12 CHAPTER Inventory Models

This chapter presents the kind of analysis which develops mathematical models of inventory processes. Efforts will be made to develop not a single general model but a wide variety of models each for a specific situation.

An *Inventory* consists of *usable but idle resources* such as men, machines, materials or money. When the resource involved is a material, the inventory is also called *'stock'* . An inventory problem is said to the exist if either the resources are subjected to control or if there is at least one such cost that decreases as inventory increases. The objective is to minimize total (actual or expected) cost. However, in situations where inventory affects demand, the objective may also be to maximize profit.

12.1 NECESSITY FOR MAINTAINING INVENTORY

Though inventory of materials is an idle resource (since the materials lie idle and are not to be used immediately), almost every organisation must maintain it for efficient and smooth running of its operations. Without it no business activity can be performed, whether it is a service organisation like a hospital or a bank or it is a manufacturing or trading organisation. If an enterprise has no inventory of materials at all, on receiving a sales order it will have to place order for purchase of raw materials, wait of their receipt and then start production. The customer will thus have to wait for a long time for the delivery of the goods and may turn to other suppliers resulting in loss of business for the enterprise. Most organisations have 20 to 25 per cent of the total funds devoted to inventory. It may even increase to 70 per cent in case of pharmaceutical, chemical and paints industries. Maintaining an inventory is necessary because of the following reasons :

1. It helps in smooth and efficient running of an enterprise. It decouples the production from customers and vendors and simplifies the otherwise complex organisation for manufacture and reduces the co-ordination effort.
2. It provides service to the customer at a short notice. Timely deliveries can fetch more goodwill and orders.
3. In the absence of inventory, the enterprise may have to pay high prices because of piecemeal purchasing. Maintaining of inventory may earn price discount because of bulk purchasing. It also takes advantage of favourable market.
4. It reduces product costs since there is an added advantage of batching and long, uninterrupted production runs.
5. It acts as a buffer stock when raw materials are received late and shop rejections are too many.
6. Process and movement inventories (also called pipeline stocks) are quite necessary in big enterprises where significant amounts of times are required to tranship items from one location to another.
7. Bulk purchases will entail less orders and, therefore, less clerical costs. This applies to goods produced within the organisation as well. Less orders, as a result of larger lots, will entail lesser machine setups and other associated costs.
8. An organisation may have to deal with several customers and vendors who are not necessarily near it. Inventories, therefore, have to be built to meet the demand at least during the transit time.

9. It helps in maintaining economy by absorbing some of the fluctuations when the demand for an item fluctuates or is seasonal.

However, too often inventories are wrongly used as a substitute for management. For example, if there are large finished goods inventories, inaccurate sales forecasting by marketing deptt. may never be apparent. Similarly, a production foreman who has large in-process inventories may be able to hide his poor planning since there is always something to manufacture. Furthermore, inventory means unproductive 'tied up' capital of the enterprise. The capital could be usefully utilised in other ventures as well. With large inventory, there is always likelihood of obsolescence too. Also maintenance of inventory costs additional money to be spent on personnel, equipment, insurance, etc. Thus excess inventories are not at all desirable. This necessitates controlling the inventories in the most useful way.

Causes of Poor Inventory Control

1. Overbuying without regard to the forecast or proper estimate of demand to take advantage of favourable market.
2. Overproduction or production of goods much before the customer requires them.
3. Overstocking may also result from the desire to provide better service to the customers. Bulk production to cut down production costs will also result in large inventories.
4. Cancellation of orders and minimum quantity stipulations by the suppliers may also give rise to large inventories.

12.1-1 Classifications of Inventories

Inventories are generally classified into the following types :

1 Direct Inventories

They include items that are directly used for production and are classified as :

(*a*) *Production Inventory* : Items such as raw materials, components and subassemblies used to produce the final product.

(*b*) *Work-in-Process Inventory* : Items in semi-finished form or products at different stages of production.

(*c*) *Finished Goods Inventory* : This includes the final products ready for dispatch to consumers or distributors.

(*d*) *MRO Inventory* : Maintenance, repair and operating items such as spare parts and consumable stores that do not go into the final product but are consumed during the production process.

(*e*) *Miscellaneous Inventory* : All other items such as scrap, obsolete and unsaleable products, stationery and other items used in office, factory and sales department, etc.

2 Indirect Inventories

Indirect inventories may be classified as :

(*a*) *Transit or Pipeline Inventories* : Also called *movement inventories*, they consist of items that are currently under transportation *e.g.*, coal being transported from coalfields to a thermal plant.

(*b*) *Buffer Inventories* : They are required as protection against the uncertainties of supply and demand. A company may well know the average demand of an item that it needs; however, the actual demand may turn out to be quite different–it may well exceed the average value. Similarly, the average delivery period (lead time) may be known but due to some unforeseen reasons, the actual delivery period could be much more. Such situations require extra stock of the item to reduce the number of stock-outs or back-orders. This extra stock in excess of the average demand during the lead time is called *buffer stock* (or *safety stock* or *cushion stock*).

(*c*) *Decoupling Inventories* : They are required to decouple or disengage the different parts of the production system. For an item that requires processing on a series of different machines with different processing times, it is a must to have decoupling inventories of the item in between the various machines for smooth and continuous production. The decoupling inventories act as shock absorbers in case of varying work-rates, machine breakdowns or failures, etc.

(*d*) *Seasonal Inventories* : Some items have seasonal demands *e.g.*, demand of woollen textiles in winter, coolers and air conditioners in summer, raincoats in rainy season, etc. Inventories for such items have to be maintained to meet their high seasonal demand.

(*e*) *Lot Size Inventories* : Items are usually purchased in lots to

(*i*) avail price discounts

(*ii*) reduce transportation and purchase costs

(*iii*) minimize handling and receiving costs.

Lot size or *cycle inventories* are, therefore, held by purchasing items in lots rather than their exact quantities required. For example, a textile industry may buy cotton in bulk during cotton season rather than buying it everyday.

(*f*) *Anticipation Inventories* : They are held to meet the anticipated demand. Purchasing of crackers well before Diwali, fans before the approaching summer, piling up of raw material in the face of imminent transporters' strike are examples of anticipation inventories.

12.2 INVENTORY COSTS

The four costs considered in inventory control models are :

1. Purchase costs
2. Inventory carrying or stock holding costs
3. Procurement costs (for bought-outs) or setup costs (for made-ins) and
4. Shortage costs (due to disservice to the customers).

12.2-1 Purchase Costs

It is the price that is paid for purchasing/producing an item. It may be constant per unit or may vary with the quantity purchased / produced. If the cost / unit is constant, it does not affect the inventory control decision. However, the purchase cost is definitely considered when it varies as in quantity discount situations.

12.2-2 Inventory Carrying Costs (or Stock Holding Costs or Holding Costs or Storage Costs)

They arise on account of maintaining the stocks and the interest paid on the capital tied up with the stocks. They vary directly with the size of the inventory as well as the time for which the item is held in stock. Various components of the stockholding cost are :

1. *Cost of money or capital tied up in inventories.* This is, by far, the most important component. Money borrowed from the banks may cost interest of about 12%. But usually the problem is viewed in a slightly different way *i.e.,* how much the organisation would have earned, had the capital been invested in an alternative project such as developing a new product, etc. It is generally taken somewhere around 15% to 20% of the value of the inventories.
2. *Cost of storage space.* This consists of rent for space. Besides space expenses, this will also include heating, lighting and other atmospheric control expenses. Typical values may vary from 1 to 3% .
3. *Depreciation and deterioration costs.* They are especially important for fashion items or items undergoing chemical changes during storage. Fragile items such as crockery are liable to damage, breakage, etc. 0.2% to 1% of the stock value may be lost due to damage and deterioration.
4. *Pilferage cost.* It depends upon the nature of the item. Valuables such as gun metal bushes and expensive tools may be more tempting, while there is hardly any possibility of heavy casting or forging being stolen. While the former must be kept under lock and key, the latter may be simply dumped in the stockyard. Pilferage cost may be taken as 1% of the stock value.

5. *Obsolescence cost.* It depends upon the nature of the item in stock. Electronic and computer components are likely to be fast outdated. Changes in design also lead to obsolescence. It may be possible to quantify the percentage loss due to obsolescence and it may be taken as 5% of the stock value.
6. *Handling costs.* These include all costs associated with movement of stock, such as cost of labour, overhead cranes, gantries and other machinery used for this purpose.
7. *Record-keeping and administrative cost.* There is no use of keeping stocks unless one can easily know whether or not the required item is in stock. This signifies the need of keeping funds for record-keeping and necessary administration.
8. *Taxes and Insurance.* Most organisations have insurance cover against possible loss from theft, fire, etc. and this may cost 1% to 2% of the invested capital.

Inventory carrying cost C_1 is expressed either as per cent/unit time (*e.g.,* 20% per year) or in terms of monetary value/unit /unit time (*e.g.,* ₹ 5/unit/ year).

Example : If the average stock during a year is of value ₹ 20,000, the inventory carrying costs, being, say, equal to 20%, amount to ₹ 20,000 × $\frac{20}{100}$ = ₹ 4,000.

12.2-3 Procurement Costs or Setup Costs

These include the fixed cost associated with placing of an order or setting up a machinery before starting production. They include costs of purchase, requisition, follow up, receiving the goods, quality control, cost of mailing, telephone calls and other follow up actions, salaries of persons for accounting and auditing, etc. *Also called order costs or replenishment costs*, they are assumed to be independent of the quantity ordered or produced but directly proportional to the number of orders placed. At times, however, these costs may not bear any simple relationship to the number of orders. More than one stock item may be ordered on one set of the documents; the clerical staff is not divisible and without the existing staff increasing or decreasing, there may be considerable scope for changing the number of orders. In such a case, the acquisition cost relationship may be quadratic or stepped instead of a straight line. They are expressed in terms of ₹/order or ₹/setup.

12.2-4 Shortage Costs or Stock-out Costs

These costs are associated with either a delay in meeting demands or the inability to meet it at all. Therefore, shortage costs are usually interpreted in two ways. In case the unfilled demand can be filled at a later stage (backlog case), these costs are proportional to quantity that is short as well as the delay time and are expressed as ₹/unit back ordered/unit time (*e.g.* ₹ 7/unit/year). They represent loss of goodwill and cost of idle equipment. In case the unfilled demand is lost (no backlog case), these costs become proportional to only the quantity that is short. These result in cancelled orders, lost sales, profit and even the business itself.

It follows from the above discussion that if the purchase cost is constant and independent of the quantity purchased, it is not considered in formulating the inventory control policy. The total variable inventory cost in this case is given by

Total variable inventory cost = Carrying cost + Ordering cost + Shortage cost.

However, if the unit cost depends upon the quantity purchased *i.e.*, price discounts are available, the purchase cost is definitely considered in formulating the inventory control policy. The total inventory cost in this case is then given by

Total inventory cost = Purchase cost + Carrying cost + Ordering cost + Shortage cost.

12.3 INVENTORY CONTROL PROBLEM

The inventory control problem consists of determination of three basic factors :

1. When to order (produce or purchase) ?
2. How much to order ?
3. How much safety stock should be kept ?

When to order. This is related to the *lead time* (also called *delivery lag*) of an item. Lead time may be defined as the time interval between the placement of an order for an item and its receipt in stock. It may be replenishment order on an outside firm or within the works. There should be enough stock for each item so that customers' orders can be reasonably met from this stock until replenishment. This stock level, known as *reorder level*, has, therefore, to be determined for each item. It is determined by balancing the cost of maintaining these stocks and the disservice to the customer if his orders are not filled in time.

How much to order. As already discussed, each order has associated with it the ordering cost or acquisition cost. To keep it low, the number of orders should be as few as possible *i.e.,* the order size should be large. But large order size would imply high inventory carrying cost. Thus the problem of how much to order is solved by compromising between the acquisition costs and inventory carrying costs.

How much should be the safety stock. This is important to avoid overstocking while ensuring that no stock-outs take place.

The inventory control policy of an organisation depends upon the demand characteristics. The demand for an item may be independent or dependent. For instance, the demand for the different models of television sets manufactured by a company does not depend upon the demand of any other item, while the demand of its various components will depend upon the demand (and hence sale) of the television sets and may be arithmetically computed from the latter. The independent demand is usually ascertained by extrapolating the past demand history *i.e.,* by *forecasting.* The order level can be fixed from the demand forecasts and the lead time. Thus while in the case of dependent demand, simple arithmetic computations are enough to ascertain requirement of the components; in the case of independent demand items, statistical forecasting techniques have to be employed. The discussion of these forecasting techniques will be taken up later in this chapter. The family tree drawn in the next section gives an idea of the various inventory control policies.

12.4 CLASSIFICATION OF FIXED ORDER QUANTITY INVENTORY MODELS

- Inventory Models
  - Elementary Models
  - Models with price breaks
  - Models with restrictions
    - Deterministic
      - Model 1 (No shortage permitted)
        - 1 (a) Demand rate uniform, replenishment rate infinite.
        - (b) Demand rate non-uniform, replenishment rate infinite.
        - (c) Demand rate uniform, replenishment rate finite.
      - Model 2 (Shortage permitted)
        - 2 (a) Demand rate uniform, replenishment rate infinite.
        - (b) Demand rate uniform, replenishment rate infinite, time interval fixed.
        - (c) Demand rate uniform, replenishment rate finite.
    - Probabilistic
      - 3 (a) Instantaneous demand, setup cost zero, stock levels discrete, lead time zero.
        - (b) Same as 3 (a) except that stock levels are continuous.
      - 4 (a) Continuous demand, setup cost zero, stock levels discrete, lead time zero.
        - (b) Same as 4(a) except that stock levels are continuous.
      - 5 (a) Continuous demand, setup cost zero, stock levels discrete, lead time significant.
        - (b) Same as 5(a) except that stock levels are continuous.

- Demand
  - Independent
    - Fixed order quantity systems
    - Periodic review systems
      - Fixed order cycle system
      - sS system
  - Dependent

Fixed order quantity systems will now be discussed in detail. The periodic review systems will be taken up briefly towards the end of this chapter.

12.5 INVENTORY MODELS WITH DETERMINISTIC DEMAND

It is extremely difficult to formulate a single general inventory model which takes into account all variations in real systems. In fact, even if such a model were developed, it may not be analytically solvable. Thus inventory models are usually developed for some specific situations.

In this section we shall deal with situations in which demand is assumed to be fixed and completely known. Models for such situations are called *economic lot size models or economic order quantity models.*

12.5-1 Model 1 (a) Classical EOQ Model (Demand Rate Uniform, Replenishment Rate Infinite)

This is one of the simplest inventory models. A stockist has order to supply goods to customers at a uniform rate R per unit time. Hence demand is fixed and known. No shortages are allowed, consequently, the cost of shortage, C_2 is infinity. He places an order with the manufacturer every t time units, where t is fixed; and the ordering cost per order is C_3. Replenishment time is negligible *i.e.*, replenishment rate is infinite so that replenishment is instantaneous (lead time is zero). The holding cost is assumed to be proportional to the amount of inventory as well as the time inventory is held. Thus the cost of holding inventory I for time T is C_1IT, where C_1 is the cost of holding one unit in inventory for a unit of time. The cost coefficients C_1, C_2 and C_3 are assumed to be constants. The stockist's problem is to determine

(*i*) How frequently he should place the order.

(*ii*) How many units should be ordered in each order.

This model is illustrated schematically in figure 12.1.

If orders are placed at intervals t, a quantity $q = Rt$ must be ordered in each order. Since the stock in small time dt is $Rt dt$, the stock in time period t will be

$$\int_0^t R\, t.dt = \frac{1}{2} Rt^2 = \frac{1}{2} qt = \text{Area of inventory triangle OAP.}$$

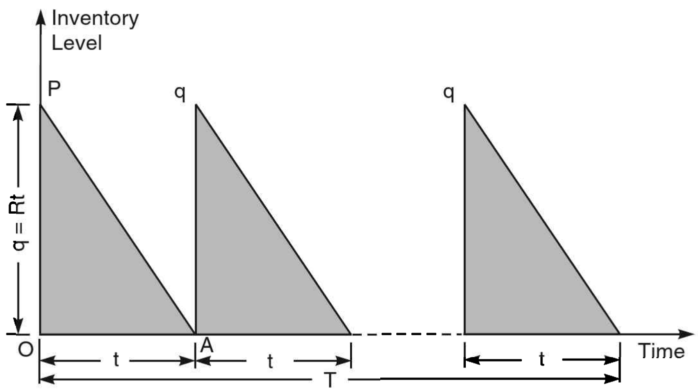

Fig. 12.1. Inventory situation for model 1 (a).

$\therefore$ Cost of holding inventory during time $t = \frac{1}{2} C_1Rt^2.$

Ordering cost to place an order = C_3.

$\therefore$ Total cost during time $t = \frac{1}{2} C_1Rt^2 + C_3.$

$$\therefore \text{ Average total cost per unit time, } C(t) = \frac{1}{2} C_1Rt + \frac{C_3}{t}. \qquad ...(12.1)$$

C will be minimum if $\frac{dC(t)}{dt} = 0$ and $\frac{d^2C(t)}{dt^2}$ is positive.

Differentiating equation (12.1) w.r.t. 't',

$$\frac{dC(t)}{dt} = \frac{1}{2} C_1R - \frac{C_3}{t^2} = 0, \text{ which gives } t = \sqrt{\frac{2C_3}{C_1R}}.$$

Differentiating equation (12.1) twice w.r.t. 't'

$\frac{d^2C(t)}{dt^2} = \frac{2C_3}{t^3}$, which is positive for value of t given by the above equation.

Thus C(t) is minimum for optimal time interval,

$$t_0 = \sqrt{\frac{2C_3}{C_1R}}. \quad ...(12.2)$$

Optimum quantity q_0 to be ordered during each order,

$$q_0 = R\, t_0 = \sqrt{\frac{2C_3R}{C_1}}, \quad ...(12.3)$$

which is known as the *optimal lot size (or economic order quantity) formula* due to R.H. Wilson. It is also called *Wilson's or square root formula or Harris lot size formula.*

Any other order quantity will result in a higher cost.

The resulting minimum average cost per unit time,

$$C_0(q) = \frac{1}{2}C_1R.\sqrt{\frac{2C_3}{C_1R}} + C_3.\sqrt{\frac{C_1R}{2C_3}}$$

$$= \frac{1}{\sqrt{2}}\sqrt{C_1C_3R} + \frac{1}{\sqrt{2}}\sqrt{C_1C_3R} = \sqrt{2C_1C_3R}. \quad ...(12.4)$$

This cost curve has the lowest value (Fig. 1.1) just above the intersection of the two cost curves viz, ordering cost curve and carrying cost curve. At the intersection point the two costs are equal.

Also the total minimum cost per unit time, including the cost of the item

$$= \sqrt{2C_1C_3R} + CR, \quad ...(12.4a)$$

where C is the cost/unit of the item.

Equation (12.1) can be written in an alternative form by replacing t by q/R as

$$C(q) = \frac{1}{2}C_1q + \frac{C_3R}{q}. \quad ...(12.5)$$

The average inventory is $\frac{q_0 + 0}{2} = \frac{q_0}{2}$ and is, thus, time independent.

It may be realized that some of the assumptions made are not satisfied in actual practice. For instance, it is seldom that a customer demand is known exactly and that replenishment time is negligible.

Corollary 1. In the above model if the order cost is $C_3 + bq$ instead of being fixed, where b is the order cost per unit item, we can prove that there is no change in the optimum order quantity due to the changed order cost.

Proof. The average cost per unit time, $C(q) = \frac{1}{2}C_1q + \frac{R}{q}(C_3 + bq)$. [From equation (12.5)]

For the minimum cost $\frac{dC(q)}{dq} = 0$ and $\frac{d^2C(q)}{dq^2}$ is positive.

i.e., $\frac{1}{2}C_1 - \frac{RC_3}{q^2} = 0$ or $q = \sqrt{\frac{2RC_3}{C_1}}$,

and $\frac{d^2C(q)}{dq^2} = \frac{2RC_3}{q^3}$, which is necessarily positive for above value of q.

$\therefore$ $q_0 = \sqrt{\frac{2C_3R}{C_1}}$, which is same as equation (12.3).

Hence there is no change in optimum order quantity as a result of change in the order cost.

Corollary 2. In model 1 (*a*) discussed above, the lead time has been assumed to be zero. Most practical problems, however, have a positive lead time L from the time the order for the item is placed until it is actually delivered. The ordering policy of the above model, therefore, must satisfy the reorder point.

If L is the lead time in days and R is the inventory consumption rate in units per day, the total inventory requirements during the lead time = LR. Thus we should place an order q as soon as the stock level becomes LR. This is called reorder point p = LR.

In practice, this is equivalent to continuously observing the level of inventory until the reorder point is obtained. That is why the economic lot size model is also called *continous review model.* Figure 12.2 shows the reorder points.

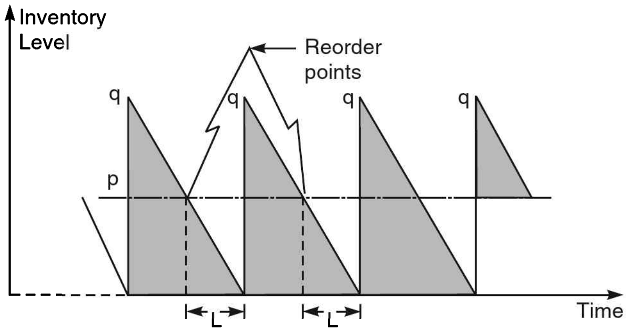

Fig. 12.2. Inventory situation with lead time.

If a buffer stock B is to be maintained, reorder point will be

$$p = B + LR. \quad ...(12.6)$$

Furthermore, if d days are required for reviewing the system,

$$p = B + LR + \frac{Rd}{2} = B + R\left(L + \frac{d}{2}\right). \quad ...(12.7)$$

Assumptions in E.O.Q. Formula

Following simplifying assumptions have been made while deriving the economic order quantity formula :

1. Demand is known and uniform (constant).
2. Shortages are not permitted ; as soon as the stock level becomes zero, it is instantaneously replenished.
3. Replenishment of stock is instantaneous or replenishment rate is infinite.
4. Lead time is zero. The moment the order is placed, the quantity ordered is received.
5. Inventory carrying cost and ordering cost per order remain constant over time. The former is linearly related to the quantity ordered and the latter to the number of orders.
6. Cost of the item remains constant over time. There are no price-breaks or quantity discounts.
7. The item is purchased and replenished in lots or batches.
8. The inventory system pertains to a single item.

Limitations of (Objections to) E.O.Q. Formula

The E.O.Q. formula has a number of limitations. It has been highly controversial since a number of objections have been raised regarding its validity. Some of them are

1. In practice the demand is neither known with certainty nor it is uniform. If the fluctuations are mild, the formula can be applicable but for large fluctuations it loses its validity. Dynamic E.O.Q. models, instead, may have to be applied.
2. The ordering cost is difficult to measure. Also it may not be linearly related to the number of orders as assumed in the derivation of the model. The inventory carrying rate is still more difficult to measure and even to define precisely.
3. It is difficult to predict the demand. Present demand may be quite different from the past history. Hardly any prediction is possible for a new product to be introduced in the market.
4. The E.O.Q. model assumes instantaneous replenishment of the entire quantity ordered. In practice, the total quantity may be supplied in parts. E.O.Q. model is not applicable in such a situation.
5. Lead time may not be zero unless the supplier is next-door and has sufficient stock of the item, which is rarely so.
6. Price variations, quantity discounts and shortages may further ivalidate the use of the E.O.Q. formula.

However, the flatness of the total cost curve around the minimum (Fig. 1.1) is an answer to many objections. Even if we deviate from E.O.Q. within reasonable limits, there is no substantial change in cost. For example, if because of inaccuracies and errors, we have selected an order quantity 20% more (or less) than q_0, the increase in total cost will be less than 2%.

EXAMPLE 12.5-1

A stockist has to supply 12,000 units of a product per year to his customer. The demand is fixed and known and the shortage cost is assumed is to be infinite. The inventory holding cost is ₹ 0.20 per unit per month and the ordering cost per order is ₹ 350. Determine

(i) The optimum lot size q_0
(ii) optimum scheduling period t_0,
(iii) minimum total variable yearly cost.

Solution

Supply rate, $$R = \frac{12,000}{12} = 1,000 \text{ units/month},$$

C_1 = ₹ 0.20 per unit per month, C_3 = ₹ 350 per order.

(*i*) $$q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 350 \times 1,000}{0.20}} = 1,870 \text{ units/order.}$$

(*ii*) $$t_0 = \sqrt{\frac{2C_3}{C_1R}} = \sqrt{\frac{2 \times 350}{0.20 \times 1,000}} = 1.87 \text{ months} = 8.1 \text{ weeks between orders.}$$

(*iii*) $$C_0 = \sqrt{2C_1C_3R} = \sqrt{2 \times .20 \times 12 \times 350 \times (1,000 \times 12)} = ₹\ 4,490 \text{ per year.}$$

EXAMPLE 12.5-2

A particular item has a demand of 9,000 units/year. The cost of one procurement is ₹100 and the holding cost per unit is ₹ 2.40 per year. The replacement is instantaneous and no shortages are allowed. Determine

(i) the economic lot size,
(ii) the number of orders per year,
(iii) the time between orders,
(iv) the total cost per year if the cost of one unit is ₹ 1. [*NIIFT Mohali, 2000*]

Solution

$$R = 9{,}000 \text{ units/year},$$
$$C_3 = ₹\ 100/\text{procurement},\ C_1 = ₹\ 2.40/\text{unit/year}.$$

(*i*) $$q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2\times100\times9{,}000}{2.40}} = 866 \text{ units/procurement.}$$

(*ii*) $$n_0 = \frac{1}{t_0} = \sqrt{\frac{C_1R}{2C_3}} = \sqrt{\frac{2.40\times9{,}000}{2\times100}} = \sqrt{108} = 10.4 \text{ orders/year.}$$

(*iii*) $$t_0 = \frac{1}{n_0} = \frac{1}{10.4} = 0.0962 \text{ years} = 1.15 \text{ months between procurement.}$$

(*iv*) $$\begin{aligned} C_0 &= 9{,}000\times1 + \sqrt{2C_1C_3R} \\ &= 9{,}000 + \sqrt{2\times2.40\times100\times9{,}000} \\ &= 9{,}000 + 2{,}080 = ₹\ 11{,}080/\text{year.} \end{aligned}$$

EXAMPLE 12.5-3

A stockist has to supply 400 units of a product every Monday to his customers. He gets the product at ₹ 50 per unit from the manufacturer. The cost of ordering and transportation from the manufacturer is ₹ 75 per order. The cost of carrying inventory is 7.5% per year of the cost of the product. Find

(i) the economic lot size,
(ii) the total optimal cost (including the capital cost),
(iii) the total weekly profit if the item is sold for ₹ 55 per unit.

[*NIIFT Mohali, 2001; P.U. B.Com. Sept., 2005*]

Solution

$$\begin{aligned} R &= 400 \text{ units/week}, \\ C_3 &= ₹\ 75/ \text{ per order}, \\ C_1 &= 7.5\% \text{ per year of the cost of the product} \\ &= ₹\ \left(\frac{7.5}{100}\times50\right) \text{ per unit per year} \\ &= ₹\ \left(\frac{7.5}{100}\times\frac{50}{52}\right) \text{ per unit per week} \\ &= ₹\ \frac{3.75}{52} \text{ per unit per week.} \end{aligned}$$

(*i*) $$q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2\times75\times400\times52}{3.75}} = 912 \text{ units/order.}$$

(*ii*) $$\begin{aligned} C_0 &= 400\times50 + \sqrt{2C_1C_3R} \\ &= 20{,}000 + \sqrt{\frac{2\times3.75}{52}\times75\times400} \\ &= 20{,}000 + 65.80 = ₹\ 20{,}065.80 \text{ per week.} \end{aligned}$$

(*iii*) Profit $P = 55 \times 400 - C_0 = 22{,}000 - 20{,}065.80 = ₹\ 1{,}934.20$ per week.

EXAMPLE 12.5-4

A stockist purchases an item at the rate of ₹ 40 per piece from a manufacturer. 2,000 units of the item are required per year. What should be the order quantity per order if the cost per order is ₹ 15 and the inventory charges per year are 20 paise ? [*J.N.T.U. Hyderabad B.Tech. Nov., 2010*]

Solution

$$R = 2{,}000 \text{ units/year},$$
$$C_3 = ₹\ 15 \ / \text{ order},$$
$$I = \text{Re. } 0.20 \ / \text{ year } \therefore\ C_1 = CI = ₹\ 0.20 \times 40 = ₹\ 8/\text{unit/year}.$$

$$\therefore \qquad q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 15 \times 2{,}000}{8}} = 87 \text{ units/order}.$$

EXAMPLE 12.5-5

The demand for a commodity is 100 units per day. Every time an order is placed, a fixed cost of ₹ 400 is incurred. Holding cost is ₹ 0.08 per unit per day. If the lead time is 13 days, determine the economic lot size and the reorder point.

Solution

$$q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 400 \times 100}{0.08}} = 1{,}000 \text{ units}.$$

Length of cycle, $$t_0 = \frac{1{,}000}{100} = 10 \text{ days}.$$

As the lead time is 13 days and cycle length is 10 days, reodering should occur when the level of inventory is sufficient to satisfy the demand for 13–10 = 3 days.

$\therefore$ Reorder point = 100 × 3 = 300 units.

It may be noted that the 'effective' lead time is taken equal to 3 days rather than 13 days. It is because the lead time is longer than t_0.

EXAMPLE 12.5-6

(*a*) *Calculate the E.O.Q. in units and total variable cost for the following items, assuming an ordering cost of ₹ 5 and a holding cost of 10%.*

Item	*Annual demand*	*Unit price (₹)*
A	*800 units*	*0.02*
B	*400 units*	*1.00*
C	*392 units*	*8.00*
D	*13,800 units*	*0.20*

(b) For the above problem, compute E.O.Q. in ₹ as well as in years of supply. Also calculate the E.O.Q. frequency for each of the four items.

Solution

(*a*) $$q_0 = \sqrt{\frac{2C_3R}{C_1}},\ C_0 = \sqrt{2C_1C_3R}.$$

Item A $$q_0 = \sqrt{\frac{2 \times 5 \times 800}{0.02 \times \frac{10}{100}}} = \sqrt{\frac{800}{0.002}} = 2{,}000 \text{ units},$$

$$C_0 = \sqrt{2 \times 5 \times 800 \times 0.02 \times \frac{10}{100}} = ₹\ 4.$$

Item B $$q_0 = \sqrt{\frac{2 \times 5 \times 400}{1.00 \times \frac{10}{100}}} = 200 \text{ units},$$

$$C_0 = \sqrt{2 \times 5 \times 400 \times 1.00 \times \frac{10}{100}} = ₹\ 20.$$

Item C $$q_0 = \sqrt{\frac{2 \times 5 \times 392}{8.00 \times \frac{10}{100}}} = 70 \text{ units,}$$

$$C_0 = \sqrt{2 \times 5 \times 392 \times 8.00 \times \frac{10}{100}} = ₹\ 56.$$

Item D $$q_0 = \sqrt{\frac{2 \times 5 \times 13{,}800}{0.20 \times \frac{10}{100}}} = 2{,}627 \text{ units,}$$

$$C_0 = \sqrt{2 \times 5 \times 13{,}800 \times 0.20 \times \frac{10}{100}} = ₹\ 52.54.$$

(*b*) *E.O.Q. in ₹*

for item A : 2,000 × 0.02 = 40,
for item B : 200 × 1= 200,
for item C : 70 × 8 = 560,
and for item D : 2,627 × 0.20 = 525.40.

E.O.Q. in years of supply

for item A : $\frac{2000}{800} = 2.5$ years,

for item B : $\frac{200}{400} = 0.5$ year,

for item C : $\frac{70}{392} = 0.18$ year,

and for item D : $\frac{2{,}627}{13{,}800} = 0.19$ year.

E.O.Q. frequency (number of orders per year)

for item A : $\frac{1}{1.25} = 0.4$,

for item B : $\frac{1}{0.5} = 2$,

for item C : $\frac{1}{0.18} = 5.6$,

and for item D : $\frac{1}{0.19} = 5.25$.

EXAMPLE 12.5-7

(a) Compute the E.O.Q. and the total variable cost for the following :

Annual demand	:	*25 units,*
unit price	:	*₹ 2.50,*
order cost	:	*₹ 4.00,*
storage rate	:	*1% per year,*
interest rate	:	*12% per year,*
obsolescence rate	:	*7% per year.*

(b) Compute the order quantity and the total variable cost that would result if an incorrect price of ₹ 1.60 were used for the item. [*NIIFT Mohali, 2001; P.T.U. MBA May, 2002*]

Solution

(*a*) $$C_1 = ₹\ \frac{(1+12+7)}{100} \times 2.50\ = ₹\ 0.50 \text{ per unit per year.}$$

$$\therefore \quad q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 4 \times 25}{0.50}} = 20 \text{ units,}$$

$$C_0 = \sqrt{2C_1C_3R} = \sqrt{2 \times 4 \times 25 \times 0.50}\ = ₹\ 10.$$

(*b*) $$q = \sqrt{\frac{2 \times 4 \times 25}{\frac{20}{100} \times 1.60}} = 25 \text{ units. This is non-optimal size.}$$

$$\text{Ordering cost} = \frac{C_3R}{q} = \frac{4 \times 25}{25} = ₹\ 4.$$

$$\text{Stock holding cost} = \frac{1}{2}C_1 . q = \frac{1}{2} . \left(\frac{20}{100} \times 2.50\right) \times 25\ = ₹\ 6.25.$$

Note that for calculating the stock holding cost, correct price is to be used. A clectrical error does not mean that the stock value changes. Price paid for the stocks shall still be ₹ 2.50 even though a less price of ₹ 1.60 is taken for E.O.Q. computations.

∴ Total variable cost/year = ₹ 10.25.

EXAMPLE 12.5-8

ABC manufacturing company purchases 9,000 parts of a machine for its annual requirement, ordering one month's usage at a time. Each part costs ₹ 20. The ordering cost per order is ₹ 15, and the carrying charges are 15% of the average inventory per year.

You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year?

[*SVSM PGDM, 2009; P.T.U.B.E. (Mech.) May, 2006; P.U.B.Com. April, 2006; Sept., 2004*]

Solution

Here, $R = 9{,}000$ parts/year,

$$\therefore \quad q = \frac{9{,}000}{12} = 750 \text{ parts.}$$

$$C = ₹\ 20 \text{ part, } C_3 = ₹\ 15 \text{ /order,}$$

$$C_1 = ₹\ 20 \times \frac{15}{100} = ₹\ 3 \text{ / part/ year.}$$

$$\text{Total annual variable cost} = \frac{q}{2} . C_1 + \frac{R}{q} . C_3$$

$$= ₹\ \left[\frac{750}{2} \times 3 + \frac{9{,}000}{750} \times 15\right] = ₹\ 1{,}305.$$

$$q_0 = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2 \times 9{,}000 \times 15}{3}} = 300 \text{ units.}$$

$$\text{Total annual variable cost} = \sqrt{2RC_1C_3} = \sqrt{2 \times 9{,}000 \times 3 \times 15}\ = ₹\ 900.$$

Hence if the company purchases 300 units each time and places 30 orders in the year, the net saving to the company will be ₹ (1,305 – 900) = ₹ 405 a year.

EXAMPLE 12.5-9

A company uses ₹ 10,000 worth of an item during the year. The ordering costs are ₹ 25 per order and carrying charges are 12.5% of the average inventory value. Find the economic order quantity, number of orders per year, time period per order and the total cost.

Solution

Here CR = ₹ 10,000, C_3 = ₹ 25/ order, I = 12.5% = 0.125.

Now E.O.Q. in units = $q_0 = \sqrt{\dfrac{2C_3R}{C_1}} = \sqrt{\dfrac{2C_3R}{CI}}$.

$\therefore$ E.O.Q. in rupees = $Cq_0 = C\sqrt{\dfrac{2C_3R}{CI}} = \sqrt{\dfrac{2C_3CR}{I}} = \sqrt{\dfrac{2 \times 25 \times 10{,}000}{0.125}} =$ ₹ 2,000.

Number of orders / year, $n_0 = \dfrac{R}{q_0} = \dfrac{CR}{Cq_0} = \dfrac{10{,}000}{2{,}000} = 5.$

Time period per order, $t_0 = \dfrac{1}{n_0} = \dfrac{1}{5}$ year = 73 days.

Annual variable cost = $\sqrt{2C_3C_1R} = \sqrt{2C_3CIR} = \sqrt{2C_3ICR}$

$= \sqrt{2 \times 25 \times 0.125 \times 10{,}000} =$ ₹ 250.

$\therefore$ Total annual cost = $CR + \sqrt{2C_3C_1R} =$ ₹ (10,000 + 250) = ₹ 10,250.

EXAMPLE 12.5-10

A manufacturing company of microwave ovens uses ₹ 75,000 worth of LED readout circuits annually in its production process. Cost per order is ₹ 45 and the carrying charges assessed against this classification of inventory are 25% of the average balance per year. This company follows an E.O.Q. purchasing system and to date has not been offered any discounts on these circuits. Now the supplier has indicated that if the company would buy its circuits four times a year in equal quantities, a discount of 1.5% off the list price would be given in return. Would you advise this company to accept this offer ? In order to maintain the present total cost, what should be the minimum discount acceptable to the company if four orders of equal sizes are placed in a year?

[*C.A. (Final) Nov., 1989*]

Solution

Here, CR = ₹ 75,000, C_3 = ₹ 45/order, I = 25% = 0.25.

$\therefore$ $C_0 = \sqrt{2RC_3CI} = \sqrt{2 \times 75{,}000 \times 45 \times .25}$ = ₹ 1,299.04.

$\therefore$ Total cost = ₹ (75,000 + 1,299.04) = ₹ 76,299.04.

To earn discount, quantity ordered, $q = \dfrac{R}{4}$. This is non-optimal size. Total cost when discount of 1.5% is availed of

$$= \frac{q}{2}\,CI + \frac{R}{q}\,.\,C_3 + CR = \frac{R}{4 \times 2}\,CI + \frac{R}{q}\,C_3 + CR$$

$$= CR\left(1 + \frac{I}{8}\right) + \frac{R}{q}\,.\,C_3 = CR\left(1 + \frac{I}{8}\right) + 4C_3$$

$$= ₹\left[0.985 \times 75{,}000\left(1 + \frac{0.25}{8}\right) + 4 \times 45\right]$$

= ₹ (76,183.60 + 180) = ₹ 76, 363.60.

Therefore, the company should not accept the offer of 1.5% discount. Let x be the minimum per cent discount acceptable to the company.

Then $$76{,}299.04 = \frac{100 - x}{100} \times 75{,}000 \left(1 + \frac{0.25}{8}\right) + 4 \times 45$$

$$= \frac{100 - x}{100} \times 75{,}000 \times \frac{33}{32} + 180$$

or $$100 - x = \frac{76{,}119.04 \times 100 \times 32}{75{,}000 \times 33} = 98.416$$

or $$x = 1.584\%.$$

EXAMPLE 12.5-11

A company is considering the feasibility of changing suppliers for coupling hardware. Presently the company has an optimal purchasing policy with Ace Hardware at a discount of 1%. Current yearly purchases are ₹ 81,000 and the Administrative Charges are ₹ 125 per purchase and the carrying charges are 25% of the average inventory level.

Bids received from other suppliers are: Nutz Co. offers 5% discount if ordered twice a year and Grabbers Co. offers 3% discount if ordered four times a year. Should the company retain the present supplier or accept the proposed offers and , if so, which offer?

[*Dayalbagh Edu. Inst. Agra Dec., 2006*]

Solution

$$q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2C_3R}{CI}}$$

$$\text{E.O.Q. in rupees} = Cq_0 = C \,.\, \sqrt{\frac{2C_3R}{CI}} = \sqrt{\frac{2CC_3R}{I}} = \sqrt{\frac{2CRC_3}{I}}$$

$$= \sqrt{\frac{2 \times 81{,}000 \times 125}{0.25}} = 9{,}000. \quad (\because \text{order value} = CR = ₹\ 81{,}000.)$$

$$\text{Total cost at 1\% discount} = \frac{1}{2}C_1q + C_3 \cdot \frac{R}{q} + CR$$

$$= \frac{1}{2}\, CIq + C_3 \cdot \frac{R}{q} + CR$$

$$= \frac{1}{2} \times 9{,}000 \times 0.25 \times 0.99 + 125 \times \frac{81{,}000}{9{,}000} + 81{,}000 \times 0.99$$

$= ₹\ 82{,}428.75.$

$$\text{Total cost at 5\% discount from Nutz Co. offer} = \frac{q}{2}\,CI + C_3 . \frac{R}{q} + CR$$

$$= \frac{81{,}000}{2 \times 2} \times 0.25 \times 0.95 + 2 \times 125 + 81{,}000 \times 0.95$$

$= ₹\ 82{,}009.38.$

$$\text{Total cost at 3\% discount from Grabbers Co. offer} = \frac{q}{2}\,CI + C_3 . \frac{R}{q} + CR$$

$$= \frac{81{,}000}{4 \times 2} \times 0.25 \times 0.97 + 4 \times 125 + 81{,}000 \times 0.97$$

$= ₹\ 81{,}525.31.$

The company should, therefore, accept the offer from Grabbers Co.

EXAMPLE 12.5-12

The purchase manager of an organisation has collected the following data for one of the A-class items :

Interest on the locked-up capital	*= 20%*
Order processing cost for each order	*= ₹ 100*
Inspection cost per lot	*= ₹ 50*
Follow up cost for each order	*= ₹ 80*
Pilferage while holding inventory	*= 5%*
Other holding cost	*= 15%*
Other procurement cost for each order	*= ₹ 170*
Annual demand	*= 1,000 units*
Cost per item	*= ₹ 10*
Discount for a minimum order quantity of 500 items	*= 10%*

What should be the ordering policy of the purchase manager?

[*ICWA (Final) Dec., 1991*]

Solution

Here, annual demand, R = 1,000 units,
cost per item, C = ₹ 10,
total acquisition cost, C_3 = ₹ (100 + 50 + 80 + 170) = ₹ 400 per order,
inventory carrying rate, I = (20 + 5 + 15) = 40%,
discount for minimum quantity of 500 items = 10%.

$$q_0 \text{ for unit cost of ₹ } 10 = \sqrt{\frac{2C_3R}{CI}} = \sqrt{\frac{2\times 400\times 1{,}000}{10\times 0.40}} = 447 \text{ units.}$$

Total cost if the purchase quantity is 447 units

$$= CR + \sqrt{2C_1C_3R} = ₹\ (CR + \sqrt{2CC_3RI}\ )$$

$$= ₹\ (9 \times 1{,}000 + \sqrt{2\times 10\times 400\times 1{,}000\times 0.4}\ )$$

$$= ₹\ (9{,}000 + 1{,}788.55) = ₹\ 10{,}788.85.$$

Unit cost when quantity ordered is 500 units = ₹ (10 × .90) = ₹ 9.

$$\text{Total cost} = CR + \frac{q}{2}CI + \frac{R}{q}\cdot C_3$$

$$= 9 \times 1{,}000 + \frac{500}{2} \times 9 \times 0.4 + \frac{1{,}000}{500} \times 400$$

$$= 9{,}000 + 900 + 800 = ₹\ 10{,}700.$$

Therefore, the purchase manager should place order for 500 items each time.

EXAMPLE 12.5-13

The purchasing manager of a distillery company is considering three sources of supply for oak barrels. The first supplier offers any quantity of barrels at ₹ 150 each. The second supplier offers barrels in lots of 150 or more at ₹ 125 per barrel. The third supplier offers barrels in lots of 250 or more at ₹ 100 each. The distillery uses 1,500 barrels a year at a constant rate. Carrying costs are 40 per cent, and it costs the purchasing agent ₹ 400 to place an order. Calculate the total annual cost for the orders placed to the probable suppliers and find out the supplier to whom the orders should be placed. [*P.U.B.Com. April, 2008; April, 2006; B.E. (E. & Ec.) April, 2008; C.A. (Final) May, 1991*]

Solution

Here, annual demand, R = 1,500 barrels,
inventory rate, I = 40% = 0.4,
ordering cost, C_3 = ₹ 400 / order.

The cost/unit is shown in the table below.

Supplier	*Quantity of barrels*	*Cost/unit*
First	Any quantity	₹150
Second	150 and above	₹ 125
Third	250 and more	₹ 100

Total annual costs are calculated below for the three suppliers :

First supplier $q_0 = \sqrt{\dfrac{2RC_3}{CI}} = \sqrt{\dfrac{2 \times 1,500 \times 400}{150 \times 0.4}}$ = 141.4 barrels (feasible).

$$\text{Total annual cost} = CR + \sqrt{2RC_3CI}$$
$$= 150 \times 1,500 + \sqrt{2 \times 1,500 \times 400 \times 150 \times 0.4}$$
$$= 2,25,000 + 8,484 = ₹\ 2,33,484.$$

Second supplier $q_0 = \sqrt{\dfrac{2 \times 1,500 \times 400}{125 \times 0.4}}$ = 154.92 barrels (feasible)

$$\text{Total annual cost} = CR + \sqrt{2RC_3CI}$$
$$= 125 \times 1,500 + \sqrt{2 \times 1,500 \times 400 \times 125 \times 0.4}$$
$$= 1,87,500 + 7,746 = ₹\ 1,95,246.$$

Third supplier $q_0 = \sqrt{\dfrac{2 \times 1,500 \times 400}{100 \times 0.4}}$ = 173.2 barrels.

(not feasible since min. quantity is 250 barrels)

∴ Minimum batch size for which order can be placed = 250 barrels.

∴ $$\text{Total annual cost} = CR + \frac{q}{2} CI + C_3 . \frac{R}{q}$$
$$= 100 \times 1,500 + \frac{250}{2} \times 100 \times 0.4 + 400 \times \frac{1,500}{250}$$
$$= 1,50,000 + 5,000 + 2,400 = ₹\ 1,57,400.$$

Thus the order for 250 oak barrels each time should be placed with the third supplier as it involves the lowest annual cost.

EXAMPLE 12.5-14

A company plans to consume 700 pieces annually of a particular component. Past records indicate that its purchasing department spent ₹ 12,500 for placing 15,000 purchase orders. The average inventory was valued at ₹ 50,000 and the total storage cost was ₹ 7,500, which included wages, rent, taxes, insurance, etc. related to store department. The company borrows capital at 10 per cent a year. If the cost of the component is ₹ 12 and lot size is 10, determine the

(a) purchase price/year,
(b) purchase expenses/year,
(c) storage expenses/year,
(d) capital cost/ year,
(e) total cost/year.

Solution

(*a*) Purchase price/year = ₹ 12 × 700 = ₹ 8,400.

(*b*) Ordering cost/order $= ₹\ \dfrac{12,500}{15,000} = ₹\ 0.83.$

$\therefore$ Purchase expenses/year = ₹ $\left(0.83 \times \frac{700}{10}\right)$ = ₹ 58.33.

(*c*) Inventory carrying rate, $I = \frac{7,500}{50,000} \times 100 = 15\%$ / year.

$\therefore$ Storage expenses/year $= \frac{q}{2}$ CI = ₹ $\left(\frac{10}{2} \times 12 \times \frac{15}{100}\right)$ = ₹ 9.

(*d*) Capital cost/year $= \frac{q}{2}$. C.I′ = ₹ $\left(\frac{10}{2} \times 12 \times \frac{10}{100}\right)$ = ₹ 6.

(*e*) Total cost/year = ₹ [8,400 + 58.33 + 9 + 6] = ₹ 8,473.33.

EXAMPLE 12.5-15

A chemical company is considering the optimal batch size for reorder of concentrated sulphuric acid. The management accountant has supplied the following information :

(*i*) *The purchase price of H_2SO_4 is ₹ 150 per gallon.*

(*ii*) *The clerical and data processing costs are ₹ 500 per order.*

All the transport is done by rail. A charge of ₹ 2,000 is made each time the special line to the factory is opened. A charge of ₹ 20 per gallon is also made. The company uses 40,000 gallons per year. Maintenance costs of stocks are ₹ 200 per gallon per year.

Each gallon requires 0.5 sq. ft. of storage space. If warehouse space is not used, it can be rented out to another company at ₹ 200 per sq. ft. per annum. Available warehouse space is 1,000 sq. ft., the overhead costs being ₹ 5,000 per annum. Calculate

(a) the economic reorder size.

(b) the minimum total annual cost of holding and reordering stock. [*P.U. MBA, 2004*]

Solution

Here, annual demand, R = 40,000 gallons,

Ordering cost, C_3 = ₹ (500 + 2,000) = ₹ 2,500/ order.

If the warehouse space is rented out, the company can get ₹ 200 per sq. ft. If it uses the space for storing H_2SO_4, it is not able to realise that amount.

$\therefore$ Carrying cost, C_1 = ₹ (200 + 0.5 × 200) = ₹ 300 / gallon / year.

Note that the rail transport cost of ₹ (20 × 40,000) = ₹ 8,00,000 as well as the overhead cost of ₹ 5,000 are fixed costs and are irrelevant for finding the economic order size.

(*a*) $\therefore \quad q_0 = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2 \times 40,000 \times 2,500}{300}} = 817$ gallons.

(*b*) Total annual cost = $CR + \sqrt{2RC_3C_1}$ + Rail transport cost + Storage overhead cost

$$= ₹\left[150 \times 40,000 + \sqrt{2 \times 40,000 \times 2,500 \times 300} + 20 \times 40,000 + 5,000\right]$$

$$= ₹\left[60,00,000 + 2,44,950 + 8,00,000 + 5,000\right] = ₹\ 70,49,950.$$

EXAMPLE 12.5-16

A company uses an item at a uniform rate of 2,000 units per year. Delivery is instantaneous and no shortages are permitted. The ordering, receiving and hauling cost is ₹ 13 per order, while inspection cost is ₹ 12 per order. Interest costs ₹ 0.056 and deterioration and obsolescence costs ₹ 0.004 respectively per year for each item actually held in inventory plus ₹ 0.02 per year based on the maximum number of units in inventory. Calculate the E.O.Q. If lead time is 25 days, find reorder level. [*P.T.U. B.Tech. April, 2012*]

Solution

Here, R = 2,000 units/year, C_3 = ₹ (13 + 12) = ₹ 25/order. Inventory carrying cost based on actual *i.e.*, average inventory = ₹ (0.056 + 0.004) = ₹ 0.06/unit/year.

Inventory carrying cost based on maximum inventory = ₹ 0.02/unit/year.

$$\text{Annual variable cost} = C_3 \cdot \frac{R}{q} + 0.06 \cdot \frac{q}{2} + 0.02 \cdot q = \frac{25 \times 2{,}000}{q} + 0.05q.$$

For E.O.Q., $\dfrac{25 \times 2{,}000}{q} = 0.05q$ (Carrying cost = Ordering cost)

or $$q^2 = \frac{25 \times 2{,}000}{0.05} \text{ or } q = \sqrt{\frac{25 \times 2{,}000}{0.05}} = 1{,}000 \text{ units.}$$

∴ E.O.Q. = 1,000 units.

$$\text{Reorder level} = \text{R} \times \text{L} = 2{,}000 \times \frac{25}{365} = 137 \text{ units.}$$

EXAMPLE 12.5-17

A wholesaler supplies 30 stuffed dolls each weekday to various shops. Dolls are purchased from the manufacturer in lots of 120 each of ₹ 1,200 per lot. Every order incurs a handling charge of ₹ 60 plus a freight charge of ₹ 250 per lot. Multiple and fractional lots also can be ordered, and all orders are filled the next day. The incremental cost is ₹ 0.60 per year to store a doll in inventory. The wholesaler finances inventory investments by paying its holding company 2% monthly for borrowed funds. Find E.O.Q. and frequency of orders assuming 250 weekdays in a year. [*P.T.U. B.Tech. April, 2012; I.C.W.A. Dec., 1984*]

Solution

Annual demand, R = 30 × 250 = 7,500 units.

$$\text{Unit cost of purchase, C} = ₹\ \frac{1{,}200}{120} = ₹\ 10.$$

Ordering cost, C_3 = ₹ (60 + 250) = ₹ 310 / order.

$$\text{Inventory carrying cost, } C_1 = ₹ \left(0.60 + 10 \times \frac{2 \times 12}{100}\right) = ₹\ 3/\text{unit}/\text{year}.$$

$$\therefore \quad q_0 = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2 \times 7{,}500 \times 310}{3}} = 1{,}245 \text{ units.}$$

$$n_0 = \frac{R}{q_0} = \frac{7{,}500}{1{,}245} \simeq 6 \text{ orders/year.}$$

$$t_0 = \frac{1}{n_0} = \frac{1}{6} \text{ years} = 2 \text{ months.}$$

∴ Order of 1,245 units should be placed every two months.

EXAMPLE 12.5-18

A company assembles a component which uses a part bought from an outside supplier at a cost of ₹ 5 per unit. Each month 18,000 components are produced at a steady rate throughout. There is a lead time of one month and the current practice for the company is to order 72,000 units at a time. This order is placed when the stock level falls to 48,000.

The supplier offers a 10% discount if orders are placed in lot sizes of 2,00,000 units and he is allowed a two months delivery period. If the company wanted to change to these terms, it would require additional storage which would cost ₹ 24,000 per year and would incur additional *handling charges of ₹ 0.05 per unit. The company would maintain the level of protection afforded by the use of safety stock.*

The cost of placing an order is ₹ 200 and the cost of receiving delivery of an order is ₹ 800. The cost of capital normally used for inventory decisions is 12% per annum. You are required

(*a*) *to calculate and state the annual cost following the current practice.*

(*b*) *to calculate and state the minimum annual cost if the extra storage space were available so that the discount for larger quantities could be considered.*

Solution

(*a*) Here, q = 72,000 units, C_3 = ₹ (200 + 800) = ₹ 1,000/order,

C = ₹ 5/unit, I = 0.12, R = 18,000 × 12 = 2,16,000 units / year.

Now ROL = lead time demand + safety stock = LTD + SS

∴ 48,000 = 1 × 18,000 + SS

or SS = 30,000 units.

$$\therefore \text{ Total annual cost } = \left(\frac{q}{2} + \text{SS}\right)\text{CI} + \text{C}_3 \cdot \frac{R}{q} + CR$$

$$= ₹ \left[\left(\frac{72,000}{2} + 30,000\right) \times 5 \times 0.12 + 1,000 \times \frac{2,16,000}{72,000} + 5 \times 2,16,000\right]$$

= ₹ (39,600 + 3,000 + 10,80,000) = ₹ 11,22,600.

(*b*) *If orders of E.O.Q. are placed*

$$q_0 = \sqrt{\frac{2RC_3}{CI}} = \sqrt{\frac{2 \times 2,16,000 \times 1,000}{5 \times 0.12}} = 26,833 \text{ units.}$$

$$\therefore \text{ Total annual cost } = \left(\frac{q}{2} + \text{SS}\right)\text{CI} + \text{ C}_3 \frac{R}{q} + CR$$

$$= ₹ \left[\left(\frac{26,833}{2} + 30,000\right) \times 5 \times 0.12 + 1,000 \times \frac{2,16,000}{26,833} + 5 \times 2,16,000\right]$$

= ₹ (26,050 + 8,050 + 10,80,000) = ₹ 11,14,100.

If orders of 2,00,000 units are placed

Here, C = ₹ (5×0.9) = ₹ 4.50 / unit,

q = 2,00,000 units,

LT = 2 months,

storage charges = ₹ 24,000/year,

handling charges = ₹ 0.05/unit.

∴ C_3 = ₹ (1,000 + 0.05 × 2,00,000) = ₹ 11,000/order.

Now ROL = LTD + SS.

∴ 48,000 = 2 × 18,000 + SS

or SS = 12,000 units.

$$\therefore \quad \text{Total annual cost } = \left(\frac{q}{2} + SS\right)\text{CI} + C_3 \frac{R}{q} + \text{CR} + \text{annual charges}$$

$$= ₹ \left[\left(\frac{2,00,000}{2} + 12,000\right) \times 4.50 \times 0.12 + 11,000 \times \frac{2,16,000}{2,00,000} + 4.50 \times 2,16,000 + 24,000\right]$$

= ₹ (1,12,000 × 0.54 + 11,000 × 1.08 + 4.50 × 2,16,000 + 24,000)

= ₹ (60,480 + 11,880 + 9,72,000 + 24,000) = ₹ 10,68,360.

∴ Minimum annual cost of ₹ 10,68,360 is incurred if order quantity is 2,00,000 units.

EXERCISES 12.1

1. Define inventory. What are the advantages and disadvantages of having inventories?
[P.T.U. MCA, 2010; M.B.A. Feb., 2009; B.Tech. (Mech.) 2009; P.U.B.Com. April, 2007; Pbi. U. MCA, 2001]
2. How will you control the inventories of a manufacturing organisation? Discuss the various inventory costs associated with this organisation. *[P.T.U. B.Com., 2010; B.E. (Mech.) May, 2011, 2006; P.U. M.Com., 2002; Pbi. U. MCA, 1997; C.A. (Final) 1995]*
3. Explain the following terms in inventory management :
(*i*) Carrying cost (*ii*) Shortage costs (*iii*) Ordering cost.
[P.U.B.Com. April, 2007, 2004, 2000, 1996; G.J.U. MBA Nov., 2003; P.T.U. B.Tech. (Mech.) May, 2011; 2010; May, 2006; Dayalbagh Edu. Inst. Agra M.Tech. Dec., 2012; 2007]
4. Write short notes on the following :
(*i*) Necessity for inventory control *[P.U. MBA Feb., 2009]*
(*ii*) Functions performed by inventory *[J.N.T.U. Hyderabad B.Tech. Nov., 2010; P.T.U. B.Tech. (Mech.) May, 2012; 2008]*
(*iii*) Inventory costs and their components. *[Univ. of Madras MBA April, 2012; P.T.U. MCA, 2010; B.Tech. (Mech.) Dec., 2006; GNDU B.Com., 1996, 95; NIIFT Mohali, 1999, 98]*
5. Most of the businessmen view inventory as a necessary evil. Do you agree with this? Explain.
[GNDU B.Com., 1996]
6. What are advantages of proper inventory management ? *[Mumbai U. MBA, 2010]*
7. Classify inventory. *[J.N.T.U. Hyderabad B.Tech. Nov., 2010]*
8. Derive the EOQ formula $q_0 = \sqrt{\dfrac{2C_3R}{C_1}}$, where the symbols have usual meanings. State the assumptions and limitations in this formula. *[J.N.T.U. Hyderabad B.Tech. (C.Sc.) Dec., 2011; P.T.U.B.Tech. (Mech.) Dec., 2011; Dec., 2006; P.U. M.Com. March, 2006; 2004; B.Com., 2010; Sept., 2006; April, 2007; 2004; C.A.(Final) 2003; ICWA, 1996; GNDU B.Com., 1996]*
9. Represent in the form of a table the various types of inventory control models.
[IGNOU MBA June, 2007; Dec., 2006; P.U. B.Com. March, 2006; Sept., 2005]
10. Explain economic lot size. *[P.T.U. MBA, 2009]*
11. Explain the basic characteristics of an inventory system. *[P.T.U. MBA, 2009]*
12. State the assumptions underlying the basic EOQ formula. *[P.T.U. MBA, 2008]*
13. In each of the following cases, stock is replenished instantaneously and no shortage is allowed. Find the economic lot size, the associated total cost and length of time between two orders.
(*a*) C_3 = ₹ 100, C_1 = Re. 0.05 and R = 30 units/year.
(*b*) C_3 = ₹ 50, C_1 = Re. 0.05 and R = 30 units/year.
(*c*) C_3 = ₹ 100, C_1 = Re. 0.01 and R = 40 units/year.
(*d*) C_3 = ₹ 100, C_1 = Re. 0.04 and R = 20 units/year. (*Ans.* (*a*) $q_0 = 346$, $t_0 = 11.55$, $C_0(q)$ = ₹ 17.30.)
14. In each case of exercise no. 11, determine the reorder level if lead time is 14 units.
15. A company purchases 10,000 items per year for use in its production shop. The unit cost is ₹ 10 per year, holding cost is ₹ 0.80 per month and cost of making a purchase is ₹ 200. Determine the following if no shortages are allowed :
(*i*) The optimum order quantity,
(*ii*) the optimum total yearly cost,
(*iii*) the number of orders per year,
(*iv*) the time between orders. *[NIIFT Mohali, 1998]*
16. A certain item costs ₹ 235 per ton. The monthly requirement is 5 tons and each time the stock is replenished, there is a setup cost of ₹ 1,000. The cost of carrying of inventory has been estimated at 10% of the value of the stock per year. What is the optimal order quantity?
[J.N.T.U Hyderabad B.Tech. (C.Sc.) Dec., 2011; Delhi M.Sc. (Math.) 1973]

17. The XYZ manufacturing company has determined, from an analysis of its accounting and production data for part number 625, that its cost to purchase is ₹ 36 per order and ₹ 2 per part. Its inventory carrying charges are 18% of the average inventory. The demand for this part is 10,000 units per annum. Find
(*a*) what should the economic order quantity be ?
(*b*) what is the optimal no. of days' supply per optimum order? [*Delhi M.B.A., 1975*]
(*Ans.* 1.144 units, 0.1414 year.)

18. An aircraft company uses rivets at an approximate consumption rate of 2,500 kg per year. The rivets cost ₹ 30 per kg and the company personnel estimate that it costs ₹ 130 to place an order and the inventory carrying cost is 10% per year. How frequently should orders for rivets be placed and what quantities should be ordered? [*Meerut M.Sc. (Stat.) 1976*]
(*Ans.* 466 kg, 5.3/year.)

19. Consider the inventory system with the following data in usual notations : R = 1,000 units/year, I = 0.30, P = ₹ 0.50 per unit, C_3 = ₹ 10, L = 2 years (lead time) and C_1 = IP. Determine
(*i*) optimal order quantity,
(*ii*) reorder point,
(*iii*) minimum average cost. [*Delhi, 1968*]
(*Ans.* 365 units, 2,000 units, ₹ 54.80.)

20. An eye stockist has to supply 400 units of a product every Monday to his customers. He gets the product at ₹ 50 per unit from the manufacturer. The cost of ordering and transportation from the manufacturer in ₹ 50 per order. The cost of carrying inventory is 75% per year of the cost of the product. Find
(*i*) the economic lot size,
(*ii*) the total optimal cost (including the capital cost). [*P.T.U. B.Tech. (Mech.) 2010*]
(*Ans.* (*i*) 235.5 units, (*ii*) ₹ 2,169.84/week.)

21. A computer company sells a particular type of personal computer. It costs the store ₹ 25,000 each time it places an order with the manufacturer. The annual carrying cost is ₹ 9,000. The manager estimates the annual demand of P.C.'s to be 1,200 units. Determine the optimal order quantity and the total minimum inventory cost. [*Osmania U. MBA, 2010*]
(*Ans. 81.6 units;* ₹ *7,34, 757.50.*)

22. A shopkeeper has a uniform demand of an item at the rate of 50 items per month. He buys it from a supplier at the cost of ₹ 6 per item and the cost of ordering is ₹ 10 each time. If the stock holding costs are 20% per year of the stock value, how frequently should he replenish his stock ? Suppose the supplier offers a 5% discount on orders between 200 and 900 items and a 10% discount on orders exceeding or equal to 1,000 items, can the shopkeeper reduce his costs by taking advantage of either of these discounts ? [*P.U.B.Com., 2010, 2004*]
(*Ans.* 5% discount should be accepted.)

23. A company uses 8,000 units of a product as raw material, costing ₹ 10 per unit. The administrative cost per purchase in ₹ 40. The holding costs are 28% of the average inventory. The company is following an optimal purchasing policy and places orders according to the EOQ. It has been offered a quantity discount of 10% if it purchases its entire requirement only four times a year. Should the company accept this offer ? If not, what minimum discount should the company demand ?
[*G.N.D.U. B.Com. April, 2008*]

24. The annual demand for an item is 3,200 units. The unit cost is ₹ 6 and inventory carrying charges are 25% per annum. If the cost of one procurement is ₹ 150, determine
(*i*) Economic order quantity.
(*ii*) Number of orders per year.
(*iii*) Time between orders.
(*iv*) The optimal cost. [*G.N.D.U.B.Com. April, 2007*]

25. A company requires 10,000 units of materials per annum. The cost per order is ₹ 50 and the storage cost is ₹ 5 per unit of average inventory. What quantity should be ordered to minimize the total cost ? Also find out the total variable inventory cost. [*G.N.D.U.B.Com. April, 2010*]

26. A factory requires 16,000 units of a part, costing ₹ 2 per unit. If carrying cost is 8% of the inventory value and ordering cost is ₹ 40 per order, calculate EOQ, optimum number of orders per annum, the total holding cost, the reorder level and the length of inventory cycle. What would be the saving if ordering is done as per EOQ instead of thrice per year ? [*G.N.D.U.B.Com. April, 2002*]
(*Ans.* 2,829 units, 5.66 orders, ₹ 226.27, 64.5 days; ₹ 94.)

27. The production department for a company requires 3,600 kg of raw material for manufacturing a particular item per year. It has estimated that the cost of placing an order is ₹ 36 and the cost of carrying inventory is 25% of the investment in the inventories. The price is ₹ 10 per kg. The purchase manager wishes to determine an ordering policy for raw material.
[*R.T.M. Nagpur U.B.E. (Mech.) 2011; J.N.T.U. Hyderabad B.Tech. (C.Sc) Dec., 2011*]

28. Derive the expression for EOQ of Wilson - Harris Inventory Model. A company uses 10,000 units per year of an item. The purchase price is ₹ 1 per item. Ordering cost is ₹ 25 per order. Carrying cost per year is 12% of the inventory value. Find EOQ and the number of orders per year. If the lead time is 4 weeks and assuming 50 working weeks per year, find the reorder point.
[*J.N.T.U. Hyderabad B.Tech. (C.Sc.) Dec., 2011*]

29. A company uses 24,000 units of a raw material which costs ₹ 12.50 per unit. Placing each order costs ₹ 22.50 and the carrying cost is 5.4% per year of the average inventory. Find the economic order quantity and the total inventory cost (including the cost of the material). Should the company accept the offer made by the supplier of a discount of 5% on the cost price on a single order of 24,000 units ?
[*J.N.T.U. Hyderabad B.Tech. (C.Sc.) Dec., 2011; April, 2011; Nov., 2010*]

30. A product 'X' is purchased by a company from outside suppliers. The consumption is 10,000 units per year. The cost of the item is ₹ 5 per unit and the ordering cost is estimated to be ₹ 100 per order. The cost of carrying inventory is 25%. If the consumption rate is uniform, determine the economic purchasing quantity. [*J.N.T.U. Hyderabad B.Tech. May, 2011*]

31. (*a*) Classify inventory.
(*b*) Find the economic lot size, that associates with total cost and the length of time between two orders, given that the setup cost is ₹ 100, daily holding cost per unit of inventory is 5 paise and daily demand is approximately 30 units. [*J.N.T.U. Hyderabad B.Tech. Nov., 2010*]

32. Find the economic order quantity for the data given below :

Annual demand :	2,400 units
Unit cost of the item :	₹ 2 per units
Ordering cost :	₹ 30 per order
Inventory holding cost :	20% of the unit cost.

[*J.N.T.U. Hyderabad B.Tech. May, 2009*]

33. A company, which raises speciality cattle for low-fat beef, feeds its cattle on a strictly monitored schedule. The ranch uses 12,000 pounds of grain a year. The cost of ordering is $ 10 per order, and the cost of carrying inventory is $ 0.06 per pound per annum. What is the economic order quantity ? What is the minimum total inventory cost ? How many times should the ranch order grain during the year ? The ranch operates 360 days a year. What is the order cycle time in days between orders ? Is carrying cost greater than, equal to, or less than the ordering cost ?
[*Dayalbagh Edu. Inst. Agra B.Sc. Engg. & M.Tech. Dec., 2011*]
(*Ans.* 2,000 pounds, $ 120, 6/year, 60 days, equal.)

34. Determine the EOQ for the following data :

Annual demand	= ₹ 1,00,000 worth of an item
Ordering cost	= 1% of order value
Carrying cost	= 20% of average inventory value.

[*P.U. B.Com., 2002*]
(*Ans.* ₹ 10,000.)

[Hint. $q_0 = \sqrt{\dfrac{2C_3R}{C_1}} = \sqrt{\dfrac{2R \cdot \dfrac{Cq_0}{100}}{CI}} = \sqrt{\dfrac{2 \times 1,00,000 \times \dfrac{q_0}{100}}{20/100}} = \sqrt{10,000\, q_0}$

$\therefore \quad q_0^2 = 10,000\, q_0$ or $q_0 =$ ₹ 10,000 (order value).]

35. An item is used at a uniform rate of 50,000 units per year. No shortage is allowed and delivery is at an infinite rate. The ordering, receiving and handling cost is ₹ 13 per order, while inspection cost is ₹ 12 per order. Interest costs ₹ 0.056 and obsolescence costs ₹ 0.004 respectively per year for each item actually held in inventory plus ₹ 0.02 per year per unit based on the maximum number of units in inventory. Calculate EOQ. If lead time is 20 days, find reorder level. [*IGNOU MCA, 2003*]
(*Ans.* 5,590 units; 2,778 units.)

36. A motor company purchases 9,000 motor spare parts for its annual requirement, ordering one month usage at a time. Each spare part costs ₹ 20. The ordering cost/order is ₹ 15 and the carrying charges are 15% of the average inventory per year.
You have been asked to suggest more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year?
[*Dayalbagh Edu. Inst. Agra MBA May, 2013; G.N.D.U. B.Com. April, 2006; P.T.U.B.Tech. (Mech.) May, 2006; P.U. B.Com., 2002, 1999*]
(*Ans.* Purchase 300 parts/ order; Annual savings ₹ 405.)

37. Thompson Tooling has a department of Defence Contract for 1,50,000 bushings a year. Thompson orders the metal for bushings in lots of 40,000 units from a supplier. It costs \$ 40 to place an order and the estimated carrying charge is 20% of the unit cost, which is \$ 0.15. Thompson wants to know what per cent their order quantity varies from optimal and what this variation is costing them, if any. Hence calculate the extra cost incurred by Thompson for not ordering in optimal lots. [*R.C.C. CHD., 2002*]
(*Ans.* 50% ; \$ 150.)

38. A manufacturer uses ₹ 25,000 worth of an item during the year. He has estimated the ordering cost as ₹ 50 per order and carrying costs as 12.5% of average inventory value. Find the optimal order size, number of orders per year, time period per order and total cost. [*IGNOU MBA, 2002*]
[**Hint.** Here R = ₹ 25,000/year, C_3 = ₹ 50 / order, I = 12.5% / year.

$$\therefore \quad q_0 = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2.CR.C_3}{CC_1}} = \sqrt{\frac{2.CR.C_3}{C.CI}} = \frac{1}{C}\sqrt{\frac{2.CR.C_3}{I}}.$$

$$\therefore \quad Cq_0 = \sqrt{\frac{2.CR.C_3}{I}} = \sqrt{\frac{2 \times 25{,}000 \times 50}{12.5/100}} = ₹\ 3{,}162.\ \text{(order value)}$$

$$n_0 = \frac{R}{q_0} = \frac{25{,}000}{3{,}162} = 7.9/\text{year}.$$

$$t_0 = \frac{1}{n_0} = \frac{12}{7.9} = 1.52 \text{ months}.$$

$$C_0 = \sqrt{2\,R\,C_3\,C_1} = \sqrt{2 \times 25{,}000 \times 50 \times \frac{12.5}{100}} = ₹\ 559.]$$

39. Frequently the manager of an inventory system must make the decision whether to purchase or manufacture an item. Suppose that an item may be purchased for ₹ 25 per unit and manufactured at the rate of 10,000 units per year for ₹ 22 each. However, if purchased, the ordering cost is only ₹ 5 compared to a setup cost of ₹ 50 if it is manufactured. The yearly demand for this item is 2,500 units and the inventory holding cost rate is 10 per cent. What suggestion (make or buy) would you make to the manager in this case? [*R.C.C.CHD. MCA, 2001*]
(*Ans.* C_P = ₹ 62,750; C_M = ₹ 55,642; make.)

40. A company uses 1,200 units per month of an electronic component each costing ₹ 2. Placing each order costs ₹ 50 and the carrying cost is 6% per year of the average inventory.
(*i*) Find EOQ.
(*ii*) If the company gets 5% discount on single order, should it accept the discount offer?
(*iii*) Find discount percentage which matches EOQ ordering for a single order.
[*P.U. B.Com. April, 2003*]

41. A factory requires 1,500 units of an item per month, each costing ₹ 27. The cost per order is ₹ 150 and the inventory carrying charges work out to be 20% of the average inventory. Find out the EOQ and the number of orders per year. Would you accept a 2% price discount on a minimum supply quantity of 1,200 units? Compare the total cost in both the cases. [*DOEACC, 1997*]

42. In a warehouse the independent demand for a commonly used bolt is 500 units per month. The ordering cost is ₹ 30 per order placed. The carrying cost is 25% per year and each bolt costs 50 paise.
(*i*) Calculate the lot size for this product. State assumptions made in using formula.
(*ii*) How often is this product to be purchased?
(*iii*) If ordering cost is reduced to ₹ 5 per order, how will it change lot size and frequency of purchasing ? [*IGNOU MBA June, 1998*]
(*Ans.* (*a*) (*i*) 1,697 units (*ii*) 3.5/year; 3.4 months. (*b*) (*i*) 693 units (*ii*) 8.66/ year; 1.38 months.)

43. A company works 50 weeks in a year. For a certain part, included in the assembly of several parts, there is an annual demand of 10,000 units. This part may be available from either an outside supplier or a subsidiary company. The following data relating to the part are given:

		From outside supplier (₹)	*From subsidiary company (₹)*
Purchase price/unit	:	12	13
Cost of placing an order	:	10	10
Cost of receiving an order	:	20	15
Storage and all carrying costs, including capital cost/unit/year	:	2	2

(*i*) What purchase quantity from which source would you recommend ?
(*ii*) What would be the minimum total cost ? [*ICWA June, 1989*]
(*Ans.* (*i*) 548 units/order from outside supplier. (*ii*) ₹ 1,21,096.)

44. A purchase manager places order each time for a lot of 500 numbers of a particular item. From the available data the following results are obtained:

Inventory carrying cost = 40%,
ordering cost per order = ₹ 600,
cost per unit = ₹ 50,
annual demand = 1,000 units.

Find out the loss to the organisation due to his ordering policy. [*ICWA (Final) Dec., 1989*]
(*Ans.* ₹ 1,301.)

45. (*a*) What are the different costs associated with inventory control systems ? How are they obtained? Give both the analytical and graphical methods of determining economic order quantity.
(*b*) For one of the bought-out items, the following are the relevant data:

Ordering cost = ₹ 500, holding cost = 40%,
cost per item = ₹ 100, annual demand = 1,000.

The purchase manager placed 5 orders of equal quantity in one year in order to avail the discount of 5% on the cost of the items. Work out the gain or loss to the organisation due to his ordering policy for this item. [*ICWA (Final) Dec., 1990*]
(*Ans.* Gain of ₹ 5,024.55.)

46. The Inventory Company, after an analysis of its accounting and production records has determined that it uses ₹ 36,000 per year of a component part purchased at ₹ 18 per part. The purchasing cost is ₹ 40 per order and its annual inventory carrying charges are $16\frac{2}{3}$% of the average inventory.

(*i*) Determine the most economic quantity to order at one time.
(*ii*) Determine the most economic number of times to order per year.
(*iii*) Determine the average days' supply for ordering the most economic quantity (year = 365 days).
(*iv*) Determine the optimum amount of rupees' worth of the units per order. [*C.A. (Final) Dec., 1990*]
(*Ans.* (*i*) 231 units (*ii*) 8.66 (*iii*) 42.15 days (*iv*) ₹ 4,158.)

47. Given the following data for a particular inventory item :

Usage	:	500 units/week
Ordering cost	:	₹ 40/order
Carrying cost	:	₹ 1/unit/week
Lead time	:	3 weeks
Price	:	₹ 50/unit.

Determine the following :
(*i*) E.O.Q. (*ii*) Cycle time (*iii*) Reorder point
(*iv*) Average inventory level
(*v*) Average weekly ordering cost
(*vi*) Average weekly carrying cost
(*vii*) Average weekly total cost including the cost of the item.
[*Dayalbagh Edu. Inst. Agra M.B.A. Dec., 2012, 2007*]
(*Ans.* 200 units, 0.4 days, 1,500 units, 100 units, ₹ 100, ₹ 100, ₹ 25,200.)

12.5-2 Model 1 (b) (Demand Rate Non-uniform, Replenishment Rate Infinite)

In this model all assumptions are same as in model 1 (*a*) with the exception that instead of uniform demand rate R, we are given some total demand D, to be satisfied during some long time period T. Thus demand rates are different in different order cycles.

Let q be the fixed quantity ordered each time the order is placed.

Number of orders, $\quad N = \frac{D}{q}.$

If t_1 is the time interval between orders 1 and 2, t_2 the time interval between orders 2 and 3 and so on, the total time T will be

$$= t_1 + t_2 + \ldots + t_n \qquad \ldots(12.8)$$

This model is illustrated schematically in figure 12.3.

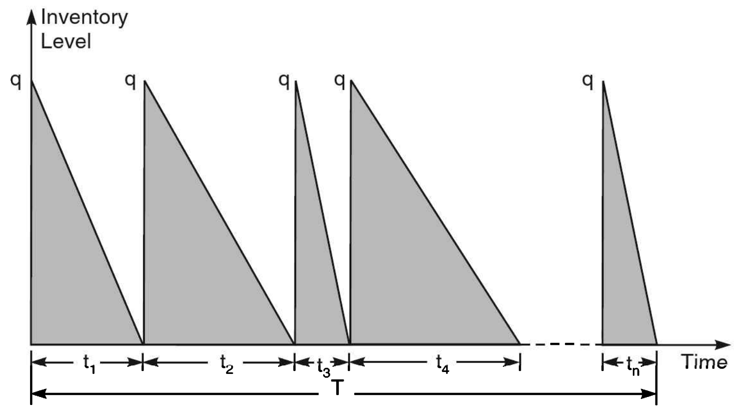

Fig. 12.3. Inventory situation for different rates of demand in different cycles.

Holding costs for time period T will be

$$= \left(\frac{1}{2}qt_1\right)C_1 + \left(\frac{1}{2}qt_2\right)C_1 \ldots + \left(\frac{1}{2}qt_n\right)C_1$$

$$= \frac{1}{2}q\,C_1\,(t_1 + t_2 + \ldots + t_n) = \frac{1}{2}q\,C_1 T,$$

and the ordering cost will be

$$= C_3 \,.\, N$$

$$= C_3 \,.\, \frac{D}{q}, \text{ where } C_3 \text{ is the ordering cost per order.}$$

$\therefore$ Total cost equation for fixed order size q will be

$$C(q) = \frac{1}{2}q\,C_1 T + C_3 \frac{D}{q}. \qquad \ldots(12.9)$$

For minimum cost, $\frac{d}{dq}[C(q)] = 0$ and $\frac{d^2}{dq^2}[C(q)]$ should be positive.

$\therefore$ Differentiating equation (12.9) w.r.t. q,

$$\frac{d}{dq}[C(q)] = \frac{1}{2}C_1 T - C_3 \frac{D}{q^2} = 0 \qquad \therefore\ q = \sqrt{\frac{2C_3 D}{C_1 T}} = \sqrt{\frac{2C_3 \,.\, D/T}{C_1}}$$

and $\quad \frac{d^2}{dq^2}[C(q)] = \frac{2C_3 D}{q^2}$, which is positive for the value of q given above.

$\therefore$ Optimal lot size, $\quad q_0 = \sqrt{\dfrac{2C_3(D/T)}{C_1}}$, ...(12.10)

and minimum total cost,

$$C_0(q) = \frac{1}{2}C_1 . T . \sqrt{\frac{2C_3(D/T)}{C_1}} + C_3 D \sqrt{\frac{C_1}{2C_3(D/T)}}$$

$$= \sqrt{2C_1C_3(D/T)}. \qquad ...(12.11)$$

From equations (12.10) and (12.11) we find that results for this model can be obtained if the uniform demand rate R in model 1 (*a*) is replaced by *average* demand rate D/T.

12.5-3 Model 1 (c) (Demand Rate Uniform, Replenishment or Production Rate Finite)

In the classical EOQ model the replenishment rate was assumed to be infinite; the entire quantity ordered was delivered in a single lot. This is possible only for ***bought-out items*** and is simply unthinkable for ***made-in items***. Such items are produced by the production department of the organisation at a constant rate and are also supplied to the customers at a constant rate. When the production starts, a fixed number of units are supposed to be added to inventory each day till the production run is completed; simultaneously, the items will be demanded at a constant rate, as stipulated earlier. Obviously, the rate at which they are produced has to be higher than the consumption rate, for only then can there be the built-up of inventory.

It is assumed that run sizes are constant and that a new run will be started whenever inventory is zero. Let

R = number of items required per unit time,
K = number of items produced per unit time,
C_1 = cost of holding per item per unit time,
C_3 = cost of setting up a production run,
q = number of items produced per run, $q = Rt$,
t = interval between runs.

Figure 12.4 shows the variation of inventory with time.

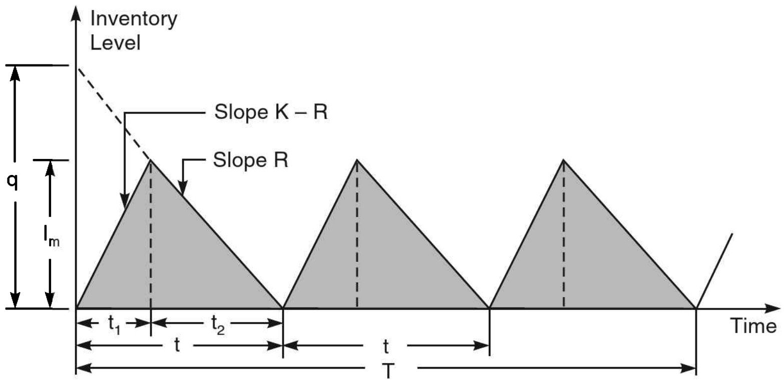

Fig. 12.4. Inventory situation with finite rate of production.

Here, each production run of length t consists of two parts t_1 and t_2, where

(*i*) t_1 is the time during which the stock is building up at a constant rate of K - R units per unit time,

(*ii*) t_2 is the time during which there is no production (or supply or replenishment) and inventory is decreasing at a constant demand rate R per unit time.

Let I_m be the maximum inventory available at the end of time t_1 which is expected to be consumed during the remaining period t_2 at the demand rate R.

Then $$I_m = (K - R)\, t_1 \quad \text{or} \quad t_1 = \frac{I_m}{K - R}. \qquad ...(12.12)$$

Now the total quantity produced during time t_1 is q and the quantity consumed during the same period is Rt_1, therefore the remaining quantity available at the end of time t_1 is

$$I_m = q - Rt_1 \qquad ...(12.13)$$

$$= q - \frac{R \,.\, I_m}{K - R}$$

$$\therefore \quad I_m\left(1 + \frac{R}{K - R}\right) = q \quad \text{or} \quad I_m = \frac{K - R}{K} q. \qquad ...(12.14)$$

Now holding cost per *production run i.e., for time period t*

$$= \frac{1}{2} \,.\, I_m \,.\, t \,.\, C_1$$

and setup cost *per production run* = C_3.

$\therefore$ Total average cost *per unit time*,

$$C\,(I_m, t) = \frac{1}{2}\, I_m\, C_1 + C_3/t \qquad ...(12.15)$$

or $$C(q, t) = \frac{1}{2}\left(\frac{K - R}{K} \,.\, q\right) C_1 + \frac{C_3}{t}$$

or $$C(q) = \frac{1}{2}\left(\frac{K - R}{K} \,.\, q\right) C_1 + \frac{C_3}{q/R} \qquad (\because \; q = Rt)$$

$$= \frac{1}{2} \frac{(K - R)}{K} C_1 \,.\, q + \frac{C_3 R}{q}. \qquad ...(12.16)$$

For minimum value of $C(q)$,

$$\frac{d}{dq}[C(q)] = \frac{1}{2}\frac{K - R}{K} \,.\, C_1 - \frac{C_3 R}{q^2} = 0, \text{ which gives}$$

$$q = \sqrt{\frac{2C_3RK}{(K - R)C_1}} = \sqrt{\frac{2C_3}{C_1} \,.\, \frac{RK}{K - R}} = \sqrt{\frac{K}{K - R}} \,.\, \sqrt{\frac{2C_3R}{C_1}}$$

and $$\frac{d^2}{dq^2}[C(q)] = \frac{2C_3R}{q^3}, \text{ which is positive.}$$

Now, when the replenishment rate is finite, the economic order quantity q_0 is called *optimum lot size*.

$\therefore$ Optimum lot size, $$q_0 = \sqrt{\frac{2C_3}{C_1} \,.\, \frac{RK}{K - R}} = \sqrt{\frac{K}{K - R}} \,.\, \sqrt{\frac{2C_3R}{C_1}}. \qquad ...(12.17)$$

Optimum time interval, $$t_0 = \frac{q_0}{R} = \sqrt{\frac{K}{K - R}} \,.\, \sqrt{\frac{2C_3}{C_1R}}. \qquad ...(12.18)$$

Optimum average cost/unit time, C_0

$$= \frac{1}{2} \frac{(K - R)}{K} C_1 \,.\, \sqrt{\frac{2C_3}{C_1} \,.\, \frac{RK}{K - R}} + C_3R \sqrt{\frac{C_1(K - R)}{2C_3 \,.\, RK}}$$

$$= \sqrt{2C_1C_3R \,.\, \frac{K - R}{K}} = \sqrt{\frac{K - R}{K}} \,.\, \sqrt{2C_1C_3R}. \qquad ...(12.19)$$

Particular Cases: (*i*) If K = R, then $C_0 = 0$, which means that there will be no holding cost and no setup cost

(*ii*) If $K = \infty$, *i.e.*, production rate is infinite, this model reduces to model 1 (*a*).

EXAMPLE 12.5-17

A company has a demand of 12,000 units/year for an item and it can produce 2,000 such items per month. The cost of one setup is ₹ 400 and the holding cost/unit/month is ₹ 0.15. Find the optimum lot size and the total cost per year, assuming the cost of 1 unit as ₹ 4. Also find the maximum inventory, manufacturing time and total time.

[*J.N.T.U. Hyderabad B.Tech. April, 2011; Mumbai U. MBA, 2010; Nagpur U. M.B.A., 1998; NIIFT Mohali, 1999*]

Solution

$$R = 12{,}000 \text{ units/year},$$
$$K = 2{,}000 \times 12 = 24{,}000 \text{ units/year},$$
$$C_3 = ₹\ 400/\text{setup},$$
$$C_1 = ₹\ 0.15 \times 12 = ₹\ 1.80/\text{unit/year}.$$

(*i*)
$$q_0 = \sqrt{\frac{K}{K-R}} \cdot \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{24{,}000}{24{,}000-12{,}000}} \cdot \sqrt{\frac{2\times 400\times 12{,}000}{1.80}}$$
$$= 3{,}264 \text{ units/setup}.$$

(*ii*)
$$C_0 = 12{,}000 \times 4 + \sqrt{2C_1C_3R} \cdot \sqrt{\frac{K-R}{K}}$$
$$= 48{,}000 + \sqrt{2\times 1.80\times 400\times 12{,}000\times \frac{12{,}000}{24{,}000}}$$
$$= 48{,}000 + 2{,}940 = ₹\ 50{,}940/\text{ year}.$$

(*iii*) Max. inventory
$$I_{m_0} = \frac{K-R}{K} \cdot q_0 = \frac{24{,}000-12{,}000}{24{,}000} \times 3{,}264 = 1{,}632 \text{ units}.$$

(*iv*) Manufacturing time
$$t_1 = \frac{I_{m0}}{K-R} = \frac{1{,}632}{12{,}000} = 0.136 \text{ years}.$$

(*v*) Total time
$$t_0 = \frac{q_0}{R} = \frac{3{,}264}{12{,}000} = 0.272 \text{ years}.$$

EXAMPLE 12.5-18

An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the setup cost is ₹ 100 per setup and holding cost is ₹ 0.01 per unit of item per day, find the economic lot size for one run, assuming that shortages are not permitted. Also find the time of cycle and minimum total cost for one run. [*Bombay B.Sc. (Stat.) 1975; Pune M.B.A., 1983*]

Solution

Here, K = 50 items per day, R = 25 items per day,

C_3 = ₹ 100 per setup, C_1 = ₹ 0.01 per unit per day.

$$\therefore \quad q_0 = \sqrt{\frac{2C_3R}{C_1}} \cdot \sqrt{\frac{K}{K-R}} = \sqrt{\frac{2\times 100\times 25}{0.01}\times\frac{50}{25}} = 1{,}000 \text{ items}.$$

$$t_0 = \frac{q_0}{R} = \sqrt{\frac{2C_3}{C_1R}} \cdot \sqrt{\frac{K}{K-R}} = \sqrt{\frac{2\times 100}{0.01\times 25}\times\frac{50}{25}} = 40 \text{ days}.$$

$$\text{Minimum daily cost} = \sqrt{2C_1C_3R} \cdot \sqrt{\frac{K-R}{K}} = ₹\ \sqrt{2\times 0.01\times 100\times 25\times\frac{25}{50}} = ₹\ 5.$$

∴ Minimum total cost per run = ₹ 5 × 40 = ₹ 200.

12.5-4 Model 2 (a) (Demand Rate Uniform, Replenishment Rate Infinite, Shortages Allowed)

In the earlier models the shortages and hence back ordering was not permitted. Hence the models involved a trade-off between carrying cost and ordering cost. However, in actual practice shortages may take place and hence shortage cost also needs to be considered. Shortages may also be allowed to derive certain advantages. One advantage of allowing shortages is to increase the cycle time, and hence spreading the ordering (or setup) cost over a long period, thereby reducing the total ordering cost over the planning period. Another advantage is decreased net stock in inventory, resulting in reduced inventory carrying cost.

This model is just the extension of model 1 (*a*), allowing shortages. Let

R = number of items required per unit time *i.e.,* demand rate,
C_1 = cost of holding the item per unit time,
C_2 = shortage cost per item per unit time,
C_3 = ordering cost/order,
q = number of items ordered in one order,
q = Rt,
t = interval between orders,
I_m = number of items that form inventory at the beginning of time interval t.

Lead time is assumed to be zero. Figure 12.5 shows the variation of inventory with time.

It is assumed that when shortages occur and customers are not served immediately, they leave their orders with the supplier and these back orders are filled as soon as the stock is received, such as point *D* in the Fig. Out of the total quantity q received, all shortages equal to an amount *S* are first taken care and the remaining quantity $I_m = q - s$ forms the inventory for the next cycle.

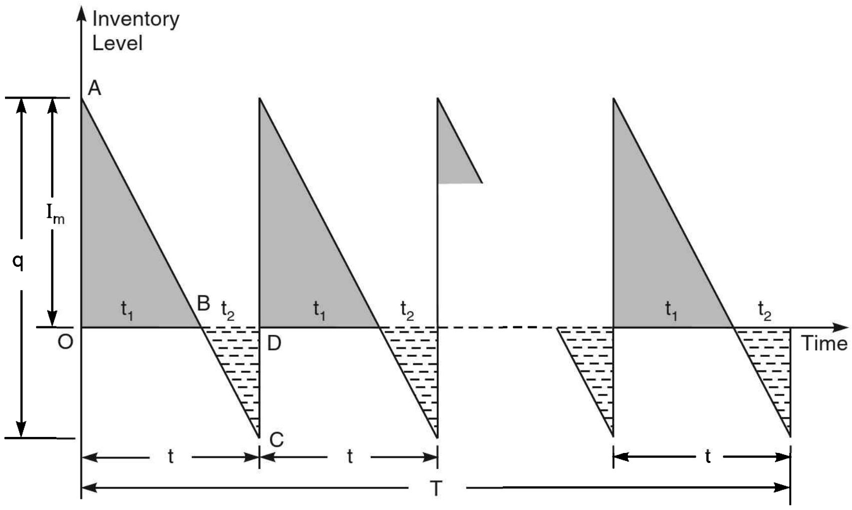

Fig. 12.5. Inventory situation for model 2 (a).

Here, the total time period T is divided into n equal time intervals, each of value t. The time interval t is further divided into two parts t_1 and t_2.

$$i.e., \qquad t = t_1 + t_2,$$

where t_1 is the time interval during which items are drawn from inventory and t_2 is the interval during which the items are not filled. Using the relationship of similar triangles,

$$\frac{t_1}{t} = \frac{I_m}{q}. \qquad \therefore\ t_1 = \frac{I_m \cdot t}{q},$$

and $\dfrac{t_2}{t} = \dfrac{q - I_m}{q}$. $\therefore\ t_2 = \dfrac{q - I_m}{q} \cdot t$.

Now total inventory during time t = area of ΔOAB = $\frac{1}{2}\, I_m \cdot t_1$.

$\therefore$ Inventory holding cost during time $t = \frac{1}{2}\, C_1\, I_m \cdot t_1$.

Similarly, total shortage during time t = area of ΔBCD = $\frac{1}{2}(q - I_m)\, t_2$.

$\therefore$ Shortage cost during time $t = \frac{1}{2}\, C_2\, (q - I_m)\, t_2$,

and ordering cost during time $t = C_3$.

$\therefore$ Total cost during time $t = \frac{1}{2}\, C_1\, I_m\, t_1 + \frac{1}{2}\, C_2\, (q - I_m)\, t_2 + C_3$

or total average cost per unit time,

$$C(I_m, t) = \frac{1}{t}\left[\frac{1}{2}C_1 I_m t_1 + \frac{1}{2}C_2 (q - I_m) t_2\right] + \frac{C_3}{t} \qquad \text{...(12.20)}$$

$$= \frac{1}{t}\left[\frac{1}{2}C_1 I_m \cdot \frac{I_m \cdot t}{q} + \frac{1}{2}C_2 (q - I_m) \cdot \frac{q - I_m}{q} \cdot t\right] + \frac{C_3}{t}.$$

$$\therefore \quad C(I_m, q) = \frac{1}{2}C_1 \frac{I_m^2}{q} + \frac{1}{2}C_2 \cdot \frac{(q - I_m)^2}{q} + \frac{C_3 \cdot R}{q}. \qquad \text{...(12.21)}$$

Total average cost per unit time C (I_m, q) being a function of two variables I_m and q, has to be partially differentiated w.r.t. I_m and q separately and then put equal to zero.

i.e., $\dfrac{[\partial C(I_m, q)]}{\partial I_m} = 0$, which gives

$$\frac{1}{2}C_1 \cdot \frac{2I_m}{q} + \frac{1}{2}C_2 \cdot \frac{2(q - I_m)}{q} \cdot (-1) + 0 = 0$$

or $\dfrac{C_1}{q} I_m - \dfrac{C_2}{q}(q - I_m) = 0$

or $\dfrac{C_1 + C_2}{q} \cdot I_m = C_2$ or $I_m = \dfrac{C_2}{C_1 + C_2} \cdot q$

and $\dfrac{\partial^2}{\partial I_m^2}[C(I_m, q)] = \dfrac{C_1}{q} + \dfrac{C_2}{q} = \dfrac{C_1 + C_2}{q}$, which is positive.

$$\therefore \text{ Optimum value of } I_m,\ I_{m0} = \frac{C_2}{C_1 + C_2} q. \qquad \text{...(12.22)}$$

Similarly, $\dfrac{\partial}{\partial q}[C(I_m, q)] = 0$, which gives

$$\frac{1}{2}C_1 I_m^2\left(-\frac{1}{q^2}\right) + \frac{1}{2}C_2 \cdot \frac{q \cdot 2 \cdot (q - I_m) - (q - I_m)^2 \cdot 1}{q^2} - \frac{C_3 R}{q^2} = 0$$

or $C_1 I_m^2 - C_2 \{(q - I_m) \cdot (2q - q + I_m)\} + 2\, C_3\, R = 0$

or $C_1 I_m^2 - C_2 \cdot (q^2 - I_m^2) + 2C_3 R = 0$

or $C_2 q^2 = (C_1 + C_2)\, I_m^2 + 2C_3 R$

or $$C_2 q^2 = (C_1 + C_2) \,.\, \frac{C_2^2}{(C_1 + C_2)^2} \,.\, q^2 + 2C_3R = \frac{C_2^2}{C_1 + C_2} \,.\, q^2 + 2C_3R$$

or $$\left(C_2 - \frac{C_2^2}{C_1 + C_2}\right) q^2 = 2C_3R$$

or $$q^2 = \frac{C_1 + C_2}{C_1 C_2} \,.\, 2C_3R \quad \text{or} \quad q = \sqrt{\frac{C_1 + C_2}{C_1 C_2}} \,.\, \sqrt{2C\ R},$$

and it can be proved that $\frac{\partial^2}{\partial q^2}[C(I_m, q)]$ is positive.

∴ Optimal value of q,

$$q_0 = \sqrt{\frac{C_1 + C_2}{C_1 C_2}} \,.\, \sqrt{2C_3R} = \sqrt{\frac{C_1 + C_2}{C_2}} \,.\, \sqrt{\frac{2C_3R}{C_1}}. \qquad \text{...(12.23)}$$

∴ From equation (12.22),

$$I_{m0} = \sqrt{\frac{C_2}{C_1 (C_1 + C_2)}} \,.\, \sqrt{2C_3R} = \sqrt{\frac{C_2}{C_1 + C_2}} \,.\, \sqrt{\frac{2C_3R}{C_1}}. \qquad \text{...(12.24)}$$

Substituting values of I_{m0} and q_0 in equation (12.21), we get the minimum average cost per unit time as

$$C_0(I_m, q) = \frac{C_1}{2}\left\{\frac{C_2}{C_1 + C_2}\right\}^2 . \, q + \frac{C_2}{2} \,.\, \frac{\left\{q - \frac{c_2}{c_1 + c_2} q\right\}^2}{q} + C_3 \frac{R}{q}$$

$$= \frac{C_1}{2} \,.\, \frac{C_2^2}{(C_1 + C_2)^2} \,.\, q + \frac{C_2}{2} \,.\, \frac{C_1^2}{(C_1 + C_2)^2} \,.\, q + C_3 \,.\, \frac{R}{q}$$

$$= \frac{C_1 C_2}{2 \,.\, (C_1 + C_2)^2} \,.\, q \,.\, (C_1 + C_2) + C_3 \,.\, \frac{R}{q}$$

$$= \frac{C_1 C_2}{2 \,.\, (C_1 + C_2)} \,.\, \sqrt{\frac{C_1 + C_2}{C_1 C_2}} \,.\, \sqrt{2C_3R} + C_3R \,.\, \frac{\sqrt{C_1 C_2}}{\sqrt{C_1 + C_2}} \,.\, \frac{1}{\sqrt{2C_3R}}$$

$$= \frac{1}{\sqrt{2}} \sqrt{\frac{C_1 C_2}{C_1 + C_2}} \,.\, \sqrt{C_3R} + \frac{1}{\sqrt{2}} \sqrt{\frac{C_1 C_2}{C_1 + C_2}} \,.\, \sqrt{C_3R}.$$

∴ $$C_0(I_m, q) = \sqrt{\frac{C_1 C_2}{C_1 + C_2}} \,.\, \sqrt{2C_3R}$$

$$= \sqrt{\frac{C_2}{C_1 + C_2}} \sqrt{2C_1C_3R} \qquad \text{...(12.25)}$$

Optimal time interval t between runs is given by

$$t_0 = \frac{q_0}{R} = \sqrt{\frac{C_1 + C_2}{C_1 C_2}} \,.\, \sqrt{\frac{2C_3}{R}}$$

$$= \sqrt{\frac{C_1 + C_2}{C_2}} \,.\, \sqrt{\frac{2C_3}{C_1 R}} \qquad \text{...(12.26)}$$

Particular cases (*i*) If shortages arc prohibited *i.e.*, $C_2 = \infty$, equations (12.23), (12.25) and (12.26) reduce to equations (12.3), (12.4) and (12.2) for model 1 (*a*), which must be true since model 1 (*a*) is a special case of model 2(*a*).

(*ii*) If $C_2 \neq \infty$ $\sqrt{\dfrac{C_2}{C_1 + C_2}} \cdot \sqrt{2C_1C_3R}$ from equation (12.25) < $\sqrt{2C_1C_3R}$ from equation (12.4).

∴ Total expected costs associated with policy of model 2 (*a*) are $\sqrt{\dfrac{C_2}{C_1 + C_2}}$ of the costs associated with policy of model 1 (*a*).

12.5-5 Model 2 (b) (Demand Rate Uniform, Replenishment Rate Infinite, Shortages Allowed, Time Interval Fixed)

Time interval t is fixed which means that inventory is to be replenished after every fixed time t. All other assumptions of model 2 (*a*) hold good.

Here, $t_1 = \dfrac{I_m \cdot t}{q}$, and $t_2 = \dfrac{q - I_m}{q} \cdot t$.

Total inventory during time $t = \dfrac{1}{2} I_m t_1$.

∴ Total inventory holding cost during time $t = \dfrac{1}{2} C_1 I_m t_1$.

Similarly, total shortage during time $t = \dfrac{1}{2} \cdot (q - I_m) t_2$.

∴ Total shortage cost during time $t = \dfrac{1}{2} C_2 \cdot (q - 1_m) t_2$.

Now ordering cost C_3 and time interval t are both constant, hence the average cost per unit time $\dfrac{C_3}{t}$ is also constant and hence is not to be considered.

∴ Total average cost per unit time,

$$C(I_m) = \frac{1}{t}\left[\frac{1}{2}C_1 I_m t_1 + \frac{1}{2}C_2 (q - I_m) t_2\right]$$

$$= \frac{1}{t}\left[\frac{1}{2}C_1 I_m \cdot \frac{I_m t}{q} + \frac{1}{2} C_2 (q - I_m) \cdot \frac{q - I_m}{q} \cdot t\right].$$

$$\therefore \quad C(I_m) = \left[\frac{C_1}{2q} \cdot I_m^2 + \frac{C_2}{2q}(q - I_m)^2\right]. \quad \ldots(12.27)$$

Now $C(I_m)$ will be optimal if $\dfrac{d}{d I_m}[C(I_m)] = 0$ and $\dfrac{d^2}{d I_m^2}[C(I_m)]$ is positive.

$\dfrac{d}{d I_m}[C(I_m)] = 0$ gives

$$\frac{C_1}{2q} \cdot 2I_m + \frac{C_2}{2q} \cdot 2(q - I_m)(-1) = 0$$

or $\quad C_1 I_m - C_2(q - I_m) = 0$

or $\quad I_m = \dfrac{C_2}{C_1 + C_2} \cdot q,$

and $\quad \dfrac{d^2}{d I_m^2}[C(I_m)] = \dfrac{C_1 + C_2}{q}$, which is positive.

∴ Optimum order quantity is given by

$$\left.\begin{aligned} I_{m0} &= \frac{C_2}{C_1 + C_2} \cdot q \\ &= \frac{C_2}{C_1 + C_2} Rt. \end{aligned}\right] \qquad \text{...(12.28)}$$

The minimum average cost per unit time from equation (12.27) is given by

$$\begin{aligned} C_0(I_m) &= \frac{C_1}{2q} \cdot \left(\frac{C_2}{C_1 + C_2} \right)^2 \cdot q^2 + \frac{C_2}{2q} \cdot \left(q - \frac{C_2}{C_1 + C_2} \cdot q \right)^2 \\ &= \frac{1}{2} C_1 q \cdot \left(\frac{C_2}{C_1 + C_2} \right)^2 + \frac{1}{2} C_2 q \cdot \left(\frac{C_1}{C_1 + C_2} \right)^2 \\ &= \frac{1}{2} \cdot \frac{C_1 C_2}{(C_1 + C_2)^2} \cdot q\,(C_1 + C_2) \end{aligned}$$

$$\left.\begin{aligned} &= \frac{1}{2} \cdot \frac{C_1 C_2}{C_1 + C_2} \cdot q \\ &= \frac{1}{2} \cdot \frac{C_1 C_2}{C_1 + C_2} \cdot Rt. \end{aligned}\right] \qquad \text{...(12.29)}$$

From equation (12.28) we observe that unless C_1 is zero, optimum order level I_m is less than the demand q during the time interval t. Therefore, it is advantageous to plan for shortages.

12.5-6 Model 2 (c) (Demand Rate Uniform, Production Rate Finite, Shortages Allowed)

This model has the same assumptions as in model 2(*a*) except that production rate is finite. Figure 12.6 shows the variation of inventory with time.

Referring to Figure 12.6, we find that inventory is zero in the beginning. It increases at constant rate (K – R) for time t_1 until it reaches a level I_m. There is no replenishment during time t_2, inventory decreases at constant rate R till it becomes zero. Shortage starts piling up at constant rate R during time t_3 until this backlog reaches a level s. Lastly, production starts and backlog is filled at a constant rate K – R during time t_4 till the backlog becomes zero. This completes one cycle; the total time taken during this cycle is

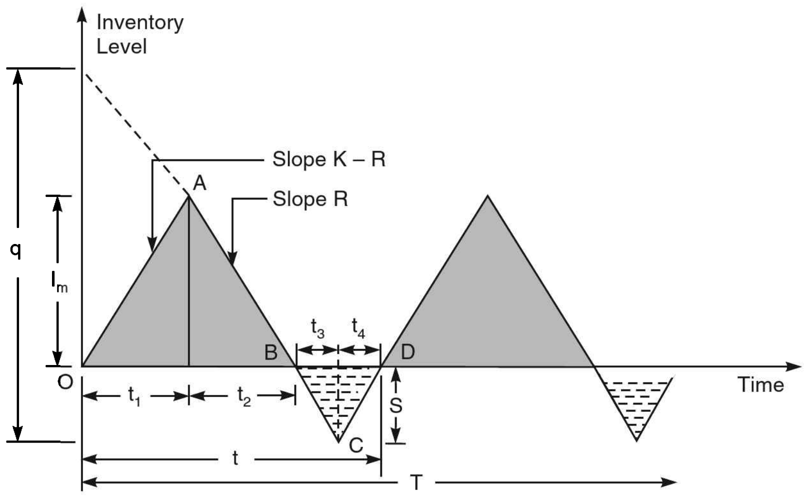

Fig. 12.6. Inventory situation for model 2 (c).

$$t = t_1 + t_2 + t_3 + t_4.$$

This cycle repeats itself over and over again.

Now holding cost during time interval t

$$= C_1 \,.\, \text{area of } \Delta \text{ OAB}$$

$$= C_1 \,.\, \frac{1}{2} \,.\, I_m (t_1 + t_2),$$

and shortage cost during time interval t

$$= C_2 \,.\, \text{area of } \Delta \text{ BCD}$$

$$= C_2 \,.\, \frac{1}{2} \,.\, s \,.\, (t_3 + t_4).$$

Also setup cost during time interval $t = C_3$.

Total average cost per unit time,

$$C = \frac{\frac{1}{2} \cdot C_1 I_m (t_1 + t_2) + \frac{1}{2} C_2 \cdot s \cdot (t_3 + t_4) + C_3}{t_1 + t_2 + t_3 + t_4}. \qquad ...(12.30)$$

Now C is a function of six variables I_m, s, t_1, t_2, t_3, and t_4 but we can derive relationships which determine the values of I_m, t_1, t_2, t_3 and t_4 in terms of only two variables q and s. An inventory policy is given when we know how much to produce *i.e.*, q and when to start production, which can be found if s is known.

Now $\quad I_m = (K - R)\, t_1, \qquad ...(12.31)$

also $\quad I_m = R\, t_2. \qquad ...(12.32)$

$\therefore \quad (K - R)\, t_1 = R\, t_2. \qquad ...(12.33)$

Further, $\quad s = R\, t_3, \qquad ...(12.34)$

also $\quad s = (K - R)\, t_4. \qquad ...(12.35)$

$\therefore \quad (K - R)\, t_4 = R\, t_3. \qquad ...(12.36)$

Adding equations (12.33) and (12.36) we get

$$(K - R)\,(t_1 + t_4) = R(t_2 + t_3). \qquad ...(12.37)$$

The manufacturing rate multiplied by the manufacturing time gives the manufactured quantity.

$$\therefore \quad q = K\, t_1 + K\, t_4 = (t_1 + t_4)\, K.$$

$$\therefore \quad t_1 + t_4 = \frac{q}{K}. \qquad ...(12.38)$$

Adding equations (12.32) and (12.34),

$$I_m + s = R\,(t_2 + t_3)$$

$$= (K - R)\,(t_1 + t_4)$$

$$= \frac{q}{K}\,(K - R). \qquad \text{[Using equation (12.38)]}$$

$$\therefore \quad I_m = q\left(1 - \frac{R}{K}\right) - s. \qquad ...(12.39)$$

From equations (12.31) and (12.32),

$$t_1 + t_2 = \frac{I_m}{K - R} + \frac{I_m}{R}$$

$$= \left\{q\left(1 - \frac{R}{K}\right) - s\right\}\left\{\frac{1}{K - R} + \frac{1}{R}\right\}. \qquad ...(12.40)$$

[Using equation (12.39)]

Similarly, $$t_3 + t_4 = \frac{s}{K-R} + \frac{s}{R} = s\left(\frac{1}{K-R} + \frac{1}{R}\right). \quad ...(12.41)$$

and $$t_1 + t_2 + t_3 + t_4 = \left(\frac{1}{K-R} + \frac{1}{R}\right)\left\{q \cdot \frac{K-R}{K}\right\} = \frac{q}{R}. \quad ...(12.42)$$

Substituting values of I_m, $t_1 + t_2$, $t_3 + t_4$ and $t_1 + t_2 + t_3 + t_4$ in equation (12.30), we get

$$C\,(q,\, s) = \frac{R}{q}[\frac{1}{2} \cdot C_1 \cdot \left\{q\left(1-\frac{R}{K}\right) - s\right\} \cdot \left\{q\left(1-\frac{R}{K}\right) - s\right\}$$

$$\left\{\frac{1}{K-R} + \frac{1}{R}\right\} + \frac{1}{2}C_2 \cdot s\left(\frac{1}{K-R} + \frac{1}{R}\right) \cdot s] + \frac{R}{q}C_3$$

$$= \frac{R}{q}\left[\frac{1}{2} \cdot \left(\frac{1}{K-R} + \frac{1}{R}\right)\right]\left[C_1 \cdot \left\{q\left(1-\frac{R}{K}\right) - s\right\}^2 + C_2 \cdot s^2\right] + \frac{R}{q} \cdot C_3.$$

$$\therefore \quad C(q,\, s) = \frac{R}{2q} \cdot \frac{K}{R\,(K-R)} \cdot \left[C_1 \left\{\left(q \cdot \frac{K-R}{K}\right) - s\right\}^2 + C_2 s^2\right] + \frac{R}{q}C_3$$

or $$C(q,\, s) = \frac{1}{2q} \cdot \frac{K}{K-R} \cdot \left[C_1 \left\{q \cdot \frac{K-R}{K} - s\right\}^2 + C_2 s^2\right] + \frac{R}{q}C_3. \quad ...(12.43)$$

Now cost $C\,(q,s)$ will be minimum if

$$\frac{\partial}{\partial q}\,[C(q,\, s)] = 0, \quad \frac{\partial^2}{\partial q^2}\,[C(q,s)] > 0$$

and $$\frac{\partial}{\partial s}\,[C(q,\, s)] = 0, \quad \frac{\partial^2}{\partial s^2}\,[C(q,s)] > 0.$$

Differentiating equation (12.43) partially w.r.t.s,

$$\frac{\partial}{\partial s}\,[C(q,\, s)] = \frac{1}{2q} \cdot \frac{K}{K-R} \cdot \left[2C_1\left(q \cdot \frac{K-R}{K} - s\right)(-1) + 2C_2 s\right] = 0$$

or $$2C_1 \cdot \left(q \cdot \frac{K-R}{K}\right) = 2\,(C_1 + C_2) \cdot s$$

or $$s = q \cdot \frac{K-R}{K} \cdot \frac{C_1}{C_1 + C_2}.$$

$$\frac{\partial^2}{\partial s^2}[C(q,\, s)] = \frac{1}{2q} \cdot \frac{K}{K-R}(2C_1 + 2C_2), \text{ which is positive.}$$

$$\therefore \quad s_0 = q \cdot \frac{K-R}{K} \cdot \frac{C_1}{C_1 + C_2} \quad ...(12.44)$$

Differentiating equation (12.43) partially w.r.t. q,

$$\frac{\partial}{\partial q}[C(q,s)] = -\frac{1}{2q^2} \cdot \frac{K}{K-R} \cdot \left[C_1\left(q \cdot \frac{K-R}{K} - s\right)^2 + C_2 s^2\right]$$

$$+ \frac{1}{2q} \cdot \frac{K}{K-R}\left[2C_1 \cdot \left(q \cdot \frac{K-R}{K} - s\right) \cdot \frac{K-R}{K}\right] - \frac{RC_3}{q^2} = 0,$$

which on simplification gives,

$$q = \sqrt{2C_3 \cdot \frac{C_1 + C_2}{C_1 C_2}} \cdot \sqrt{\frac{KR}{K - R}}.$$

It can be proved that $\frac{\partial^2}{\partial q^2}$ [C(*q*,*s*)] is positive, so that

$$q_0 = \sqrt{2C_3 \cdot \frac{(C_1 + C_2)}{C_1 C_2}} \cdot \sqrt{\frac{KR}{K - R}}$$

$$= \sqrt{\frac{C_1 + C_2}{C_2}} \cdot \sqrt{\frac{K}{K - R}} \cdot \sqrt{\frac{2C_3 R}{C_1}}. \quad \text{...(12.45)}$$

From equation (12.44),
$$s_0 = \sqrt{2C_3 \cdot \frac{C_1}{(C_1 + C_2)\, C_2}} \cdot \sqrt{\frac{R\,(K - R)}{K}}. \quad \text{...(12.46)}$$

Substituting values of q_0 and s_0 in equation (12.30), and simplifying we get

$$C_0(q,s) = \sqrt{\frac{2C_1 C_2}{C_1 + C_2}} \cdot C_3 \sqrt{\frac{R(K - R)}{K}}$$

$$= \sqrt{\frac{C_2}{C_1 + C_2}} \cdot \sqrt{\frac{K - R}{K}} \cdot \sqrt{2C_1 C_3 R}. \quad \text{...(12.47)}$$

Optimum time interval t_0 is given by

$$t_0 = \frac{q_0}{R} = \sqrt{2C_3 \cdot \frac{(C_1 + C_2)}{C_1 C_2} \cdot \frac{K}{R(K - R)}}$$

$$= \sqrt{\frac{C_1 + C_2}{C_2}} \cdot \sqrt{\frac{K}{K - R}} \cdot \sqrt{\frac{2C_3}{C_1 R}} \quad \text{...(12.48)}$$

and
$$I_{m0} = q_0 \left(1 - \frac{R}{K}\right) - s_0 \quad \text{[Equation (12.39)]}$$

$$= \sqrt{\frac{2C_2 \cdot C_3}{C_1 (C_1 + C_2)}} \cdot \sqrt{\frac{R\,(K - R)}{K}}$$

$$= \sqrt{\frac{C_2}{C_1 + C_2}} \cdot \sqrt{\frac{K - R}{K}} \cdot \sqrt{\frac{2C_3\, R}{C_1}}. \quad \text{...(12.49)}$$

Particular cases

(*i*) If $K = \infty$, *i.e.*, production rate is infinity, equations (12.45), (12.49) and (12.47) giving q_0, I_{m0} and C_0 reduce to equations (12.23), (12.24) and (12.25) for model 2(*a*).

(*ii*) If $C_2 = \infty$, *i.e.*, no shortages are allowed, equations (12.45), (12.48) and (12.47) reduce to (12.17), (12.18) and (12.19) for model 1(*c*).

(*iii*) If $K = \infty$, $C_2 = \infty$, this model becomes model 1(*a*) and equations (12.45), (12.47) and (12.48) reduce to equations (12.3), (12.4) and (12.2) respectively.

EXAMPLE 12.5-19

Find the results of example 12.5-2 if in addition to the data given in that problem the cost of shortage is also given as ₹ 5 per unit per year.

Solution

$$R = 9{,}000 \text{ units/year, } C_3 = ₹\ 100\text{/procurement,}$$
$$C_1 = ₹\ 2.40\text{/unit/year, } C_2 = ₹\ 5\text{/unit/year.}$$

(*i*) From equation (12.23),

$$q_0 = \sqrt{\frac{C_1 + C_2}{C_2}} \cdot \sqrt{\frac{2C_3 R}{C_1}} = \sqrt{\frac{2.40 + 5}{5}} \cdot \sqrt{\frac{2 \times 100 \times 9{,}000}{2.40}}$$
$$= \sqrt{11{,}10{,}000} = 1{,}053 \text{ units/procurement.}$$

(*ii*) From equation (12.25),

$$C_0(I_m, q) = 9{,}000 \times 1 + \sqrt{\frac{C_2}{C_1 + C_2}} \sqrt{2C_1C_3R} = 9{,}000$$
$$+ \sqrt{\frac{5}{2.4 + 5}} \cdot \sqrt{2 \times 2.4 \times 100 \times 9{,}000}$$
$$= ₹\ (9{,}000 + 1{,}710) = ₹\ 10{,}710\text{/year.}$$

(*iii*) Number of orders/year, $n_0 = \dfrac{9{,}000}{1{,}053} = 8.55.$

(*iv*) Time between orders,

$$t_0 = \frac{1}{n_0} = \frac{1}{8.55} = 0.117 \text{ year} = 1.4 \text{ months.}$$

EXAMPLE 12.5-20

The data for this example are same as that of example 12.5-14 except that the shortage cost of one unit is ₹ 20 per year. Find the various results.

Solution

$$R = 12{,}000 \text{ units/year,}$$
$$K = 2{,}000 \times 12 = 24{,}000 \text{ units/year,}$$
$$C_3 = ₹\ 400\text{/setup,}$$
$$C_1 = ₹\ 0.15 \times 12 = ₹\ 1.80 \text{ per unit/year,}$$
$$C_2 = ₹\ 20 \text{ per year.}$$

(*i*) Using equation (12.45),

$$q_0 = \sqrt{\frac{2C_3R}{C_1}} \cdot \sqrt{\frac{C_1 + C_2}{C_2}} \cdot \sqrt{\frac{K}{K - R}}$$
$$= \sqrt{\frac{2 \times 400 \times 12{,}000}{1.80}} \cdot \sqrt{\frac{1.80 + 20}{20}} \cdot \sqrt{\frac{24{,}000}{24{,}000 - 12{,}000}}$$
$$= \sqrt{\frac{2 \times 400 \times 12{,}000}{1.80}} \cdot \sqrt{\frac{10.9}{10}} \cdot \sqrt{2} = 3{,}410 \text{ units.}$$

(*ii*) From equation (12.47),

$$C_0(q, s) = 12{,}000 \times 4 + \sqrt{2C_1 C_3 R} \cdot \sqrt{\frac{C_2}{C_1 + C_2}} \cdot \sqrt{\frac{K - R}{K}}$$
$$= 48{,}000 + \sqrt{2 \times 1.80 \times 400 \times 12{,}000} \cdot \sqrt{\frac{20}{20 + 1.80}} \times \sqrt{\frac{24{,}000 - 12{,}000}{24{,}000}}$$
$$= 48{,}000 + \sqrt{2 \times 1.80 \times 400 \times 12{,}000} \cdot \sqrt{\frac{10}{10.9}} \cdot \frac{1}{\sqrt{2}} = ₹\ 50,\ 185\text{/ year.}$$

(*iii*) Using equation (12.49),

$$I_{m_0} = \sqrt{\frac{2C_3R}{C_1}} \cdot \sqrt{\frac{C_2}{C_1 + C_2}} \cdot \sqrt{\frac{K - R}{K}}$$

$$= \sqrt{\frac{2 \times 400 \times 12{,}000}{1.80}} \times \sqrt{\frac{20}{1.80 + 20}} \cdot \sqrt{\frac{24{,}000 - 12{,}000}{24{,}000}}$$

$$= \sqrt{\frac{2 \times 400 \times 12{,}000}{1.80} \times \frac{10}{10.9} \times \frac{1}{2}} = 1.564 \text{ units/production run.}$$

(*iv*) Manufacturing time interval, $t_1 + t_4$

$$= \frac{q_0}{K} \quad \text{[equation (12.38)]}$$

$$= \frac{3{,}410}{24{,}000} = 0.1421 \text{ year} = 1.7 \text{ months.}$$

(*v*) Total time interval, t_0 $= \frac{q_0}{R}$ [equation (12.48)]

$$= \frac{3{,}410}{12{,}000} = 0.2842 \text{ year} = 3.4 \text{ months.}$$

EXAMPLE 12.5-21

A contractor supplies diesel engines to a truck manufacturer at the rate of 20 per day. He has to pay a penalty of ₹ 10 per engine per day for missing the scheduled delivery date. Holding cost of a complete engine is ₹ 12 per month. The manufacturing of engines starts with the beginning of the month and is completed at the end of the month. What should be the inventory level at the beginning of each month ?

Solution

R = 20 engines/day, C_2 = ₹ 10 per engine per day,
C_1 = ₹ 12 per month = 12/30 = ₹ 0.40/day, t = 1 month = 30 days.

Using equation (12.28), $I_{m_0} = \frac{C_2}{C_1 + C_2} \cdot q \quad \frac{C_2}{C_1 + C_2} \cdot Rt$

$$= \frac{10}{0.40 + 10} \times 20 \times 30 = 577 \text{ engines/month.}$$

Example 12.5-22

A dealer supplies you the following information with regard to a product dealt-in by him:

Annual demand	:	*10,000 units*
Ordering cost	:	*₹ 10 per order*
Inventory carrying cost	:	*20% of value of inventory per year*
Price	:	*₹ 20 per unit.*

The dealer is considering the possibility of allowing some back-order (stock-out) to occur. He has estimated that the annual cost of back-ordering will be 25% of the value of inventory.

(*i*) *What should be the optimum number of units of the product he should buy in one lot?*
(*ii*) *What quantity of the product should be allowed to be back-ordered, if any ?*
(*iii*) *What would be the maximum quantity of inventory at any time of the year ?*
(*iv*) *Would you recommend to allow back-ordering ? If so, what would be the annual cost saving by adopting the policy of back-ordering ?* [*P.U.M.F.C., 2002*]

Solution

Here R = 10,000 units/year,
C_3 = ₹ 10/order,

$$C_1 = CI = ₹\ 20 \times \frac{20}{100} = ₹\ 4/\text{unit/year},$$

$$C_2 = ₹\ 20 \times \frac{25}{100} = ₹\ 5/\text{unit/year}.$$

(*i*) q_0, allowing back-order $= \sqrt{\frac{C_1 + C_2}{C_2}} \cdot \sqrt{\frac{2RC_3}{C_1}}$

$$= \sqrt{\frac{4+5}{5}} \cdot \sqrt{\frac{2 \times 10{,}000 \times 10}{4}} = 300 \text{ units/order.}$$

(*ii*) Maximum quantity to be back-ordered, $s_0 = q_0 - I_{m_0}$,

where $\quad I_{m_0} = \sqrt{\frac{C_2}{C_1 + C_2}} \cdot \sqrt{\frac{2RC_3}{C_1}}$

$$= \sqrt{\frac{5}{4+5}} \cdot \sqrt{\frac{2 \times 10{,}000 \times 10}{4}} = 167 \text{ units.}$$

$\therefore \quad s_0 = 300 - 167 = 133$ units.

(*iii*) Maximum inventory, $\quad I_{m_0} = 167$ units.

(*iv*) Annual cost without back-order $= \sqrt{2RC_3C_1} = \sqrt{2 \times 10{,}000 \times 10 \times 4} = ₹\ 894.$

Annual cost allowing back-order $= \sqrt{\frac{C_2}{C_1 + C_2}} \cdot \sqrt{2RC_3C_1}$

$$= \sqrt{\frac{5}{4+5}} \cdot \sqrt{2 \times 10{,}000 \times 10 \times 4} = ₹\ 667.$$

$\therefore$ Saving in annual cost if back-order is allowed = ₹ 227.

EXERCISES 12.2

1. Formulate an inventory model in which demand is not uniform and production rate is infinite. [*P.U.MFC-1 April, 1999*]
2. Derive economic lot size formula for made-in items when lead time is zero and shortages are not allowed. [*P.U.M.Com. Sept., 2004; B.Com. Sept., 2004; P.T.U. B.Tech., 2001, 2000*]
3. (*a*) What do you understand by a fixed time period, and fixed quantity models ? When would you use each of them ?
   (*b*) Derive the equation for EOQ under certainty.
   (*c*) How will this equation change under condition of uncertainty ? [*P.U. MBA, 1997*]
4. Derive EOQ model for deterministic demand when replenishment rate is infinite and shortages are permitted. [*J.N.T.U. Hyderabad B.Tech. May, 2011; P.T.U. B.Tech., 2001*]
5. Derive economic lot size formula for an inventory model with finite production rate and shortages permitted. [*K.U.M.Sc. (Math.) 2001*]
6. Discuss the concept of economic lot size. How is it determined ? What happens when total carrying cost per year is not equal to total ordering cost per year ? [*P.U.B.Com. Sept., 2001*]

Section 12.5-3

7. A contractor has to supply 10,000 bearings per day to an automobile manufacturer. He finds that, when he starts a production run, he can produce 25,000 bearings per day. The cost of holding a bearing in stock for one year is 2 paise, and the setup cost of a production run is ₹ 18. What is the optimum lot size and how frequently should production run be made?
[*ICWA Dec., 1993; Meerut M.Sc. (Math.) 1969; Delhi M.Com., 1975*]
(*Ans.* 1,05,000; 10.5 days.)

8. A product is produced at the rate of 50 items per day. The demand occurs at the rate of 30 items per day. Given that setup cost per order = ₹ 100 and holding cost per unit per unit time = ₹ 0.05, find the economic lot size and the associated total cost per cycle assuming that no shortage is allowed.

(*Ans.* 225, ₹ 11.)

9. A company manufactures refrigeration units in batches. The estimated demand is 10,000 units/year. It costs ₹ 100 to setup the manufacturing process and the carrying cost is ₹ 0.50 per unit per year. Once the production process has been setup, 80 units can be manufactured daily. The demand during the production day has been 60 units per day. How many units should the company produce in each batch ? How long will the production cycle last ? [*IGNOU MBA Dec., 1997*]

(*Ans.* 4,000 units; 50 days.)

10. The manager of a company manufacturing car parts has entered into a contract of supplying 1,000 nos. per day of a particular part to a car manufacturer. He finds that his plant has a capacity of producing 2,000 nos. per day of the part. The cost of the part is ₹ 50, cost of holding stock is 12% per annum and setup cost per production run is ₹ 100. What should be run size for each production run and total optimum cost/month ? How frequently should production runs be made ? Shortage is not permissible. [*Mumbai U.MBA, 2010; Baroda Univ. B.E., 1973*]

11. A product is sold at the rate of 50 pieces per day and is manufactured at the rate of 250 pieces per day. The setup cost of the machines is ₹ 1,000 and the storage cost is found to be ₹ 0.0015 per piece per day. With labour charges of ₹ 3.20 per piece, material cost at ₹ 2.10 per piece and overhead cost of ₹ 4.10 per piece, find the minimum cost batch size if the interest charges are 8 per cent (assume 300 working days in a year). Also compute the optimal number of orders in a year. [*J.U. MBA, 2004*]

[Hint. $R = 50 \times 300 = 15{,}000$ pieces / year, $K = 250 \times 300 = 75{,}000$ pieces / year,

$C_3 =$ ₹ 1,000/setup, $C_1 =$ ₹ $[0.0015 \times 300 + 0.08\,(3.20 + 2.10 + 4.10)]$
$=$ ₹ 12.02/year.

$$\therefore\ q_0 = \sqrt{\frac{K}{K-R}}\sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{75{,}000}{75{,}000-15{,}000}}\sqrt{\frac{2\times 1{,}000\times 15{,}000}{12.02}}$$

$= 5{,}586$ pieces.

$$n_0 = \frac{R}{q_0} = \frac{15{,}000}{5{,}586} \simeq 3 \text{ cycles/year.]}$$

12. A company can produce a part it uses in an assembly operation at the rate of 50 an hour. The company operates 8 hours a day, 300 days a year. Daily usage of the part is 300 parts. The run size is 6,000 parts. The annual holding cost is ₹ 20 per unit and the setup cost is ₹ 1,000 per setup.
(*a*) How many runs per year will be there ?
(*b*) How many parts/day are being added to inventory, while production is occurring ?
(*c*) Assuming that production begins when there are no parts on hand, what is the maximum number of parts in inventory ?
(*d*) Every so often, preventive maintenance, which requires 6 working days, must be performed on it. Does this interrupt production cycles, or is there enough time between cycles to perform the maintenance ? Explain. [*Dayalbagh Edu. Inst. Agra B.B.M. May, 2008*]

[Hint. (*a*) $n = \dfrac{300 \times 300}{6{,}000} = 15.$

(*b*) Parts added/day to inventory = 50 × 8 – 300 = 100 /day.

(*c*) $I_0 = \dfrac{p-d}{p}\,.\,q = \dfrac{100}{400} \times 6{,}000 = 1{,}500$ parts.

(*d*) For demand of 300 parts / day, inventory of 1,500 parts will be able to meet the demand for 5 days. As preventive maintenance requires 6 days, production cycles will be interrupted.]

13. Find the most economic batch quantity of a product on a machine if the production rate of the item on the machine is 200 pieces/day and the demand is uniform at the rate of 100 pieces/day. The setup cost is ₹ 200 per batch and the cost of holding one item in inventory is ₹ 0.81 per day. How will the batch quantity vary if the machine production rate was infinite ? [*J.N.T.U. Hyderabad B.Tech. April, 2011; Nov., 2010*]

Section 12.5-4

14. A contractor undertakes to supply diesel engines to a truck manufacturer at the rate of 25 per day. He finds that the cost of holding a completed engine in stock is ₹ 16 per month, and there is a clause in the contract penalising him ₹ 10 per engine per day late for missing the scheduled delivery date. Production of engines is in batches, and each time a new batch is started there are setup costs of ₹ 10,000. How frequently should batches be started, and what should be the initial inventory level at the time each batch is completed ? [*Baroda B.Sc. (Math.) 1978*]
(*Ans.* 40 days, 943 engines.)
15. A manufacturer receives an order for 6,890 items to be delivered over a period of a year as follows:
at the end of the 1st week: 5 items,
at the end of 2nd week: 10 items,
at the end of 3rd week: 15 items, etc.
The cost of carrying inventory is ₹ 2.60 per item per year and the cost of a setup is ₹ 450 per production run.
Compute costs for the following policies:
(1) Make all 6,890 at start of the year.
(2) Make 3,445 now and 3,445 in 6 months.
(3) Make 1/12th of the order each month.
(4) Make 1/52 of the order each week. (*Ans.* ₹ 12,000, ₹ 8,000, ₹ 9,000, ₹ 27,000 (approx.))
16. A manufacturer received an annual contract for supplying 4,000 gears to be delivered over a period of one year. Deliveries are to be affected as under:
First quarter –3,500, second quarter –4,500, third quarter –2,000, fourth quarter –4,000. The manufacturer wants to plan out his production on his vital machine which costs ₹ 590 for setting up. The cost of gear is ₹ 60 and inventory carrying cost comes to 10% per year. Calculate the annual cost for producing this quantity in a number of equal lots. What will be the minimum cost over the year ? Explain and derive the formula used. [*Baroda Univ., 1975*]
17. (*a*) Discuss in detail the inventory costs which are to be considered for determination of economic order quantity.
(*b*) The demand for a product is 25 units per month, and the items are withdrawn uniformly. The setup cost each time a production run is made is ₹ 15. The inventory holding cost is ₹ 0.30 per item per month. Assuming that shortages are not allowed, determine how often to make a production run and what size should it be. Prove the formula used. If shortages cost ₹ 1.50 per item/month, determine how often to make production run and what size should it be.
[*P.U.B.Com. Sept., 2000; Baroda Univ. B.E., 1973*]
(*Ans.* 50 units, 2 months; 54 units, 2.16 months.)
18. The annual demand for an automobile component nut is 36,000 units. The carrying cost is ₹ 0.5 per unit per year. The ordering cost is ₹ 25 per order and the shortage cost is ₹ 15 per unit per year. Find the optimal values of :
(*i*) Economic order quantity (*ii*) Maximum inventory (*iii*) Cycle time (*iv*) No. of orders.
[*J.N.T.U. Hyderabad B.Tech. (Mech.) May, 2012*]

Section 12.5-5–12.5-6

19. A product is produced at the rate of 50 items per day. The demand occurs at the rate of 30 items per day. Given that
setup cost per order = ₹ 100,
holding cost per item per unit time = ₹ 0.05,
and shortages being allowed, what is the shortage cost per unit under optimal conditions if the lot size is 600 items. (*Ans.* ₹ 1/220.)
20. Consider the following data:

Unit cost	:	₹ 100,
order cost	:	₹ 160,
inventory carrying cost	:	₹ 20,
back-order cost	:	₹ 10,
annual demand	:	1,000 units.

Compute the following:

(*i*) Minimum cost order quantity

(*ii*) Time between orders

(*iii*) Maximum number of back-orders

(*iv*) Maximum inventory level

(*v*) Overall annual cost. [*C.A. (Final) May, 1990*]

(*Ans.* (*i*) 219 units (*ii*) 2.63 months (*iii*) 146 units (*iv*) 73 units (*v*) ₹ 1,01,460.59.)

21. The demand for an item is 16,000 units per year. Its production rate is 900 units per month. The carrying cost is ₹ 400 per unit per year and the setup cost is ₹ 3,000 per setup. The penalty cost is ₹ 1,000 per unit per year. Find out :

(*i*) Economic order quantity

(*ii*) Number of orders per year

(*iii*) Time between two consecutive orders. [*J.N.T.U. Hyderabad B.Tech. (Mech.) May, 2012*]

22. The demand for an item in a company is 18,000 units per year, and the company can produce the item at a rate of 3,000 per month. The cost of one setup is ₹ 500 and the holding cost of one unit per month is 15 paise. The shortage cost of one unit is ₹ 20 per year. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and the time between setups. [*J.N.T.U. Hyderabad B.Tech. May, 2011*]

12.6 INVENTORY MODELS WITH PROBABILISTIC DEMAND

The models discussed in the previous sections are only artificial since in practical situations, demand is hardly known precisely. In most situations demand is probabilistic since only *probability distribution* of future demand, rather than the exact value of demand itself, is known. The probability distribution of future demand is usually determined from the data collected from past experience. In such situations we choose policies that minimize the *expected* costs rather than the actual costs. Expected costs are obtained by multiplying the actual costs for a particular situation with the probability of occurrence of that situation and then either summing or integrating according as the probability distribution is discrete or continuous.

12.6-1 Model 3(a) (Instantaneous Demand, Setup Cost Zero, Stock Levels Discrete and Lead Time Zero)

This model deals with the inventory situation of items that require one time purchase only. Perishable items such as cut flowers, cosmetics, seasonal items such as calendars and diaries and spare parts fall under this category.

In this model the item is ordered at the beginning of the period to meet the demand during that period, the demand being instantaneous as well as discrete in nature. At the end of the period, there are two types of costs involved : over-stocking cost and under-stocking cost. They represent opportunity losses incurred when the number of units stocked is not exactly equal to the number of units actually demanded.

Let R = discrete demand rate with probability p_R,

I_m = discrete stock level for time interval t,

t = constant interval between orders,

C_1 = over-stocking cost (over-ordering cost). This is opportunity loss associated with each unit left unsold

$= C + C_h - V$,

C_2 = under-stocking cost (under-ordering cost). This is opportunity loss due to not meeting the demand.

$= S - C - C_h/2 + C_s$,

where C is the unit cost price, C_h the unit carrying cost, C_s the unit shortage cost, S the unit selling price and V is the salvage value. If value of any parameter is not given, it is taken as zero.

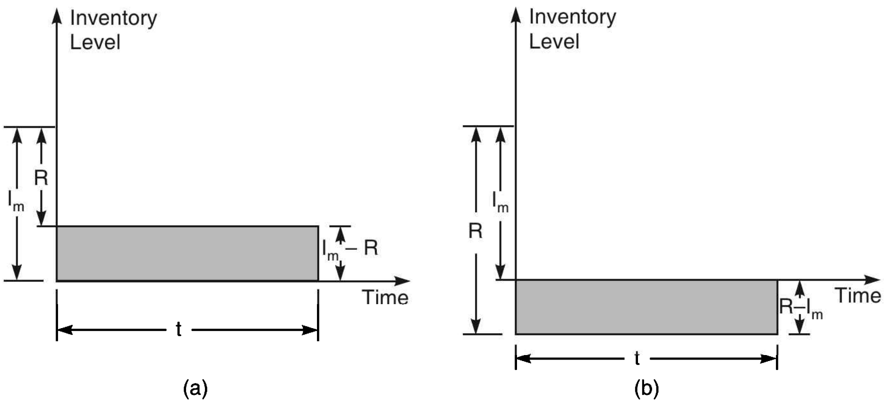

Fig. 12.7. Inventory situation for model 3(a), (a) $R \le I_m$ (b) $R > I_m$.

Production is assumed to be instantaneous and lead time negligibly small. The problem is to determine the optimal inventory level I_m, where $R \le I_m$ or $R > I_m$ at the beginning of each time interval. The variation of inventory with time for these two cases is shown in Fig. 12.7(*a*) and (*b*).

When $R \le I_m$, as shown in figure 12.7(*a*), there are no shortages; when $R > I_m$, as shown in figure 12.7(*b*), shortages occur.

Then the optimal order quantity I_{m_0} is determined when value of cumulative probability distribution exceeds the ratio $\frac{C_2}{C_1 + C_2}$ by computing

$$p_{R \le I_m - 1} < \frac{C_2}{C_1 + C_2} < p_{R \le I_m}.$$

Example 12.6-1

A trader stocks a particular seasonal product at the beginning of the season and cannot reorder. The item costs him ₹ 25 and he sells it at ₹ 50 each. For any item that cannot be met on demand, the trader has estimated a goodwill cost of ₹ 15. Any item unsold will have a salvage value of ₹ 10. Holding cost during the period is estimated to be 10% of the price. The probability of demand is as follows :

Units stocked	:	*2*	*3*	*4*	*5*	*6*
Probability of demand	:	*0.35*	*0.25*	*0.20*	*0.15*	*0.05*

Determine the optimal number of items to be stocked.

Solution

Here, C = ₹ 25, S = ₹ 50, C_h = ₹ (0.10 × 25) = ₹ 2.50, C_s = ₹ 15 and V = ₹ 10.

∴ $C_1 = C + C_h - V$ = ₹ (25 + 2.50 – 10) = ₹ 17.50,

$C_2 = S - C - C_h/2 + C_s$ = ₹ $\left(50 - 25 - \frac{2.50}{2} + 15\right)$ = ₹ 38.75.

Cumulative probability of demand is now calculated.

Units stocked	:	2	3	4	5	6
Probability of demand, p_R	:	0.35	0.25	0.20	0.15	0.05
Cumulative probability of demand $\sum_{R=0}^{I_m} p_R$	:	0.35	0.60	0.80	0.95	1.00

Now $$\frac{C_2}{C_1 + C_2} = \frac{38.75}{17.50 + 38.75} = 0.69.$$

This ratio lies between cumulative probabilities of 0.60 and 0.80, which suggests that the value of I_m must lie between 3 and 4 since

$$0.60 < 0.69 < 0.80.$$

$$\therefore \qquad I_{m_0} = 4 \text{ units.}$$

EXAMPLE 12.6-2

A newspaper boy buys papers for 5 paise each and sells them for 6 paise each. He cannot return unsold newspapers. Daily demand R for newspapers follows the distribution :

R	:	*10*	*11*	*12*	*13*	*14*	*15*	*16*
p_R	:	*0.05*	*0.15*	*0.40*	*0.20*	*0.10*	*0.05*	*0.05*

If each day's demand is independent of the previous day's, how many papers should be ordered each day?

Solution

Let I_m be the number of newspapers ordered per day and R be the demand for it *i.e.*, the number that are actually sold per day.

Now $\qquad C_1 =$ ₹ 0.05,

$\qquad C_2 = S - C =$ ₹ (0.06 – 0.05) = ₹ 0.01.

The probabilities for demand are

R	:	10	11	12	13	14	15	16
p_R	:	0.05	0.15	0.40	0.20	0.10	0.05	0.05
$\sum_{R=0}^{I_m} p_R$	:	0.05	0.20	0.60	0.80	0.90	0.95	1.00

The desired optimum value for I_m is determined by the double inequality,

$$p_{R \le I_m - 1} < \frac{C_2}{C_1 + C_2} < p_{R \le I_m}.$$

Now
$$\frac{C_2}{C_1 + C_2} = \frac{0.01}{0.01 + 0.05} = \frac{1}{6} = 0.167.$$

This suggests that I_m must lie between 10 and 11 because

$$0.05 < 0.167 < 0.20.$$

$$\therefore \qquad I_{m_0} = 11.$$

EXAMPLE 12.6-3

Some of the spare parts of a ship cost ₹ 50,000 each. These spare parts can only be ordered together with the ship. If not ordered at the time the ship is constructed, these parts cannot bt available on need. Suppose that a loss of ₹ 4,500,000 is suffered for each spare that is needed when none is available in the stock. Further suppose that the probabilities that the spares will be needed as replacement during the life term of the class of ship discussed are

Spares required	*Probability*
0	*0.900*
1	*0.040*
2	*0.025*
3	*0.020*
4	*0.010*
5	*0.005*
6 or more	*0.000*
Total	*1.000*

How many spare parts should be procured ?

Solution

$$C_1 = ₹\ 50{,}000,$$
$$C_2 = C_s = ₹\ 4{,}500{,}000.$$
$$\frac{C_2}{C_1 + C_2} = \frac{4{,}500{,}000}{4{,}550{,}000} = 0.989.$$

Cumulative probability distribution is

Spares required	0	1	2	3	4	5	6 or more
$\sum_{R=0}^{I_m} p_R$	0.900	0.940	0.965	0.985	0.995	1.000	1.000

The desired optimum value for I_m is determined by the double inequality

$$p_{R \le I_m - 1} < \frac{C_2}{C_1 + C_2} < p_{R \le I_m}.$$

This suggests that I_m must lie between 3 and 4 because $0.985 < 0.989 < 0.995$.

$\therefore \qquad I_{m_0} = 4.$

EXAMPLE 12.6-4

(a) A firm is to order a new lathe. Its power unit is an expensive part and can be ordered only with the lathe. Each of these units is uniquely built for a particular lathe and cannot be used on any other. The firm wants to know how many spare units should be incorporated in the order for each lathe. Cost of the unit when ordered with the lathe is ₹ 700. If a spare unit is needed (because of its failure during service) and is not available, the whole lathe becomes useless. The cost of the unit made to order and the down time cost of the lathe is ₹ 9,300. The analysis of 100 similar units on similar lathes yields the following information given in table 12.1.

(b) If in the above problem the shortage cost of the part is unknown and the firm wants to maintain stock level of 4 parts, find the shortage cost.

TABLE 12.1

No. of spare units required	*No. of lathes requiring indicated number of spare units*	*Estimated probability of occurrence of indicated number of failures*
0	*87*	*0.87*
1	*5*	*0.05*
2	*3*	*0.03*
3	*2*	*0.02*
4	*1*	*0.01*
5	*1*	*0.01*
6	*1*	*0.01*
7 or more	*0*	*0.00*

Solution

(*a*) The range of optimum value of stock level, I_m is given by

$$p_{R \le I_m - 1} < \frac{C_2}{C_1 + C_2} < p_{R \le I_m}.$$

Table 12.2 gives the data of table 12.1 after reformulation.

TABLE 12.2

I_m	R	p_R	$\sum_{R=0}^{I_{m0}} p_R$
0	0	0.87	0.87
1	1	0.05	0.92
2	2	0.03	0.95
3	3	0.02	0.97
4	4	0.01	0.98
5	5	0.01	0.99
6	6	0.01	1.00
7 or more		0.00	1.00
		1.00	

Here C_1 = ₹ 700, $C_2 = C_s$ = ₹ 9,300.

$$\therefore \quad \frac{C_2}{C_1 + C_2} = \frac{9,300}{700 + 9,300} = 0.93.$$

∴ Optimum value of $I_m = 2$, since $0.92 < 0.93 < 0.95$.

(*b*) Here $I_m = 4$.

$$\therefore \quad p_{R \le 3} < \frac{C_2}{700 + C_2} < p_{R \le 4}$$

$$\text{or} \quad 0.97 < \frac{C_2}{700 + C_2} < 0.98.$$

∴ The least value of C_2 is given by

$$\frac{C_2}{700 + C_2} = 0.97 \quad \text{or} \quad C_2 = \frac{700 \times 0.97}{0.03} = ₹\ 22,633.33,$$

and the greatest value of C_2 is given by

$$\frac{C_2}{700 + C_2} = 0.98 \quad \text{or} \quad C_2 = \frac{700 \times 0.98}{0.02} = ₹\ 34,300.$$

∴ The value of shortage cost ranges from ₹ 22,633.33 to ₹ 34,300.

EXAMPLE 12.6-5

The cost of holding an item in stock is ₹ 2 per unit and the shortage cost is ₹ 8. If ₹ 2 is the purchasing cost per unit, determine the optimal order level of inventory, given the following probability distribution:

R	:	*0*	*1*	*2*	*3*	*4*	*5*
p(R)	:	*0.05*	*0.25*	*0.20*	*0.15*	*0.20*	*0.15*

Solution

Here, C_h = ₹ 2, C_s = ₹ 8, C = ₹ 2.

$$\therefore \quad C_1 = C + C_h - V = ₹\ (2 + 2 + 0) = ₹\ 4,$$

$$C_2 = S - C - \frac{C_h}{2} + V = 0 - 2 - \frac{2}{2} + 8 = ₹\ 5.$$

$$\therefore \quad \frac{C_2}{C_1 + C_2} = \frac{5}{4+5} = \frac{5}{9} = 0.56.$$

Cumulative probability distribution is

R	:	0	1	2	3	4	5
$\sum_{R=0}^{I_m} p(R)$	:	0.05	0.30	0.50	0.65	0.85	1.00

The desired optimum value for I_m is determined by the double inequality

$$p_{R \le I_m - 1} < \frac{C_2}{C_1 + C_2} < p_{R \le I_m}.$$

This suggests that I_m must lie between 2 and 3 because $0.50 < 0.56 < 0.65$.

$\therefore \quad I_{m_0} = 3.$

12.6-2 Model 3 (b) (Instantaneous Demand, No Setup Cost, Stock Levels Continuous, Lead Time zero)

In this model, all conditions are same as in model 3(*a*) except that the stock levels are continuous (rather than discrete). Therefore, probability $f(R)\, dR$ will be used instead of p_R, where $f(R)$ is the probability density function of the demand rate R.

Then the optimal order quantity I_{m_0} is determined when the value of cumulative probability distribution is equal to $\frac{C_2}{C_1 + C_2}$ by computing

$$\int_{R=0}^{I_m} f(R)\,.\,dR = \frac{C_2}{C_1 + C_2}.$$

Example 12.6-6

A baking company sells one of its types of cakes by weight. It makes a profit of 95 paise a pound on every pound of cake sold on the day it is baked. It disposes of all cakes not sold on the day they are baked at a loss of 15 paise a pound. If demand is known to be rectangular between 3,000 and 4,000 pounds, determine the optimum amount to be baked.

[*Delhi M. Sc. (Math.) 1973*]

Solution

Penalty cost/unit of oversupply, C_1 = ₹ 0.15,

Penalty cost/unit of undersupply, C_2 = ₹ 0.95,

R_1 = 3,000 pounds,

R_2 = 4,000 pounds.

$$f(R) = \frac{1}{R_2 - R_1} = \frac{1}{1{,}000}.$$

Optimum value of I_m is given by

$$\int_0^{I_m} f(R)\,.\,dR = \frac{C_2}{C_1 + C_2}.$$

$$\therefore \quad \int_{3{,}000}^{I_m} \frac{1}{1{,}000}\,.\,dR = \frac{0.95}{0.15 + 0.95} = \frac{0.95}{1.1}.$$

$$\therefore \quad \frac{1}{1{,}000}[I_m - 3{,}000] = \frac{0.95}{1.1}$$

$$\text{or} \quad I_m = \frac{950}{1.1} + 3{,}000 = 3{,}864 \text{ units.}$$

EXAMPLE 12.6-7

A baking company sells one of its types of cake by weight. It makes a profit of 95 paise a pound on every pound of cake sold on the day it is baked. It disposes of all cakes not sold on the day they are baked at a loss of 15 paise a pound. If demand is known to have probability density function

$$f(R) = 0.03 - 0.0003\,R,$$

find the optimum amount of cake the company should bake daily.

Solution

Using the relation

$$\int_0^{I_m} f(R)\,.\,dR = \frac{C_2}{C_1 + C_2}, \text{ we get}$$

$$\int_0^{I_m} (0.03 - 0.0003R)\,dR = \frac{0.95}{0.15 + 0.95} = \frac{0.95}{1.1}.$$

$$\therefore \quad 0.03\,I_m - \frac{0.0003\,I_m^2}{2} = \frac{0.95}{1.1}$$

or $\quad 0.03\,I_m - 0.00015\,I_m^2 = 0.8636$

or $\quad 3{,}000\,I_m - 15\,I_m^2 = 86{,}360$

or $\quad 200\,I_m - I_m^2 = 5{,}757$

or $\quad I_m^2 - 200\,I_m + 5{,}757 = 0$

or $\quad I_m = \dfrac{200 \pm \sqrt{(200)^2 - 4 \times 5{,}757}}{2} = 165.15$ or 34.85 pounds.

$I_m = 165.15$ pounds is not feasible since the given probability distribution of R is not applicable above 100 pounds.

$\therefore$ Optimum value of $I_m = 34.85$ pounds/day.

EXAMPLE 12.6-8

A fish stall sells a variety of fish at the rate of ₹ 5 per kg on the day of the catch. If the stall fails to sell the catch on the same day, it pays for storage at the rate of ₹ 0.30 per kg and the price fetched is ₹ 4.50 per kg on the next day. Past records show that there is an unlimited demand for fish one day old. The problem is to ascertain how much fish should be procured every day so that the total expected cost is minimum. It has been found from the past records that the daily demand follows an exponential distribution with

$$f(x) = 0.02\,e^{-0.02x},\ 0 \le x \le \infty.$$

[*Bombay B.Sc.(Stat.) 1984; B.Sc. (Appl. Comp.) 1984*]

Solution

The cost of over-stocking 1 kg of fish, C_1 = Re. [0.30 + (5 – 4.50)] = Re. 0.80.

$$\therefore \quad \int_0^{I_m} f(x)\,.\,dx = \frac{C_2}{C_1 + C_2} \text{ yields}$$

$$\int_0^{I_m} 0.02\,e^{-0.02x}\,.\,dx = \frac{5}{5 + 0.80} = 0.862$$

or $\quad 0.02\left[\dfrac{e^{-0.02x}}{-0.02}\right]_0^{I_m} = 0.862$

or $\quad -\left[e^{-0.02 I_m} - 1\right] = 0.862$

$\therefore \quad e^{-0.02\,I_m} = 0.138 \quad \therefore\ I_{m_0} = 100.$

12.6-3 Model 4 (*a*) (Continuous Demand, Setup Cost Zero, Stock Levels Discrete, Lead Time Zero)

This model is similar to model 3(*a*) with the difference that demand is continuous rather than instantaneous *i.e.*, withdrawals from stock are continuous rather than instantaneous. Also the rate of withdrawals is assumed to be constant.

The reorder time is assumed to be fixed and known; hence setup cost is not included in calculations. Production is assumed to be instantaneous and lead time negligibly small.

The problem is to determine the optimal order level I_m where $R \le I_m$ or $R > I_m$, at the beginning of each time period. The variation of inventory with time for these two cases is shown in Fig. 12.8(*a*) and (*b*).

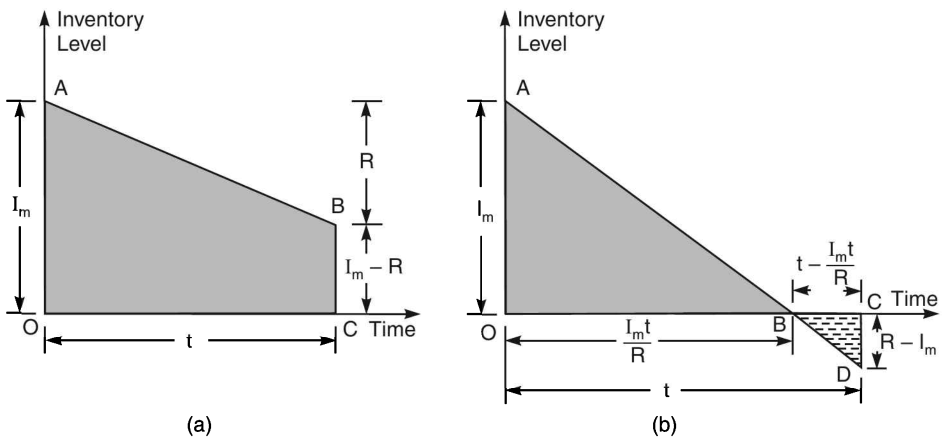

Fig. 12.8. Inventory situation for model 4(a); (a) $R \le I_m$ (b) $R > I_m$.

When $R \le I_m$ as shown in Fig. 12.8(*a*), there are no shortages; when $R > I_m$, as shown in figure 12.8(*b*), shortages occur.

It can be shown that for optimum stock level,

$$\sum_{R=0}^{I_m-1} p_R + \left(I_m - \frac{1}{2}\right) \sum_{R=I_m}^{\infty} \frac{p_R}{R} < \frac{C_2}{C_1 + C_2} < \sum_{R=0}^{I_m} p_R + \left(I_m + \frac{1}{2}\right) \sum_{R=I_m+1}^{\infty} \frac{p_R}{R}$$

EXAMPLE 12.6-9

The probability distribution of monthly sales of a certain item is as follows:

Monthly sales	0	1	2	3	4	5	6	7	8
Probability	0.01	0.04	0.25	0.30	0.23	0.08	0.05	0.03	0.01

The cost of holding inventory is ₹ 8 per unit per month. A stock of 5 items is maintained at the start of each month. If the shortage cost is proportional to both time and quantity short, find the imputed cost of shortage of unit item for unit time.

Solution

As the problem is stated in discrete units, the answer will consist of a range of values for the imputed cost. Here,

optimum inventory, $I_m = 5$ units,

holding cost, C_1 = ₹ 8 per unit per month.

Range of monthly sales = 0 to 8.

Probability p_R for sale R in each month is

p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
0.01	0.04	0.25	0.30	0.23	0.08	0.05	0.03	0.01

Range of optimum value of I_m is given by

$$\sum_{R=0}^{I_m-1} p_R + \left(I_m - \frac{1}{2}\right)\sum_{R=I_m}^{\infty} \frac{p_R}{R} < \frac{C_2}{C_1+C_2} < \sum_{R=0}^{I_m} p_R + \left(I_m + \frac{1}{2}\right)\sum_{R=I_m+1}^{\infty} \frac{p_R}{R}$$

$\therefore$ The least value of C_2 is given by

$$\sum_{R=0}^{I_m-1} p_R + \left(I_m - \frac{1}{2}\right)\sum_{R=I_m}^{\infty} \frac{p_R}{R} = \frac{C_2}{C_1+C_2}$$

or $$\sum_{R=0}^{4} p_R + \left(5 - \frac{1}{2}\right)\sum_{R=5}^{8} \frac{p_R}{R} = \frac{C_2}{8+C_2}$$

or $$(p_0+p_1+p_2+p_3+p_4) + \frac{9}{2}\left(\frac{p_5}{5}+\frac{p_6}{6}+\frac{p_7}{7}+\frac{p_8}{8}\right) = \frac{C_2}{8+C_2}$$

or $$(0.01 + 0.04 + 0.25 + 0.30 + 0.23) + \frac{9}{2}\left(\frac{0.08}{5}+\frac{0.05}{6}+\frac{0.03}{7}+\frac{0.01}{8}\right) = \frac{C_2}{8+C_2}$$

or $$0.83 + 4.5\ (0.016 + 0.0083 + 0.0043 + 0.00125) = \frac{C_2}{8+C_2}$$

or $0.83 + 4.5 \times 0.02985 = \dfrac{C_2}{8+C_2}$ or $0.9643 = \dfrac{C_2}{8+C_2}$ or $C_2 = 216$.

Similarly, the greatest value of C_2 is given by

$$\sum_{R=0}^{I_m} p_R + \left(I_m + \frac{1}{2}\right)\sum_{R=I_m+1}^{\infty} \frac{p_R}{R} = \frac{C_2}{C_1+C_2}$$

or $$\sum_{0}^{5} p_R + \left(5 + \frac{1}{2}\right)\sum_{R=6}^{8} \frac{p_R}{R} = \frac{C_2}{8+C_2}$$

or $$(p_0+p_1+p_2+p_3+p_4+p_5) + \frac{11}{2}\left(\frac{p_6}{6}+\frac{p_7}{7}+\frac{p_8}{8}\right) = \frac{C_2}{8+C_2}$$

or $$(0.01 + 0.04 + 0.25 + 0.30 + 0.23 + 0.08) + \frac{11}{2}\left(\frac{0.05}{6}+\frac{0.03}{7}+\frac{0.01}{8}\right) = \frac{C_2}{8+C_2}$$

or $0.91 + \dfrac{11}{2}(0.01385) = \dfrac{C_2}{8+C_2}$ or $0.91 + 0.076165 = \dfrac{C_2}{8+C_2}$ or $C_2 = ₹\ 570.25$.

$\therefore$ Range of values for the imputed cost C_2 is ₹ $216 < C_2 <$ ₹ 570.25.

12.6-4 Model 4 (b) (Continuous Demand, Setup Cost Zero, Continuous Stock Levels, Lead Time Zero)

In this model, all conditions are same as in model 4 (*a*) except that the stock levels are continuous (rather than discrete). Therefore, probability $f(R)\,dR$ will be used instead of p_R, where $f(R)$ is the probability density function of the demand rate R.

Then the optimal order quantity I_{m_0} is given by

$$\int_0^{I_{m_0}} f(R)\,.\,dR + \int_{I_{m_0}}^{\infty} I_m \,.\, \frac{f(R)}{R}\,.\,dR = \frac{C_2}{C_1+C_2}.$$

EXAMPLE 12.6-10

Let the probability density of demand of a certain item during a week be

$$f(x) = \begin{cases} 0.1, 0 \le x \le 10; \\ 0, \text{otherwise}. \end{cases}$$

This demand is assumed to occur with a uniform pattern over the week. Let the unit carrying cost of the item in inventory be ₹ 2 per week and unit shortage cost be ₹ 8 per week. How will you determine the optimal order level of the inventory ? [*Agra M. Stat., 1973*]

Solution

Here $f(x) = 0.1,\ 0 \le x \le 10,$
$C_1 = ₹\ 2/\text{week},$
$C_2 = ₹\ 8/\text{week}.$

As the demand is uniform over the week, the optimum order level of the inventory, I_m is given by

$$\int_0^{I_m} f(x)\ .\ dx + I_m \int_{I_m}^{\infty} \frac{f(x)}{x}\ .\ dx = \frac{C_2}{C_1 + C_2}$$

or $$\int_0^{I_m} 0.1 dx + I_m \int_{I_m}^{10} \frac{0.1}{x}\ .\ dx = \frac{8}{2+8}$$

or $$0.1\ I_m + 0.1\ I_m (\log 10 - \log I_m) = 0.8$$

or $$I_m + 2.3\ I_m - I_m \log I_m = 8$$

or $$3.3\ I_m - I_m \log I_m = 8.$$

On solving the equation by trial and error method, we get $I_m = 4.5$.

EXERCISES 12.3

1. Discuss any stochastic model of inventory management. Derive the formula of optimum level of inventory. [*Dibrugarh M.Sc. (Stat.) 1994*]
2. Discuss the problem of inventory control when the stochastic demand is uniform, production is instantaneous and lead time is negligible (discrete case). [*Meerut M.Sc. (Math.) 1995*]
3. Discuss the continuous case of a probabilistic inventory model with instantaneous demand and no setup cost. [*Dibrugarh M.Sc. (Stat.) 1994*]

Section 12.6-1 to 12.6-4

4. A newspaper boy buys papers for 30 paise and sells them for 70 paise each. He cannot return unsold newspapers. Daily demand has the following distribution:

No. of customers	:	23	24	25	26	27	28	29	30	31	32
Probability	:	0.01	0.03	0.06	0.10	0.20	0.25	0.15	0.10	0.05	0.05

If each day's demand is independent of the previous day's, how many papers should he order each day ? [*Meerut M.Sc. (Stat.) 1971, 74; Mumbai B.Sc. (Stat.) 1975*]
(*Ans.* 28.)

5. The probability distribution of monthly sales of a certain item is as follows:

Monthly sales	:	0	1	2	3	4	5	6
Probability	:	0.01	0.06	0.25	0.35	0.20	0.03	0.10

The cost of carrying inventory is ₹ 30 per unit per month and the cost of unit shortage is ₹ 70 per month. Determine the optimum stock level which minimizes the total expected cost. [*Delhi M.Sc. (Math.) 1976*]
(*Ans.* 3.)

6. An electric company is about to order a new generator for its plant. One of the essential parts of the generator is expensive and complicated. Its failure cannot be foreseen and its failure leads to the breakdown of the generator. Each of these parts is uniquely built for a particular generator and may

not be used on any other. The cost of part when ordered with generator is ₹ 4,000. If a spare part is required and is not available, the cost of having the part to order plus the cost of down time is ₹ 80,000. The planned life of the generator is 20 years and records of similar parts in similar generators give the following information:

No. of spare parts required in 20 years	*No. of generators requiring indicated no. of spares*	*Estimated probability of indicated no. of failures*
0	90	0.90
1	5	0.05
2	2	0.02
3	1	0.01
4	1	0.01
5	1	0.01
6 or more	0	0.00

How many spare parts should be incorporated in the order for each generator ? (*Ans.* 2.)

7. An ice cream company sells one of its types of ice cream by weight. If the product is not sold on the day it is prepared, it can be sold at a loss of 50 paise per pound. There is, however, an unlimited market for one day old ice cream. On the other hand the company makes a profit of ₹ 3.20 on every pound of ice cream sold on the day it is prepared. Past daily orders form a distribution with

$$f(x) = 0.02 - 0.0002\,x,\ 0 \le x \le 100.$$

How many pounds of ice cream should the company prepare every day ?

[*Baroda B.Sc. (Math). 1978; Agra M.Sc. (Stat.) 1974*]

(*Ans.* 63.3 pounds.)

8. A baking company sells cake by the pound. It makes a profit of 50 paise a pound on every pound sold on the day it is baked. It disposes of all cakes not sold on the day it is baked at a loss of 12 paise a pound. If the demand is known to be rectangular between 2,000 and 3,000 pounds, determine the optimum daily amount baked ? [*J.N.T.U. Hyderabad B.Tech. Nov., 2010; Meerut M.Sc. (Math.)* 1969, *Kuru. M.Sc. (Malh.)1977*]

(*Ans.* 2,807 pounds.)

9. An item is sold for ₹ 25 per unit and it costs ₹ 10. Unsold items can be sold for ₹ 4 each. It is assumed that there is no shortage penalty cost besides the lost revenue. The demand is known to be any value between 600 and 1,000 items.
Determine the optimum number of units of the item to be stocked. [*IGNOU MCA, 2003*]

(*Ans.* 886 units.)

10. Show that when considering the optimum level of inventory S_0, which minimizes the total expected cost in case of continuous (non-discrete) quantities, the condition to be satisfied is

$$F(S_0) = \frac{C_2}{C_1 + C_2},$$

where $$F(S_0) = \int_0^{s_0} f(r)\,.\,dr.$$

Here, $f(r)$ = the probability density function of requirement of quantity r,
C_2 = the shortage cost,
C_1 = the holding cost per unit of quantity per unit of time. [*Gujarat Univ. B.E., 1976*]

11. A shopkeeper has to decide how much quantity of bread he should stock every week. The quantity of bread demanded in any week is assumed to be a continuous random variable with a given probability function $f(x)$. Let '*a*' be the unit cost of purchasing bread and '*d*' be the unit penalty cost. Find the optimum quantity of bread to be stocked. [*I.S.I., 1962*]

12. The probability distribution of monthly sales of a certain item is as follows:

Monthly sales :	0	1	2	3	4	5	6
Probability :	0.02	0.05	0.03	0.27	0.20	0.10	0.06

The cost of carrying inventory is ₹ 10 per unit per month. The current policy is to maintain a stock of 4 items at the beginning of each month. Assuming that the cost of shortage is proportional to both time and quantity short, obtain the imputed cost of shortage of one item for one time unit.

[*Meerut M.Sc. (Stat.) 1970*]

(*Ans.* Between ₹ 17.78 and ₹ 52.50.)