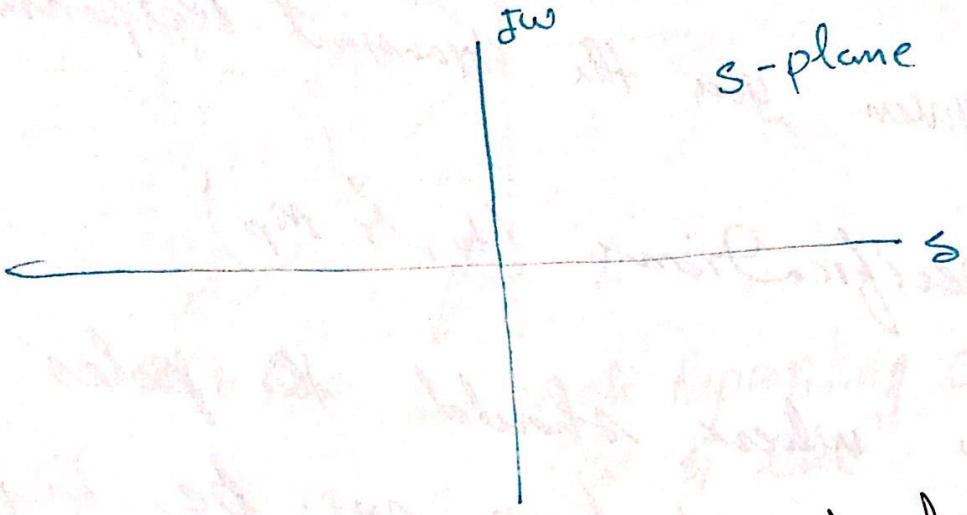
Pole Placement Region

where should we place poles in the s-plane??



why do we talk about pole placement??

Is the big picture clear??

→ Suppose you've a plant with the tf H(s) and you want to control it (operate it as you wish).

Then first by looking at the pole locations we can easily predict the response (e.g to a unit step).

(a) Is the sys stable ($\sigma \not> 0$)

(b) $\theta \geq ?$ $\theta \to 0^\circ$ means undamped sys.

or for bigger θ, $M_p \uparrow$.

(c) how much $\xi\omega_n = \sigma$ is big.

→ In short you will know whether you like the response or not.

→ If yes then OK

→ If NO then suppose someone has given you the transient Response specifications (t_r, t_s, M_p)

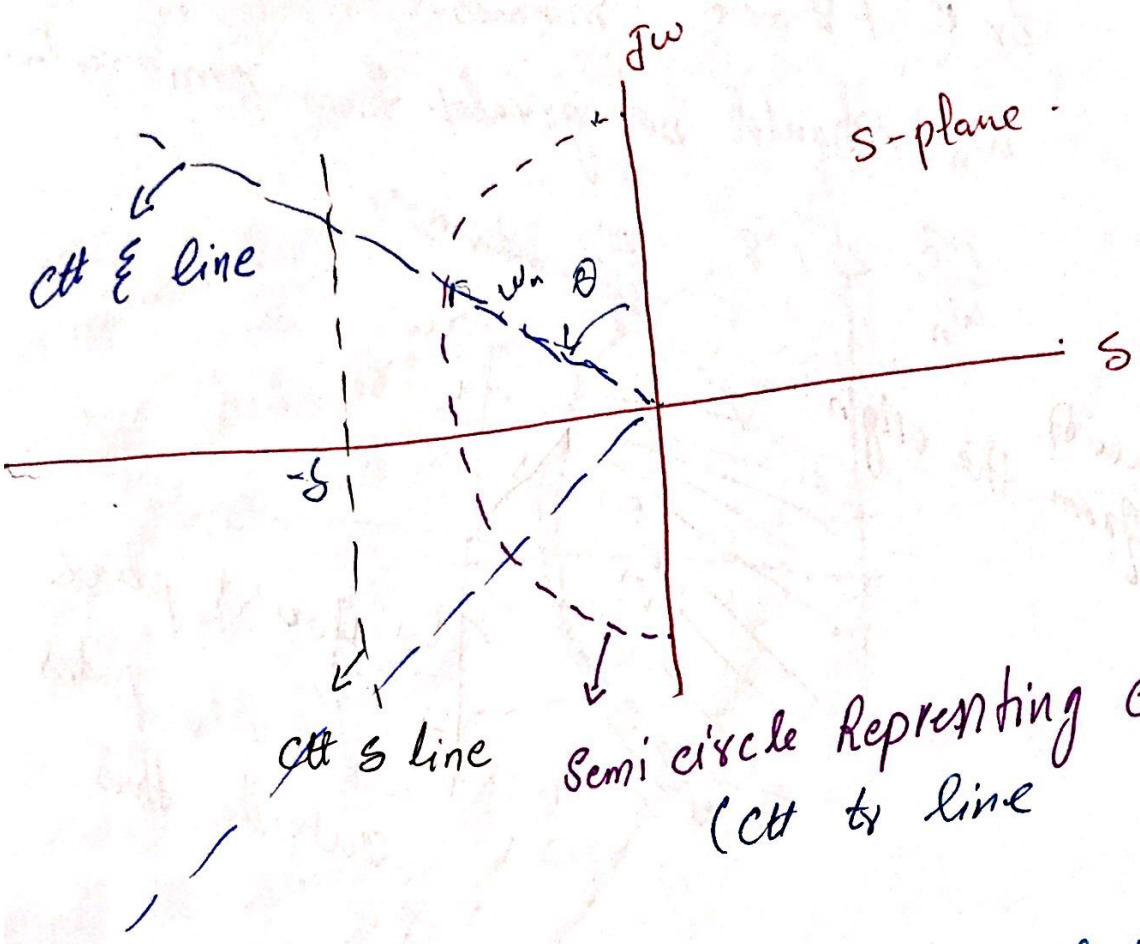
→ Now where should the poles of the closed loop sys: be, in the s-plane s.t (t_r, t_s & M_p) are satisfied.

→ Which can be achieved by feedback which places the poles at the desired locations.

→ Desired location?

Prior to defining region in terms of t_s, t_r & M_p

Note the following



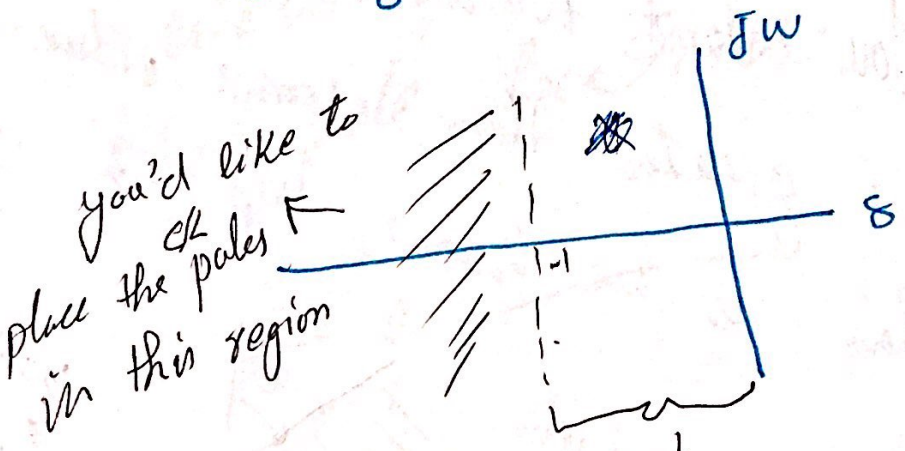
$t_r \cong \frac{1.8}{\omega_n}$

$t_s = \frac{4.6}{\sigma}$

ctt σ line Semi circle Representing ctt ω_n
(ctt t_r line

① Now assume we are given that $t_s < 4.6$ sec

$t_s = \frac{4.6}{\sigma} < 4.6 \Rightarrow \sigma > 1$

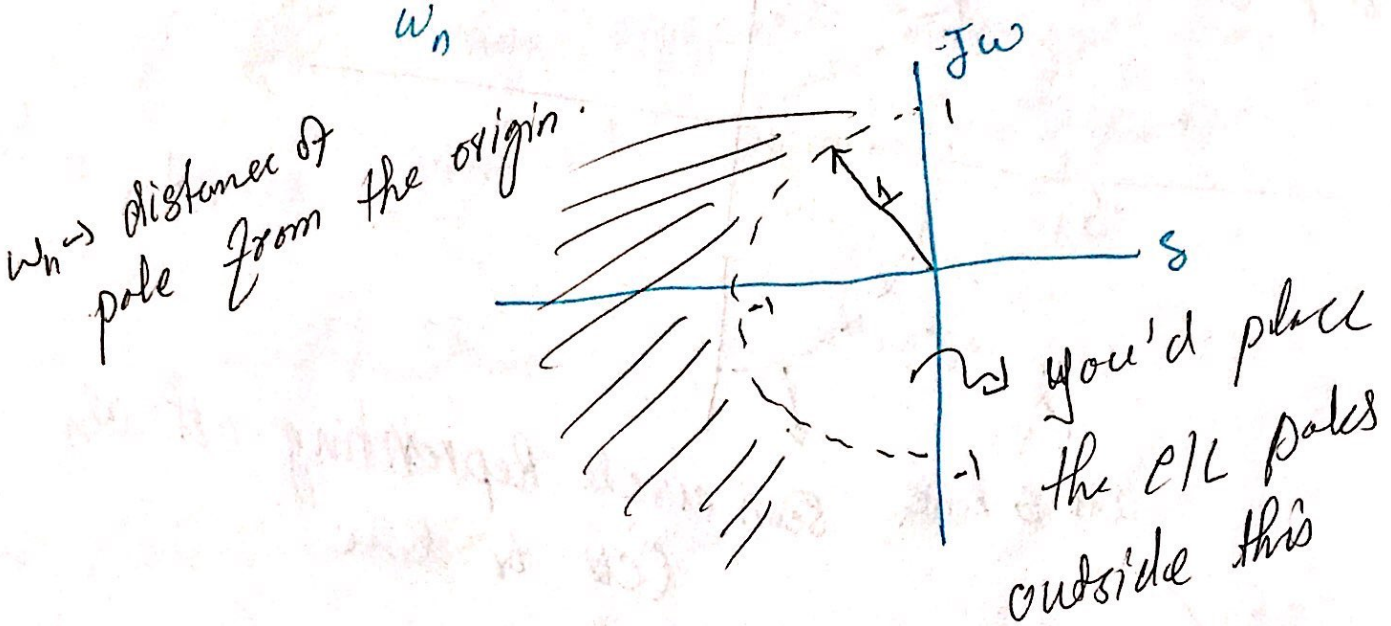


w.r.t specs this is now a forbidden region.

② $t_r < 1.8$ sec means

ω_n should be greater than some value

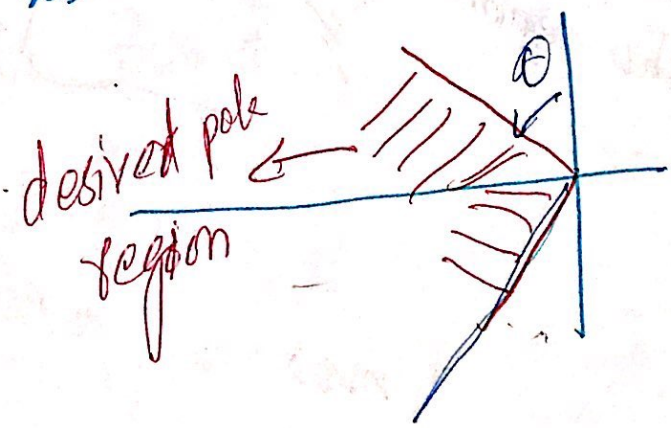
$\frac{1.8}{\omega_n} < 1.8 \Rightarrow \omega_n > 1$



semi circle.

③ You don't want the M_p to be greater than some value.

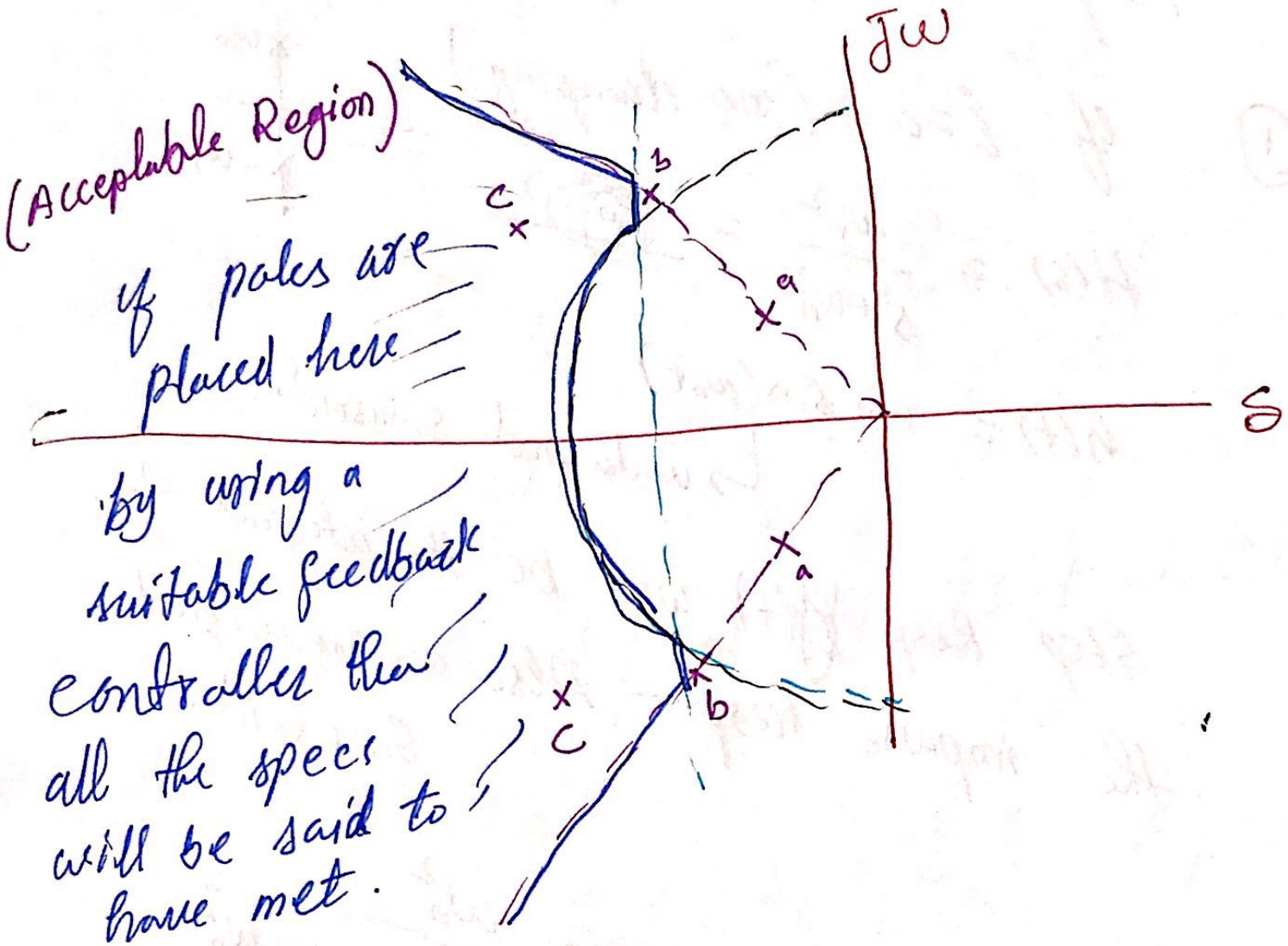
Then as we know $M_p \propto \frac{1}{\xi}$ So



$\theta = \sin^{-1}\xi$

$\eta_5 = \sin^{-1}(0.707)$

→ In general



⇒ We've related time response to the pole locations.