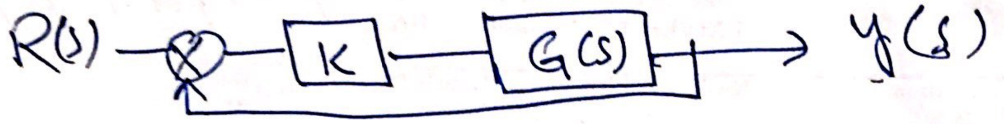
Root Locus

(1)



$G(s) = \frac{1}{s+1}$

$$\frac{Y(s)}{R(s)} = \boxed{K \frac{1}{s+1}} \qquad s = -1$$

$$\frac{Y}{R} = \frac{KG(s)}{\boxed{1 + KG(s)}}, \qquad \frac{E}{R} = \frac{1}{\boxed{1+KG(s)}}$$

↳ xtic polynomial.

$$\boxed{Y/R = \frac{K}{s+1+K}} \qquad s = -1$$

e.g.,

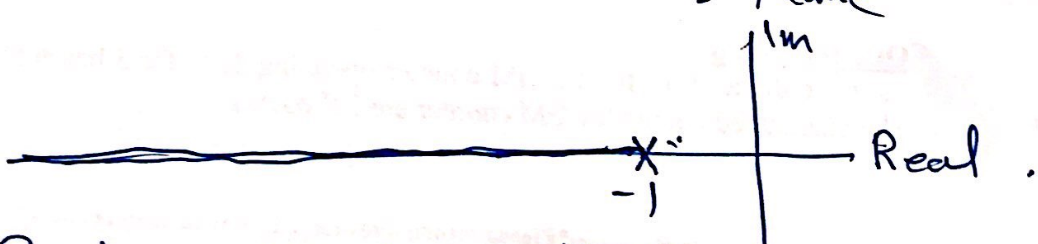
$K = 0 \quad s = -1$

$K = 1 \quad s = -2$

$K = 2 \quad s = -3$

$K \to \boxed{0 \longrightarrow \infty}$

$$\frac{Y}{R} = \frac{K}{s^2 + 10s + K} \qquad K = 1, 2, 3 \ldots$$



Root → we know that it is a solution of a poly.

Locus → Path.

K_p $\qquad$ $K(s)$ $\qquad$ $\rightarrow \boxed{K(s)} \rightarrow$

$$K(s) = K_0 \, K'(s)$$

$$1 + K(s)G(s) = 0$$

$$1 + K_0 \underbrace{K'(s)G(s)}_{L(s)} = 0$$

$$\boxed{1 + K_0 L(s) = 0}$$

↳ closed loop xtic eqn.

$$\boxed{K_0 L(s)} = \boxed{-1 + 0j}$$

$\boxed{s = \sigma + j\omega_d}$ complex no.

phase and magnitude.

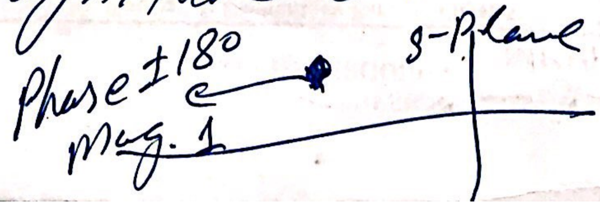
Real = Real

Im = Im

$-1 + 0j = 1\angle \pm 180^\circ$

① Phase Condition:→ Every pt. of the s-plane which has phase $= \pm 180^\circ$ is on the locus provided

② Magnitude Condition:→ Mag. is $= 1$



$$1 + K_o L(s) = 0 \rightarrow (1)$$

$$K_o L(s) = -1 = 1 \angle \pm 180^\circ \rightarrow (2)$$

A solution of (1) (i.e., values of s satisfying (1)) will give the C/L poles and as (2) indicated, these C/L poles will satisfy the two conditions

(1) Phase Condition

(2) Magnitude Condition

i.e., The values of s for which

$$|K_o L(s)| = 1 \quad \text{and} \quad \angle K_o L(s) = \pm 180^\circ$$

for some K_o, are the C/L poles.

Root Locus :→ A R/L is a graphical $\boxed{\text{solution}}$ of (2) for diff. values of K_o

Drawing the Root Locus.

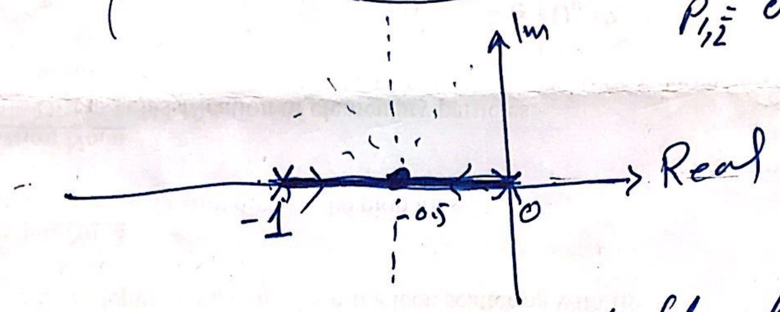
$1 + K_o L(s) = 0 \rightarrow (1)$

$L(s) =$ O/L T/F.

(1) The locus starts from O/L poles and goes to O/L zeros or infinity.

(2) ~~There~~ The number of branches of the R/L are equal to the no. of O/L poles.

$L(s) = \frac{1}{\boxed{s(s+1)}}$ $\quad P = 2$, $z = 0$, $P_{1,2} = 0, -1$



(3) The locus is to the left of an odd no. of poles and zeros.

(4) Centre and angle of asymptotes.

how many asymptotes $= P - Z$

$$\alpha = \frac{\Sigma P_i - \Sigma z_i}{P - Z} \Rightarrow \frac{0 - 1 - 0}{2 - 0}$$

$$\boxed{\alpha = \frac{-1}{2} = -0.5}$$

$j = 1$

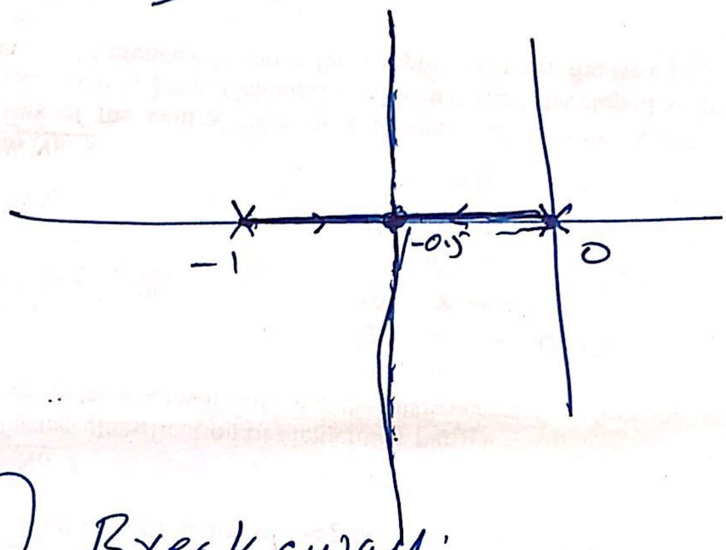
$$\phi_j = \frac{180^\circ + 360^\circ (j-1)}{P - Z}$$

$$\phi_1 = \frac{180^\circ}{2} = 90^\circ$$

$i = 2$

$$\phi_2 = \frac{180^\circ + 360^\circ}{2} = \frac{540^\circ}{2}$$

$$= 270^\circ = -90^\circ$$



5 Breakaway:

$$L(s) = \frac{1}{s^2+s} = \frac{1}{s(s+1)} = \frac{a(s)}{b(s)}$$

$$b(s)\frac{da(s)}{ds} - a(s)\frac{db(s)}{ds} = 0$$

$$0 - 1\frac{d}{ds}(s^2+s) = 0$$

$$-2s - 1 = 0$$

$$-2s = +1$$

$\boxed{s = -0.5}$ breakaway

Ex:- $L(s) = \dfrac{s+10}{s+5}$

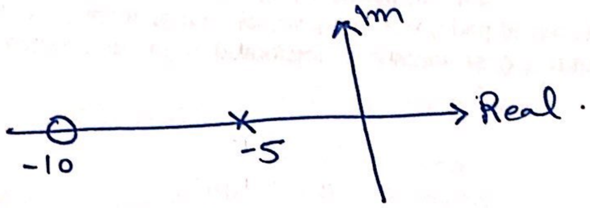
no. of poles = $P = 1$, $P_1 = -5$

" " zeros = $Z = 1$, $P_2 = -10$

no. of branches = $P = 1$

no. of assymptotes = $P - Z = 1 - 1 = 0$

1



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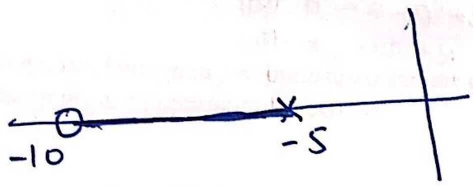

3 No need for centre & angle of assymptotes
∵ no assymptotes.

[Assymptotes are required for branches of the R/L that goes to infinity
⇒ no. branches go to ∞

4 Breakaway :- The pt. on the real axis where multiple branches of the R/L coincides and leave the real axis

$$L(s) = \frac{s+10}{s+5} = \frac{a(s)}{b(s)}$$

$$b(s)\frac{da(s)}{ds} - a(s)\frac{db(s)}{ds} = 0$$

$$(s+5)(1) - (s+10)(1) = 0$$

$$\cancel{s}+5-\cancel{s}-10 = 0$$

$$\boxed{-5 = 0}$$

This uncertainty means there is no valid breakaway.