called dead band. The effect of such dead time must be considered while the design of the controllers.

6.3 Classification of Controllers

The classification of the controllers is based on the response of the controller and mode of operation of the controller. For example in a simple temperature control of a room, the heater is to be controlled. It should be switched on or off by the controller when temperature crosses its setpoint. Such an operation of the controller is called discontinuous operation and the mode of operation is called **discontinuous mode** of controller. But in some process control systems, simple on/off decision is not sufficient. For example, controlling the steam flow by opening or closing the valve. In such case a smooth opening or closing of valve is necessary. The controller in such a case is said to be operating in a **continuous mode.**

Thus the controllers are basically classified as discontinuous controllers and continuous controllers.

The discontinuous mode controllers are further classified as ON-OFF controllers and multiposition controllers.

The continuous mode controllers are further classified as proportional controllers, integral controllers and derivative controllers.

Some continuous mode controllers can be combined to obtain composite controller mode. The examples of such composite controllers are PI, PD and PID controllers.

The most of the controllers are placed in the forward path of control system. But in some cases, input to the controller is controlled through a feedback path. The example of such a controller is rate feedback controller.

In this chapter, only continuous and composite controllers are discussed.

6.4 Continuous Controller Modes

In the discontinuous controller mode, the output of the controller is discontinuous and not smoothly varying. But in the continuous controller mode, the controller output varies smoothly proportional to the error or proportional to some form of the error. Depending upon which form of the error is used as the input to the controller to produce the continuous controller output, these controllers are classified as,

1. Proportional control mode
2. Integral control mode
3. Derivative control mode

Let us discuss these control modes in detail.

6.5 Proportional Control Mode

In this control mode, the output of the controller is simple proportional to the error e(t). The relation between the error e(t) and the controller output p is determined by constant called **proportional gain constant** denoted as K_p. The output of the controller is a linear function of the error e(t). Thus each value of the error has a unique value of the controller output. The range of the error which covers 0 % to 100 % controller output is called **proportional band.**

Now though there exists linear relation between controller output and the error, for a zero error the controller output should not be zero, otherwise the process will come to halt. Hence there exists some controller output p_o for the zero error. Hence mathematically the proportional control mode is expressed as,

$$p(t) = K_p e(t) + p_o \qquad ...(1)$$

Where K_p = Proportional gain constant

p_o = Controller output with zero error

The direct and reverse action is possible in the proportional control mode. The error may be positive or negative because error is r–b and b can be less or greater than reference setpoint r.

If the controlled variable i.e. input to the controller increases, causing increase in the controller output, the action is called **direct action.** For example the output valve is to be controlled to maintain the liquid level in a tank. So if the level increases, the valve should be opened more to maintain the level.

If the controlled variable decreases, causing increase in the controller output or increase in the controlled variable, causing decrease in the controller output, the action is called **reverse action.** For example simple heater control for maintaining temperature. If the temperature increases, the drive to the heater must be decreased.

So if e(t) is negative then $K_p e(t)$ gets subtracted from p_o and e(t) is positive, then $K_p e(t)$ gets added to p_o, this is reverse action.

So equation (1) represents the reverse action. But using negative sign to the correction term $K_p e(t)$, the direct action proportional controller can be achieved.

The proportional controller depends on the proper design of the gain K_p. For fixed p_o, if gain K_p is high then large output results for small error but narrow error band can be handled. Beyond these limits of the error, output will be saturated. If the gain is small then the response is smaller but large error band can be handled. This is shown in the Fig. 6.2.

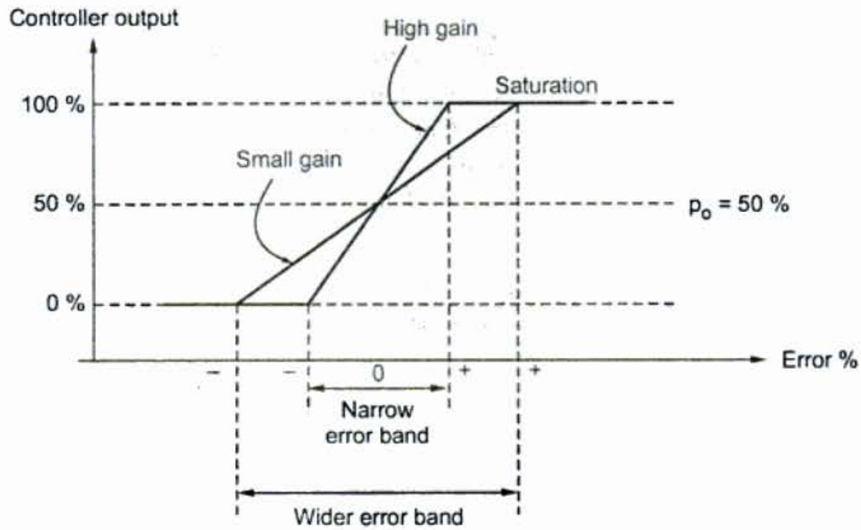


Fig. 6.2

The proportional band is mathematically defined by,

$$PB = \frac{100}{K_p}$$

6.5.1 Characteristic of Proportional Mode

The various characteristics of the proportional mode are,

1. When the error is zero, the controller output is constant equal to p_o.
2. If the error occurs, then for every 1 % of error the correction of K_p % is achieved. If error is positive, K_p% correction gets added to p_o and if error is negative, K_p% correction gets subtracted from p_o.
3. The band of error exists for which the output of the controller is between 0 % to 100 % without saturation.
4. The gain K_p and the error band PB are inversely proportional to each other.

6.5.2 Offset

The major disadvantage of the proportional control mode is that it produces an **offset error** in the output. When the load changes, the output deviates from the setpoint. Such a deviation is called offset error or steady state error. Such an offset error is shown in the Fig. 6.3. The offset error depends upon the reaction rate of the controller. Slow reaction rate produces small offset error while fast reaction rate produces large offset error.

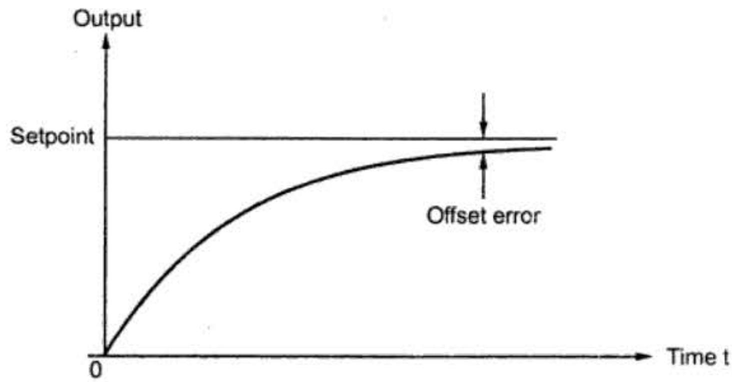


Fig. 6.3 Offset error in porportional mode

The dead time or transfer lag present in the system further worsens the result. It produces not only the large offset at the output but the time required to achieve steady state is also large.

The offset error can be minimized by the large proportional gain K_p which reduces the proportional band. If K_p is made very large, the proportional band becomes so small that it acts as an ON/OFF controller producing oscillations about the setpoint instead of an offset error.

6.5.3 Applications

The proportional controller can be suitable where,

1. Manual reset of the operating point is possible.
2. Load changes are small.
3. The dead time exists in the system is small.

6.6 Integral Control Mode

In the proportional control mode, error reduces but can not go to zero. It finally produces an offset error. It can not adapt with the changing load conditions. To avoid this, another control mode is oftenly used in the control systems which is based on the history of the errors. This mode is called **integral mode** or **reset action controller**.

In such a controller, the value of the controller output p(t) is changed at a rate which is proportional to the actuating error signal e(t). Mathematically it is expressed as,

$$\boxed{\frac{d\,p(t)}{dt} = K_i e(t)}$$

Where K_i = Constant relating error and rate

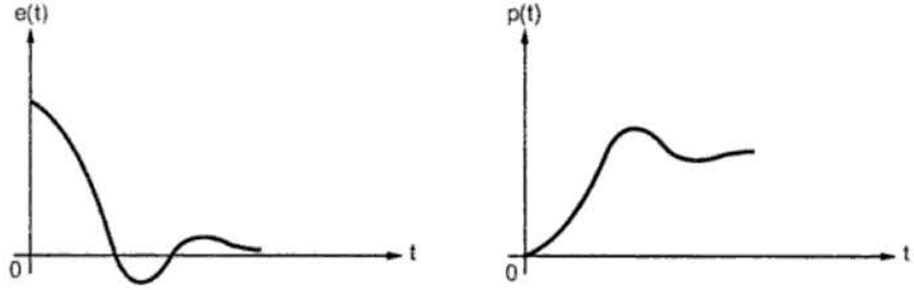
The constant K_i is also called **integral constant**. Integrating the above equation, actual controller output at any time t can be obtained as,

$$p(t) = K_i \int e(t)dt + p(0) \quad \text{... (2)}$$

Where $p(0)$ = Controller output when integral action starts i.e. at $t = 0$.

The output signal from the controller, at any instant is the area under the actuating error signal curve up to that instant. If the value of the error is doubled, the value of p(t) varies twice as fast i.e. rate of the controller output change also doubles.

If the error is zero, the controller output is not changed. The control signal p(t) can have nonzero value when the error signal e(t) is zero. This is because the output depends on the history of the error and not on the instantaneous value of the error. This is shown in the Fig. 6.4.



(a) Error (b) Controller output

Fig. 6.4 Integral mode

The scale factor or constant K_i expresses the scaling between error and the controller output. Thus a large value of K_i means that a small error produces a large rate of change of p(t) and viceversa. This is shown in the Fig. 6.5.

If there is positive error, the controller output begins to ramp up.

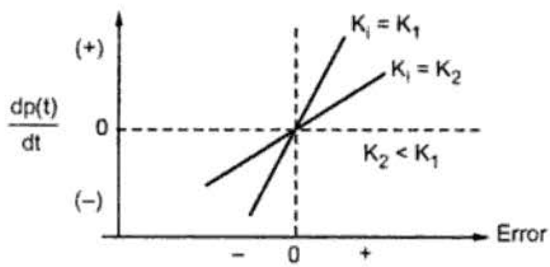


Fig. 6.5

6.6.1 Step Response of Integral Mode

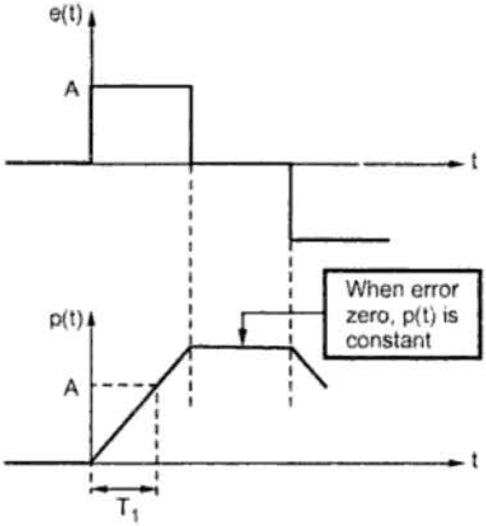


Fig. 6.6 Step response

The step response of the integral control mode is shown in the Fig. 6.6.

The integration time constant is the time taken for the output to change by an amount equal to the input error step. This is shown in the Fig. 6.6.

It can be seen that when error is positive, the output p(t) ramps up. For zero error, there is no change in the output. And when error is negative, the output p(t) ramps down.

6.6.2 Characteristics of Integral Mode

The integrating controller is relatively slow controller. It changes its output at a rate which is dependent on the integrating time constant, until the error signal is cancelled. Compared to the proportional control, the integral control requires time to build up an appreciable output. However it continues to act till the error signal disappears. This corrects the problem of the offset error in the proportional controller.

For example, let us assume that the integral controller is used to control the armature current of a d.c. motor and to keep its value constant at 500 A. As long as the armature current is less than 500 A, the armature voltage, controlled by the controller, will increase. Thus the output of the controller will increase and will continue to do so till the error becomes zero i.e. armature current becomes 500 A. Then the controller output will remain at that value reached. This is possible because the output of the controller can remain at any value within its range, if the input is zero. The controller must not be overdriven as it will not then be effective.

Thus for an integral mode,

1. If error is zero, the output remains at a fixed value equal to what is was, when the error became zero.
2. If the error is not zero, then the output begins to increase or decrease, at a rate K_i % per second for every $\pm$ 1 % of error.

In some cases, the inverse of K_i called **integral time** is specified, denoted as T_i.

$$T_i = \frac{1}{K_i} = \text{Integral time}$$

It is expressed in minutes instead of seconds.

6.6.3 Applications

The comparison of proportional and integral mode behaviour at the time of occurrence of an error signal is tabulated below,

Controller	Initial behaviour	Steady state behaviour
P	Acts immediately. Action according to K_p.	Offset error always present. Larger the K_p smaller the error.
I	Acts slowly. It is the time integral of the error signal.	Error signal always becomes zero.

Table 6.1

It can be seen that proportional mode is more favourable at the start while the integral is better for steady state response. In pure integral mode, error can oscillate about zero and can be cyclic. Hence in practice **integral mode is never used alone** but combined with the proportional mode, to enjoy the advantages of both the modes.

6.7 Derivative Control Mode

In practice the error is function of time and at a particular instant it can be zero. But it may not remain zero forever after that instant. Hence some action is required corresponding to the rate at which the error is changing. Such a controller is called derivative controller.

In this mode, the output of the controller depends on the time rate of change of the actual errors. Hence it is also called **rate action mode** or **anticipatory action mode.**

The mathematical equation for the mode is,

$$p(t) = K_d \frac{d\,e(t)}{dt}$$

Where K_d = Derivative gain constant.

The derivative gain constant indicates by how much % the controller output must change for every % per sec rate of change of the error. Generally K_d is expressed in minutes. The important feature of this type of control mode is that for a given rate of change of error signal, there is a unique value of the controller output.

The advantage of the derivative control action is that it responds to the rate of change of error and can produce the significant correction before the magnitude of the actuating error becomes too large. Derivative control thus anticipates the actuating error, initiates an early corrective action and tends to increase stability of the system improving the transient response.

6.7.1 Characteristics of Derivative Control Mode

The Fig. 6.7 shows how derivative mode changes the controller output for the various rates of change of the error.

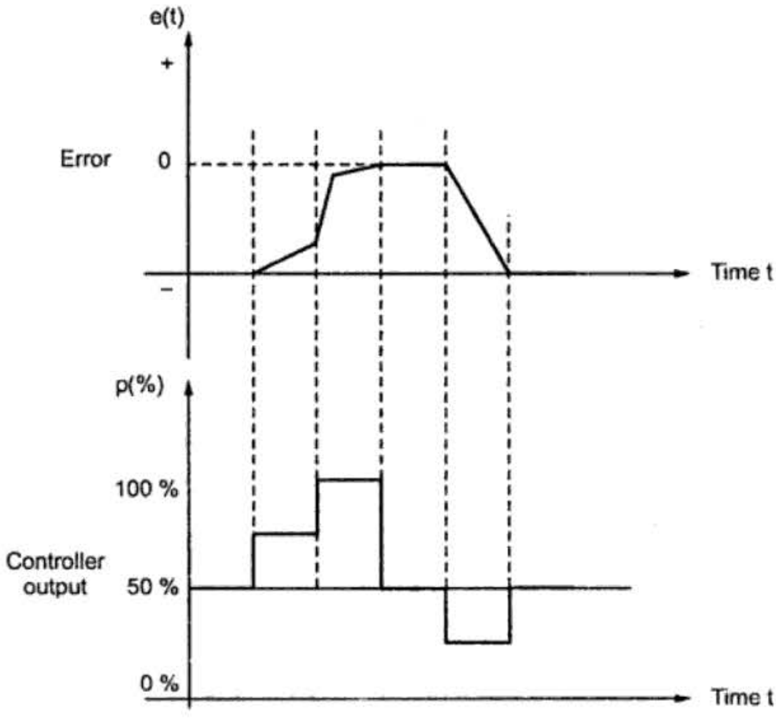


Fig. 6.7

The controller output is 50 % for the zero error. When error starts increasing, the controller output suddenly jumps to the higher value. It further jumps to a higher value for higher rate of increase of error. Then error becomes constant, the output returns to 50 %. When error is decreasing i.e. having negative slope, controller output decreases suddenly to a lower value.

The various characteristics of the derivative mode are,

1. For a given rate of change of error signal, there is a unique value of the controller output.
2. When the error is zero, the controller output is zero.
3. When the error is constant i.e. rate of change of error is zero, the controller output is zero.
4. When error is changing, the controller output changes by K_d % for even 1 % per second rate of change of error.

6.7.2 Applications

When the error is zero or a constant, the derivative controller output is zero. Hence it is never used alone. Its gain should be small because faster rate of change of error can cause very large sudden change of controller output. This may lead to the instability of the system.

6.8 Composite Control Modes

As mentioned earlier, due to offset error proportional mode is not used alone. Similarly integral and derivative modes are also not used individually in practice. Thus to take the advantages of various modes together, the composite control modes are used. The various composite control modes are,

1. Proportional + Integral mode (PI)
2. Proportional + Derivative mode (PD)
3. Proportional + Integral + Derivative mode (PID)

Let us see the characteristics of these three modes.

6.9 Proportional + Integral Mode (PI Control Mode)

This is a composite control mode obtained by combining the proportional mode and the integral mode.

The mathematical expression for such a composite control is,

$$p(t) = K_p\, e(t) + K_p K_i \int_0^t e(t)dt + p(0)$$

Where $p(0)$ = Initial value of the output at $t = 0$