CHAPTER

3

Central Tendency

Tools You Will Need

The following items are considered essential background material for this chapter. If you doubt your knowledge of any of these items, you should review the appropriate chapter or section before proceeding.

- Summation notation (Chapter 1)
- Frequency distributions (Chapter 2)

Preview

Research has now confirmed what you already suspected to be true—alcohol consumption increases the attractiveness of opposite-sex individuals (Jones, Jones, Thomas, & Piper, 2003). In the study, college-age participants were recruited from bars and restaurants near campus and asked to participate in a "market research" study. During the introductory conversation, they were asked to report their alcohol consumption for the day and were told that moderate consumption would not prevent them from taking part in the study. Participants were then shown a series of photographs of male and female faces and asked to rate the attractiveness of each face on a scale from 1 to 7. Figure 3.1 shows the general pattern of results obtained in the study. The two polygons in the figure show the distributions of attractiveness ratings for one female photograph obtained from two groups of males: those who had no alcohol and those with moderate alcohol consumption. Note that the attractiveness ratings from the alcohol group are noticeably higher than the ratings from the no-alcohol group. Incidentally, the same pattern of results was obtained for the female's ratings of male photographs.

The Problem: Although it seems obvious that the moderate-alcohol ratings are noticeably higher than the no-alcohol ratings, this conclusion is based on a general impression, or a subjective interpretation, of the figure. In fact, this conclusion is not always true. For example, there is overlap between the two groups so that some of the no-alcohol males actually rate the photograph as more attractive than some of the moderate-alcohol males. What we need is a method to summarize each group as a whole so that we can objectively describe how much difference exists between the two groups.

The Solution: A measure of *central tendency* identifies the average, or typical, score to serve as a representative value for each group. Then we can use the two averages to describe the two groups and to measure the difference between them. The results should show that average attractiveness rating from males consuming alcohol really is higher than the average rating from males who have not consumed alcohol.

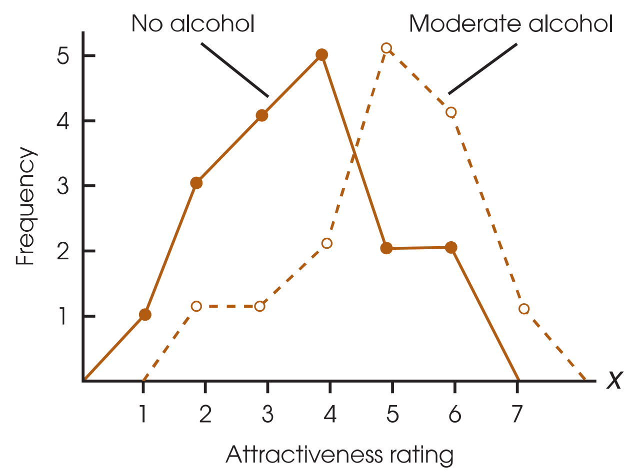


FIGURE 3.1

Frequency distributions for ratings of attractiveness of a female face shown in a photograph for two groups of male participants: those who had consumed no alcohol and those who had consumed moderate amounts of alcohol.

3.1 OVERVIEW

The general purpose of descriptive statistical methods is to organize and summarize a set of scores. Perhaps the most common method for summarizing and describing a distribution is to find a single value that defines the average score and can serve as a representative for the entire distribution. In statistics, the concept of an average, or representative, score is called *central tendency*. The goal in measuring central tendency is to describe a distribution of scores by determining a single value that identifies the center of the distribution. Ideally, this central value is the score that is the best representative value for all of the individuals in the distribution.

DEFINITION

Central tendency is a statistical measure to determine a single score that defines the center of a distribution. The goal of central tendency is to find the single score that is most typical or most representative of the entire group.

In everyday language, central tendency attempts to identify the "average" or "typical" individual. This average value can then be used to provide a simple description of an entire population or a sample. In addition to describing an entire distribution, measures of central tendency are also useful for making comparisons between groups of individuals or between sets of figures. For example, weather data indicate that for Seattle, Washington, the average yearly temperature is 53° and the average annual precipitation is 34 inches. By comparison, the average temperature in Phoenix, Arizona, is 71° and the average precipitation is 7.4 inches. The point of these examples is to demonstrate the great advantage of being able to describe a large set of data with a single, representative number. Central tendency characterizes what is typical for a large population and, in doing so, makes large amounts of data more digestible. Statisticians sometimes use the expression *number crunching* to illustrate this aspect of data description. That is, we take a distribution consisting of many scores and "crunch" them down to a single value that describes them all.

Unfortunately, there is no single, standard procedure for determining central tendency. The problem is that no single measure produces a central, representative value in every situation. The three distributions shown in Figure 3.2 should help demonstrate this fact. Before we discuss the three distributions, take a moment to look at the figure and try to identify the center or the most representative score for each distribution.

1. The first distribution [Figure 3.2(a)] is symmetrical, with the scores forming a distinct pile centered around $X = 5$. For this type of distribution, it is easy to identify the center, and most people would agree that the value $X = 5$ is an appropriate measure of central tendency.
2. In the second distribution [Figure 3.2(b)], however, problems begin to appear. Now the scores form a negatively skewed distribution, piling up at the high end of the scale around $X = 8$, but tapering off to the left all the way down to $X = 1$. Where is the center in this case? Some people might select $X = 8$ as the center because more individuals had this score than any other single value. However, $X = 8$ is clearly not in the middle of the distribution. In fact, the majority of the scores (10 out of 16) have values less than 8, so it seems reasonable that the center should be defined by a value that is less than 8.
3. Now consider the third distribution [Figure 3.2(c)]. Again, the distribution is symmetrical, but now there are two distinct piles of scores. Because the distribution is symmetrical with $X = 5$ as the midpoint, you may choose $X = 5$ as the center. However, none of the scores is located at $X = 5$ (or even close), so this value is not particularly good as a representative score. On the other hand, because there are two separate piles of scores with one group centered at $X = 2$ and the other centered at $X = 8$, it is tempting to say that this distribution has two centers. But can one distribution have two centers?

Clearly, there can be problems defining the center of a distribution. Occasionally, you will find a nice, neat distribution like the one shown in Figure 3.2(a), for which everyone agrees on the center. But you should realize that other distributions are possible and that there may be different opinions concerning the definition of the center. To deal with these problems, statisticians have developed three different methods for measuring central tendency: the mean, the median, and the mode. They are computed

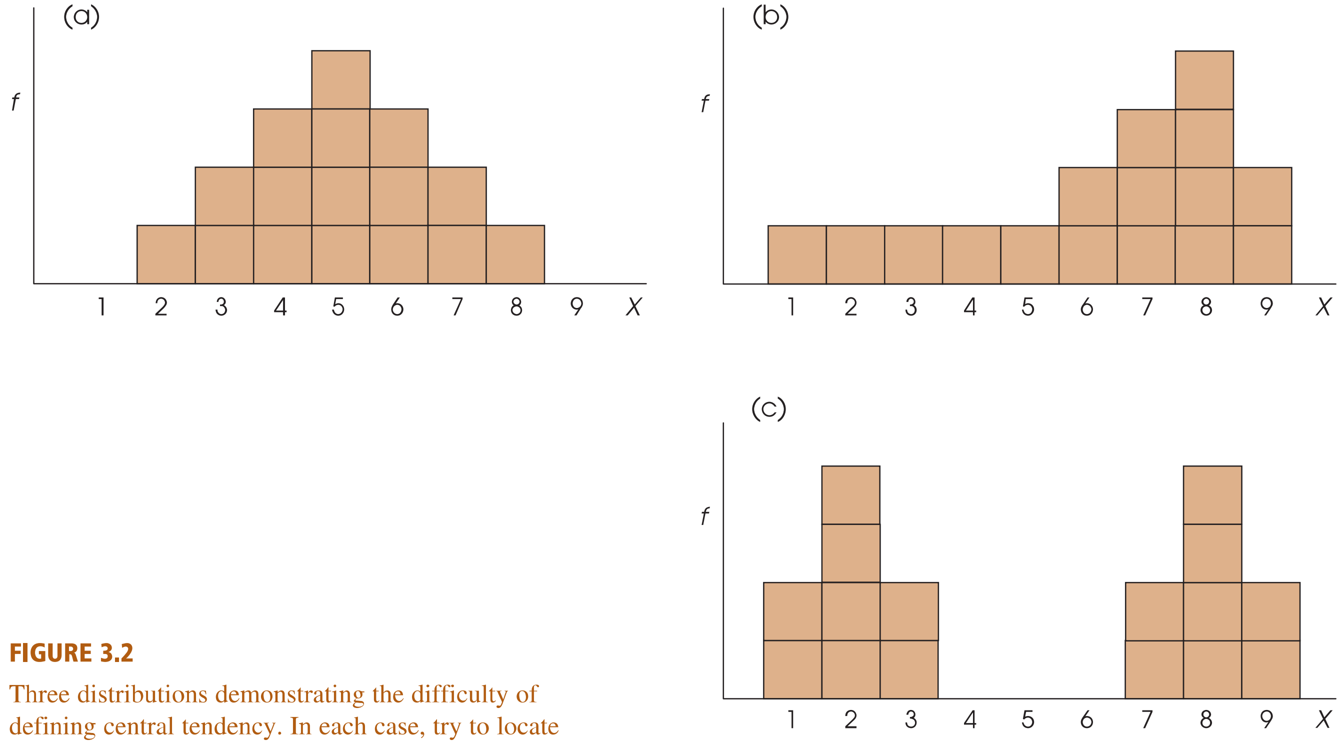


FIGURE 3.2

Three distributions demonstrating the difficulty of defining central tendency. In each case, try to locate the "center" of the distribution.

differently and have different characteristics. To decide which of the three measures is best for any particular distribution, you should keep in mind that the general purpose of central tendency is to find the single most representative score. Each of the three measures we present has been developed to work best in a specific situation. We examine this issue in more detail after we introduce the three measures.

3.2 THE MEAN

The *mean*, also known as the arithmetic average, is computed by adding all the scores in the distribution and dividing by the number of scores. The mean for a population is identified by the Greek letter mu, μ (pronounced "mew"), and the mean for a sample is identified by M or $\overline{X}$ (read "x-bar").

The convention in many statistics textbooks is to use $\overline{X}$ to represent the mean for a sample. However, in manuscripts and in published research reports the letter M is the standard notation for a sample mean. Because you will encounter the letter M when reading research reports and because you should use the letter M when writing research reports, we have decided to use the same notation in this text. Keep in mind that the $\overline{X}$ notation is still appropriate for identifying a sample mean, and you may find it used on occasion, especially in textbooks.

DEFINITION

The **mean** for a distribution is the sum of the scores divided by the number of scores.

The formula for the *population mean* is

$$\mu = \frac{\sum X}{N} \tag{3.1}$$

First, add all the scores in the population, and then divide by N. For a sample, the computation is exactly the same, but the formula for the *sample mean* uses symbols that signify sample values:

$$\text{sample mean} = M = \frac{\sum X}{n} \tag{3.2}$$

In general, we use Greek letters to identify characteristics of a population (parameters) and letters of our own alphabet to stand for sample values (statistics). If a mean is identified with the symbol M, you should realize that we are dealing with a sample. Also note that the equation for the sample mean uses a lowercase n as the symbol for the number of scores in the sample.

EXAMPLE 3.1 For a population of $N = 4$ scores,

3, 7, 4, 6

the mean is

$$\mu = \frac{\Sigma X}{N} = \frac{20}{4} = 5$$

ALTERNATIVE DEFINITIONS FOR THE MEAN

Although the procedure of adding the scores and dividing by the number of scores provides a useful definition of the mean, there are two alternative definitions that may give you a better understanding of this important measure of central tendency.

Dividing the total equally The first alternative is to think of the mean as the amount each individual receives when the total (ΣX) is divided equally among all of the individuals (N) in the distribution. This somewhat socialistic viewpoint is particularly useful in problems for which you know the mean and must find the total. Consider the following example.

EXAMPLE 3.2 A group of $n = 6$ boys buys a box of baseball cards at a garage sale and discovers that the box contains a total of 180 cards. If the boys divide the cards equally among themselves, how many cards will each boy get? You should recognize that this problem represents the standard procedure for computing the mean. Specifically, the total (ΣX) is divided by the number (n) to produce the mean, $\frac{180}{6} = 30$ cards for each boy.

The previous example demonstrates that it is possible to define the mean as the amount that each individual gets when the total is distributed equally. This new definition can be useful for some problems involving the mean. Consider the following example.

EXAMPLE 3.3 Now suppose that the 6 boys from Example 3.2 decide to sell their baseball cards on eBay. If they make an average of $M = \$5$ per boy, what is the total amount of money for the whole group? Although you do not know exactly how much money each boy has, the new definition of the mean tells you that if they pool their money together and then distribute the total equally, each boy will get \$5. For each of $n = 6$ boys to get \$5, the total must be 6(\$5) = \$30. To check this answer, use the formula for the mean:

$$M = \frac{\Sigma X}{n} = \frac{\$30}{6} = \$5$$

The mean as a balance point The second alternative definition of the mean describes the mean as a balance point for the distribution. Consider a population consisting of $N = 5$ scores (1, 2, 6, 6, 10). For this population, $\Sigma X = 25$ and $\mu = \frac{25}{5} = 5$. Figure 3.3 shows this population drawn as a histogram, with each score represented as a box that is sitting on a seesaw. If the seesaw is positioned so that it pivots at a point equal to the mean, then it will be balanced and will rest level.

The reason that the seesaw is balanced over the mean becomes clear when we measures the distance of each box (score) from the mean:

Score	Distance from the Mean
$X = 1$	4 points below the mean
$X = 2$	3 points below the mean
$X = 6$	1 point above the mean
$X = 6$	1 point above the mean
$X = 10$	5 points above the mean

Notice that the mean balances the distances. That is, the total distance below the mean is the same as the total distance above the mean:

below the mean: $4 + 3 = 7$ points

above the mean: $1 + 1 + 5 = 7$ points

Because the mean serves as a balance point, the value of the mean is always located somewhere between the highest score and the lowest score; that is, the mean can never be outside the range of scores. If the lowest score in a distribution is $X = 8$ and the highest is $X = 15$, then the mean *must* be between 8 and 15. If you calculate a value that is outside this range, then you have made an error.

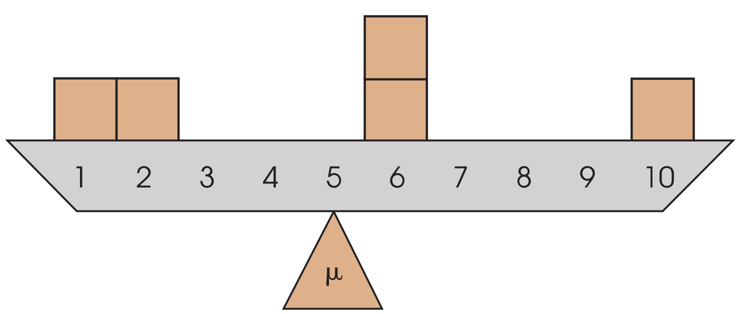


FIGURE 3.3
The frequency distribution shown as a seesaw balanced at the mean.

(Based on G. H. Weinberg, J. A. Schumaker, & D. Oltman (1981). *Statistics: An Intuitive Approach* (p. 14). Belmont, Calif.: Wadsworth.

The image of a seesaw with the mean at the balance point is also useful for determining how a distribution is affected if a new score is added or if an existing score is removed. For the distribution in Figure 3.3, for example, what would happen to the mean (balance point) if a new score were added at $X = 10$?

THE WEIGHTED MEAN

Often it is necessary to combine two sets of scores and then find the overall mean for the combined group. Suppose that we begin with two separate samples. The first sample has $n = 12$ scores and a mean of $M = 6$. The second sample has $n = 8$ and $M = 7$. If the two samples are combined, what is the mean for the total group?

To calculate the overall mean, we need two values:

1. the overall sum of the scores for the combined group (ΣX), and
2. the total number of scores in the combined group (n).

The total number of scores in the combined group can be found easily by adding the number of scores in the first sample (n_1) and the number in the second sample (n_2). In this case, there are $12 + 8 = 20$ scores in the combined group. Similarly, the overall sum for the combined group can be found by adding the sum for the first sample (ΣX_1) and the sum for the second sample (ΣX_2). With these two values, we can compute the mean using the basic equation

$$\text{overall mean} = M = \frac{\Sigma X \text{ (overall sum for the combined group)}}{n \text{ (total number in the combined group)}}$$

$$= \frac{\Sigma X_1 + \Sigma X_2}{n_1 + n_2}$$

To find the sum of the scores for each sample, remember that the mean can be defined as the amount each person receives when the total (ΣX) is distributed equally. The first sample has $n = 12$ and $M = 6$. (Expressed in dollars instead of scores, this sample has $n = 12$ people and each person gets \$6 when the total is divided equally.) For each of 12 people to get $M = 6$, the total must be $\Sigma X = 12 \times 6 = 72$. In the same way, the second sample has $n = 8$ and $M = 7$ so the total must be $\Sigma X = 8 \times 7 = 56$. Using these values, we obtain an overall mean of

$$\text{overall mean} = M = \frac{\Sigma X_1 + \Sigma X_1}{n_1 + n_2} = \frac{72 + 56}{12 + 8} = \frac{128}{20} = 6.4$$

The following table summarizes the calculations.

First Sample	Second Sample	Combined Sample
$n = 12$	$n = 8$	$n = 20$ $(12 + 8)$
$\Sigma X = 72$	$\Sigma X = 56$	$\Sigma X = 128$ $(72 + 56)$
$M = 6$	$M = 7$	$M = 6.4$

Note that the overall mean is not halfway between the original two sample means. Because the samples are not the same size, one makes a larger contribution to the total group and, therefore, carries more weight in determining the overall mean. For this reason, the overall mean we have calculated is called the *weighted mean*. In this example, the overall mean of $M = 6.4$ is closer to the value of $M = 6$ (the larger sample) than it is to $M = 7$ (the smaller sample). An alternative method for finding the weighted mean is presented in Box 3.1.

BOX 3.1 AN ALTERNATIVE PROCEDURE FOR FINDING THE WEIGHTED MEAN

In the text, the weighted mean was obtained by first determining the total number of scores (n) for the two combined samples and then determining the overall sum (ΣX) for the two combined samples. The following example demonstrates how the same result can be obtained using a slightly different conceptual approach.

We begin with the same two samples that were used in the text: One sample has $M = 6$ for $n = 12$ students, and the second sample has $M = 7$ for $n = 8$ students. The goal is to determine the mean for the overall group when the two samples are combined.

Logically, when these two samples are combined, the larger sample (with $n = 12$ scores) will make a greater contribution to the combined group than the smaller sample (with $n = 8$ scores). Thus, the larger sample will carry more weight in determining the mean for the combined group. We accommodate this fact by assigning a weight to each sample mean so that the weight is determined by the size of the sample. To determine how much weight should be assigned to each sample mean, you simply consider the sample's contribution to the combined group. When the two samples are combined, the resulting group will have a total of 20 scores ($n = 12$ from the first sample and $n = 8$ from the second). The first sample contributes 12 out of 20 scores and, therefore, is assigned a weight of $\frac{12}{20}$. The second sample contributes 8 out of 20 scores, and its weight is $\frac{8}{20}$. Each sample mean is then multiplied by its weight, and the results are added to find the weighted mean for the combined sample. For this example,

$$\text{weighted mean} = \left(\frac{12}{20}\right)(6) + \left(\frac{8}{20}\right)(7)$$

$$= \frac{72}{20} + \frac{56}{20}$$

$$= 3.6 + 2.8$$

$$= 6.4$$

Note that this is the same result obtained using the method described in the text.

COMPUTING THE MEAN FROM A FREQUENCY DISTRIBUTION TABLE

When a set of scores has been organized in a frequency distribution table, the calculation of the mean is usually easier if you first remove the individual scores from the table. Table 3.1 shows a distribution of scores organized in a frequency distribution table. To compute the mean for this distribution you must be careful to use both the X values in the first column and the frequencies in the second column. The values in the table show that the distribution consists of one 10, two 9s, four 8s, and one 6, for a total of $n = 8$ scores. Remember that you can determine the number of scores by adding the frequencies, $n = \Sigma f$. To find the sum of the scores, you must be careful to add all eight scores:

$$\Sigma X = 10 + 9 + 9 + 8 + 8 + 8 + 8 + 6 = 66$$

Note that you can also find the sum of the scores by computing ΣfX as we demonstrated in Chapter 2 (pp. 40–41). Once you have found ΣX and n, you compute the mean as usual. For these data,

$$M = \frac{\sum X}{n} = \frac{66}{8} = 8.25$$

TABLE 3.1

Statistics quiz scores for a sample of $n = 8$ students.

Quiz Score (X)	f	fX
10	1	10
9	2	18
8	4	32
7	0	0
6	1	6

LEARNING CHECK

1. Find the mean for the following sample of $n = 5$ scores: 1, 8, 7, 5, 9
2. A sample of $n = 6$ scores has a mean of $M = 8$. What is the value of ΣX for this sample?
3. One sample has $n = 5$ scores with a mean of $M = 4$. A second sample has $n = 3$ scores with a mean of $M = 10$. If the two samples are combined, what is the mean for the combined sample?
4. A sample of $n = 6$ scores has a mean of $M = 40$. One new score is added to the sample and the new mean is found to be $M = 35$. What can you conclude about the value of the new score?
   a. It must be greater 40.
   b. It must be less than 40.
5. Find the values for n, ΣX, and M for the sample that is summarized in the following frequency distribution table.

X	*f*
5	1
4	2
3	3
2	5
1	1

ANSWERS

1. $\Sigma X = 30$ and $M = 6$
2. $\Sigma X = 48$
3. The combined sample has $n = 8$ scores that total $\Sigma X = 50$. The mean is $M = 6.25$.
4. b
5. For this sample, $n = 12$, $\Sigma X = 33$, and $M = \frac{33}{12} = 2.75$.

CHARACTERISTICS OF THE MEAN

The mean has many characteristics that will be important in future discussions. In general, these characteristics result from the fact that every score in the distribution contributes to the value of the mean. Specifically, every score adds to the total (ΣX) and every score contributes one point to the number of scores (n). These two values (ΣX and n) determine the value of the mean. We now discuss four of the more important characteristics of the mean.

Changing a score Changing the value of any score changes the mean. For example, a sample of quiz scores for a psychology lab section consists of 9, 8, 7, 5, and 1. Note that the sample consists of $n = 5$ scores with $\Sigma X = 30$. The mean for this sample is

$$M = \frac{\sum X}{n} = \frac{30}{5} = 6.00$$

Now suppose that the score of $X = 1$ is changed to $X = 8$. Note that we have added 7 points to this individual's score, which also adds 7 points to the total (ΣX). After changing the score, the new distribution consists of

9, 8, 7, 5, 8

There are still $n = 5$ scores, but now the total is $\Sigma X = 37$. Thus, the new mean is

$$M = \frac{\Sigma X}{n} = \frac{37}{5} = 7.40$$

Notice that changing a single score in the sample has produced a new mean. You should recognize that changing any score also changes the value of ΣX (the sum of the scores), and, thus, always changes the value of the mean.

Introducing a new score or removing a score Adding a new score to a distribution, or removing an existing score, usually changes the mean. The exception is when the new score (or the removed score) is exactly equal to the mean. It is easy to visualize the effect of adding or removing a score if you remember that the mean is defined as the balance point for the distribution. Figure 3.4 shows a distribution of scores represented as boxes on a seesaw that is balanced at the mean, $\mu = 7$. Imagine what would happen if we added a new score (a new box) at $X = 10$. Clearly, the seesaw would tip to the right and we would need to move the pivot point (the mean) to the right to restore balance.

Now imagine what would happen if we removed the score (the box) at $X = 9$. This time the seesaw would tip to the left and, once again, we would need to change the mean to restore balance.

Finally, consider what would happen if we added a new score of $X = 7$, exactly equal to the mean. It should be clear that the seesaw would not tilt in either direction, so the mean would stay in exactly the same place. Also note that if we removed the new score at $X = 7$, the seesaw would remain balanced and the mean would not change. In general, adding a new score or removing an existing score causes the mean to change unless the that score is located exactly at the mean.

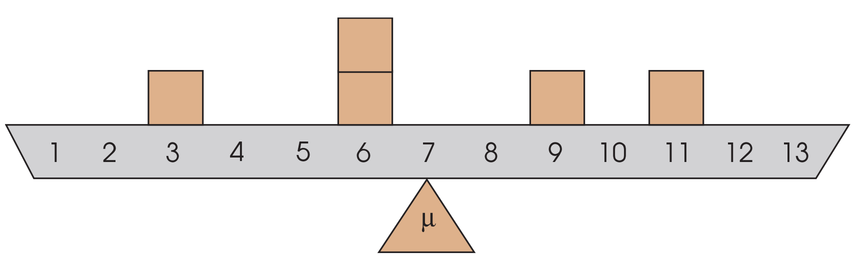
The following example demonstrates exactly how the new mean is computed when a new score is added to an existing sample.

EXAMPLE 3.4

Adding a score (or removing a score) has the same effect on the mean whether the original set of scores is a sample or a population. To demonstrate the calculation of the new mean, we will use the set of scores that is shown in Figure 3.4. This time, however, we will treat the scores as a sample with $n = 5$ and $M = 7$. Note that this sample must have $\Sigma X = 35$. What happens to the mean if a new score of $X = 13$ is added to the sample?

To find the new sample mean, we must determine how the values for n and ΣX are be changed by a new score. We begin with the original sample and then consider the effect of adding the new score. The original sample had $n = 5$ scores, so adding one new score produces $n = 6$. Similarly, the original sample had $\Sigma X = 35$. Adding a score

FIGURE 3.4
A distribution of $N = 5$ scores that is balanced with a mean of $\mu = 7$.



of $X = 13$ increases the sum by 13 points, producing a new sum of $\Sigma X = 35 + 13 = 48$. Finally, the new mean is computed using the new values for n and ΣX.

$$M = \frac{\Sigma X}{n} = \frac{48}{6} = 8$$

The entire process can be summarized as follows:

Original Sample	New Sample, Adding $X = 13$
$n = 5$	$n = 6$
$\Sigma X = 35$	$\Sigma X = 48$
$M = 35/5 = 7$	$M = 48/6 = 8$

Adding or subtracting a constant from each score If a constant value is added to every score in a distribution, the same constant is added to the mean. Similarly, if you subtract a constant from every score, the same constant is subtracted from the mean.

As mentioned in Chapter 2 (p. 38), Schmidt (1994) conducted a set of experiments examining how humor influences memory. In one study, participants were shown lists of sentences, of which half were humorous (I got a bill for my surgery—now I know why those doctors were wearing masks.) and half were nonhumorous (I got a bill for my surgery—those doctors were like robbers with the prices they charged.). The results showed that people consistently recalled more of the humorous sentences.

Table 3.2 shows the results for a sample of $n = 6$ participants. The first column shows their memory scores for nonhumorous sentences. Note that the total number of sentences recalled is $\Sigma X = 17$ for a sample of $n = 6$ participants, so the mean is $M = \frac{17}{6} = 2.83$. Now suppose that the effect of humor is to add a constant amount (2 points) to each individual's memory score. The resulting scores for humorous sentences are shown in the second column of the table. For these scores, the 6 participants recalled a total of $\Sigma X = 29$ sentences, so the mean is $M = \frac{29}{6} = 4.83$. Adding 2 points to each score has also added 2 points to the mean, from $M = 2.83$ to $M = 4.83$. (It is important to note that experimental effects are usually not as simple as adding or subtracting a constant amount. Nonetheless, the concept of adding a constant to every score is important and will be addressed in later chapters when we are using statistics to evaluate the effects of experimental manipulations.)

TABLE 3.2

Number of sentences recalled for humorous and nonhumorous sentences.

Participant	Nonhumorous Sentences	Humorous Sentences
A	4	6
B	2	4
C	3	5
D	3	5
E	2	4
F	3	5
	$\Sigma X = 17$	$\Sigma X = 29$
	$M = 2.83$	$M = 4.83$

Multiplying or dividing each score by a constant If every score in a distribution is multiplied by (or divided by) a constant value, the mean changes in the same way.

Multiplying (or dividing) each score by a constant value is a common method for changing the unit of measurement. To change a set of measurements from minutes to seconds, for example, you multiply by 60; to change from inches to feet, you divide by 12. One common task for researchers is converting measurements into metric units to conform to international standards. For example, publication guidelines of the American Psychological Association call for metric equivalents to be reported in parentheses when most nonmetric units are used. Table 3.3 shows how a sample of $n = 5$ scores measured in inches would be transformed to a set of scores measured in centimeters. (Note that 1 inch equals 2.54 centimeters.) The first column shows the original scores that total $\Sigma X = 50$ with $M = 10$ inches. In the second column, each of the original scores has been multiplied by 2.54 (to convert from inches to centimeters) and the resulting values total $\Sigma X = 127$, with $M = 25.4$. Multiplying each score by 2.54 has also caused the mean to be multiplied by 2.54. You should realize, however, that although the numerical values for the individual scores and the sample mean have changed, the actual measurements have not changed.

LEARNING CHECK

1. Adding a new score to a distribution always changes the mean. (True or false?)
2. Changing the value of a score in a distribution always changes the mean. (True or false?)
3. A population has a mean of $\mu = 40$.
   a. If 5 points were added to every score, what would be the value for the new mean?
   b. If every score were multiplied by 3, what would be the value for the new mean?
4. A sample of $n = 4$ scores has a mean of 9. If one person with a score of $X = 3$ is removed from the sample, what is the value for the new sample mean?

ANSWERS

1. False. If the score is equal to the mean, it does not change the mean.
2. True.
3. **a.** The new mean would be 45. **b.** The new mean would be 120.
4. The original sample has $n = 4$ and $\Sigma X = 36$. The new sample has $n = 3$ scores that total $\Sigma X = 33$. The new mean is $M = 11$.

TABLE 3.3

Measurements converted from inches to centimeters.

Original Measurement in Inches	Conversion to Centimeters (Multiply by 2.54)
10	25.40
9	22.86
12	30.48
8	20.32
11	27.94
$\Sigma X = 50$	$\Sigma X = 127.00$
$M = 10$	$M = 25.40$

3.3 THE MEDIAN

The second measure of central tendency we consider is called the *median*. The goal of the median is to locate the midpoint of the distribution. Unlike the mean, there are no specific symbols or notation to identify the median. Instead, the median is simply identified by the word *median*. In addition, the definition and the computations for the median are identical for a sample and for a population.

DEFINITION

If the scores in a distribution are listed in order from smallest to largest, the **median** is the midpoint of the list. More specifically, the median is the point on the measurement scale below which 50% of the scores in the distribution are located.

FINDING THE MEDIAN FOR MOST DISTRIBUTIONS

Defining the median as the *midpoint* of a distribution means that the scores are divided into two equal-sized groups. We are not locating the midpoint between the highest and lowest X values. To find the median, list the scores in order from smallest to largest. Begin with the smallest score and count the scores as you move up the list. The median is the first point you reach that is greater than 50% of the scores in the distribution. The median can be equal to a score in the list or it can be a point between two scores. Notice that the median is not algebraically defined (there is no equation for computing the median), which means that there is a degree of subjectivity in determining the exact value. However, the following two examples demonstrate the process of finding the median for most distributions.

EXAMPLE 3.5

This example demonstrates the calculation of the median when n is an odd number. With an odd number of scores, you list the scores in order (lowest to highest), and the median is the middle score in the list. Consider the following set of $N = 5$ scores, which have been listed in order:

3, 5, 8, 10, 11

The middle score is $X = 8$, so the median is equal to 8. Using the counting method, with $N = 5$ scores, the 50% point would be $2\frac{1}{2}$ scores. Starting with the smallest scores, we must count the 3, the 5, and the 8 before we reach the target of at least 50%. Again, for this distribution, the median is the middle score, $X = 8$.

EXAMPLE 3.6

This example demonstrates the calculation of the median when n is an even number. With an even number of scores in the distribution, you list the scores in order (lowest to highest) and then locate the median by finding the average of the middle two scores. Consider the following population:

1, 1, 4, 5, 7, 8

Now we select the middle pair of scores (4 and 5), add them together, and divide by 2:

$$\text{median} = \frac{4 + 5}{2} = \frac{9}{2} = 4.5$$

Using the counting procedure, with $N = 6$ scores, the 50% point is 3 scores. Starting with the smallest scores, we must count the first 1, the second 1, and the 4 before we reach the target of at least 50%. Again, the median for this distribution is 4.5, which is the first point on the scale beyond $X = 4$. For this distribution, exactly 3 scores (50%) are located below 4.5. Note: If there is a gap between the middle two scores, the convention is to define the median as the midpoint between the two scores. For example, if the middle two scores are $X = 4$ and $X = 6$, the median would be defined as 5.

The simple technique of listing and counting scores is sufficient to determine the median for most distributions and is always appropriate for discrete variables. Notice that this technique always produces a median that is either a whole number or is halfway between two whole numbers. With a continuous variable, however, it is possible to divide a distribution precisely in half so that *exactly* 50% of the distribution is located below (and above) a specific point. The procedure for locating the precise median is discussed in the following section.

FINDING THE PRECISE MEDIAN FOR A CONTINUOUS VARIABLE

Recall from Chapter 1 that a continuous variable consists of categories that can be split into an infinite number of fractional parts. For example, time can be measured in seconds, tenths of a second, hundredths of a second, and so on. When the scores in a distribution are measurements of a continuous variable, it is possible to split one of the categories into fractional parts and find the median by locating the precise point that separates the bottom 50% of the distribution from the top 50%. The following example demonstrates this process.

EXAMPLE 3.7

For this example, we will find the precise median for the following sample of $n = 8$ scores: 1, 2, 3, 4, 4, 4, 4, 6

The frequency distribution for this sample is shown in Figure 3.5(a). With an even number of scores, you normally would compute the average of the middle two scores to find the median. This process produces a median of $X = 4$. For a discrete variable,

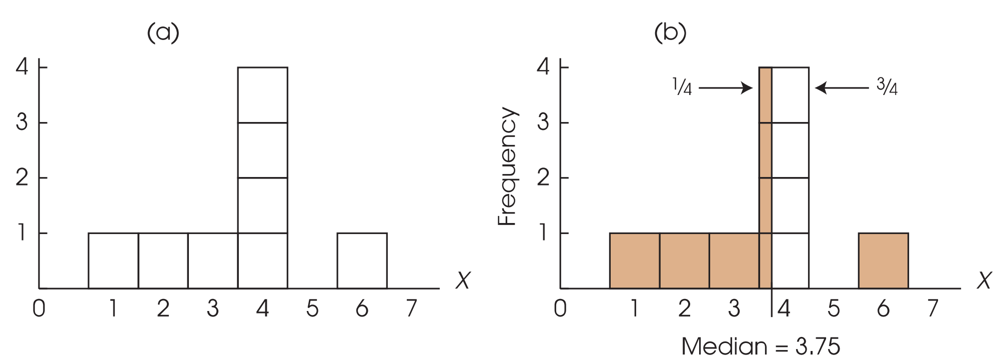


FIGURE 3.5
A distribution with several scores clustered at the median. The median for this distribution is positioned so that each of the four boxes above $X = 4$ is divided into two sections, with $\frac{1}{4}$ of each box below the median (to the left) and $\frac{3}{4}$ of each box above the median (to the right). As a result, there are exactly four boxes, 50% of the distribution, on each side of the median.

$X = 4$ is the correct value for the median. Recall from Chapter 1 that a discrete variable consists of indivisible categories, such as the number of children in a family. Some families have 4 children and some have 5, but none have 4.31 children. For a discrete variable, the category $X = 4$ cannot be divided and the whole number 4 is the median.

However, if you look at the distribution histogram, the value $X = 4$ does not appear to be the exact midpoint. The problem comes from the tendency to interpret a score of $X = 4$ as meaning exactly 4.00. However, if the scores are measurements of a continuous variable, then the score $X = 4$ actually corresponds to an interval from 3.5 to 4.5, and the median corresponds to a point within this interval.

To find the precise median, we first observe that the distribution contains $n = 8$ scores represented by 8 boxes in the graph. The median is the point that has exactly 4 boxes (50%) on each side. Starting at the left-hand side and moving up the scale of measurement, we accumulate a total of 3 boxes when we reach a value of 3.5 on the X-axis [see Figure 3.5(a)]. What is needed is 1 more box to reach the goal of 4 boxes (50%). The problem is that the next interval contains four boxes. The solution is to take a fraction of each box so that the fractions combine to give you one box. For this example, if we take $\frac{1}{4}$ of each box, the four quarters will combine to make one whole box. This solution is shown in Figure 3.5(b). The fraction is determined by the number of boxes needed to reach 50% and the number that exists in the interval.

$$\text{fraction} = \frac{\text{number needed to reach 50\%}}{\text{number in the interval}}$$

For this example, we needed 1 out of the 4 boxes in the interval, so the fraction is $\frac{1}{4}$. To obtain one-fourth of each box, the median is the point that is located exactly one-fourth of the way into the interval. The interval for $X = 4$ extends from 3.5 to 4.5. The interval width is 1 point, so one-fourth of the interval corresponds to 0.25 points. Starting at the bottom of the interval and moving up 0.25 points produces a value of $3.50 + 0.25 = 3.75$. This is the median, with exactly 50% of the distribution (4 boxes) on each side.

You may recognize that the process used to find the precise median in Example 3.7 is equivalent to the process of interpolation that was introduced in Chapter 2 (pp. 55–59). Specifically, the precise median is identical to the 50th percentile for a distribution, and interpolation can be used to locate the 50th percentile. The process of using interpolation is demonstrated in Box 3.2 using the same scores that were used in Example 3.7.

Remember, finding the precise midpoint by dividing scores into fractional parts is sensible for a continuous variable, however, it is not appropriate for a discrete variable. For example, a median time of 3.75 seconds is reasonable, but a median family size of 3.75 children is not.

THE MEDIAN, THE MEAN, AND THE MIDDLE

Earlier, we defined the mean as the “balance point” for a distribution because the distances above the mean must have the same total as the distances below the mean. One consequence of this definition is that the mean is always located inside the group of scores, somewhere between the smallest score and the largest score. You should notice, however, that the concept of a balance point focuses on distances rather than scores. In particular, it is possible to have a distribution in which the vast majority of the scores

BOX 3.2 USING INTERPOLATION TO LOCATE THE 50TH PERCENTILE (THE MEDIAN)

The precise median and the 50th percentile are both defined as the point that separates the top 50% of a distribution from the bottom 50%. In Chapter 2, we introduced interpolation as a technique for finding specific percentiles. We now use that same process to find the 50th percentile for the scores in Example 3.7.

Looking at the distribution of scores shown in Figure 3.5, exactly 3 of the $n = 8$ scores, or 37.5%, are located below the real limit of 3.5. Also, 7 of the $n = 8$ scores (87.5%) are located below the real limit of 4.5. This interval of scores and percentages is shown in the following table. Note that the median, the 50th percentile, is located within this interval.

	Scores (X)	Percentages	
Top	4.5	87.5%	
	?	50%	← Intermediate value
Bottom	3.5	37.5%	

We will find the 50th percentile (the median) using the 4-step interpolation process that was introduced in Chapter 2.

1. For the scores, the width of the interval is 1 point. For the percentages, the width is 50 points.
2. The value of 50% is located 37.5 points down from the top of the percentage interval. As a fraction of the whole interval, this is 37.5 out of 50, or 0.75 of the total interval.
3. For the scores, the interval width is 1 point and 0.75 of the interval corresponds to a distance of 0.75(1) = 0.75 points.
4. Because the top of the interval is 4.5, the position we want is 4.5 – 0.75 = 3.75

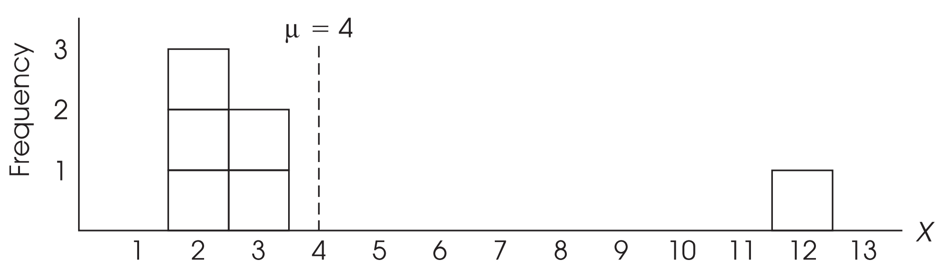
For this distribution, the 50% point (the 50th percentile) corresponds to a score of $X = 3.75$. Note that this is exactly the same value that we obtained for the median in Example 3.7.

are located on one side of the mean. Figure 3.6 shows a distribution of $N = 6$ scores in which 5 out of 6 scores have values less than the mean. In this figure, the total of the distances above the mean is 8 points and the total of the distances below the mean is 8 points. Thus, the mean is located in the middle of the distribution if you use the concept of distance to define the middle. However, you should realize that the mean is not necessarily located at the exact center of the group of scores.

The median, on the other hand, defines the middle of the distribution in terms of scores. In particular, the median is located so that half of the scores are on one side and half are on the other side. For the distribution in Figure 3.6, for example, the median is located at $X = 2.5$, with exactly 3 scores above this value and exactly 3 scores below. Thus, it is possible to claim that the median is located in the middle of the distribution, provided that the term *middle* is defined by the number of scores.

FIGURE 3.6

A population of $N = 6$ scores with a mean of $\mu = 4$. Notice that the mean does not necessarily divide the scores into two equal groups. In this example, 5 out of the 6 scores have values less than the mean.



In summary, the mean and the median are both methods for defining and measuring central tendency. Although they both define the middle of the distribution, they use different definitions of the term *middle*.

LEARNING CHECK

1. Find the median for each distribution of scores:
   a. 3, 4, 6, 7, 9, 10, 11
   b. 8, 10, 11, 12, 14, 15
2. If you have a score of 52 on an 80-point exam, then you definitely scored above the median. (True or false?)
3. The following is a distribution of measurements for a continuous variable. Find the precise median that divides the distribution exactly in half.

   Scores: 1, 2, 2, 3, 4, 4, 4, 4, 4, 5

ANSWERS

1. a. The median is $X = 7$. b. The median is $X = 11.5$.
2. False. The value of the median would depend on where all of the scores are located.
3. The median is 3.70 (one-fifth of the way into the interval from 3.5 to 4.5).

3.4 THE MODE

The final measure of central tendency that we consider is called the *mode*. In its common usage, the word *mode* means "the customary fashion" or "a popular style." The statistical definition is similar in that the mode is the most common observation among a group of scores.

DEFINITION

In a frequency distribution, the **mode** is the score or category that has the greatest frequency.

As with the median, there are no symbols or special notation used to identify the mode or to differentiate between a sample mode and a population mode. In addition, the definition of the mode is the same for a population and for a sample distribution.

The mode is a useful measure of central tendency because it can be used to determine the typical or average value for any scale of measurement, including a nominal scale (see Chapter 1). Consider, for example, the data shown in Table 3.4. These data were obtained by asking a sample of 100 students to name their favorite restaurants in

TABLE 3.4

Favorite restaurants named by a sample of $n = 100$ students. *Caution:* The mode is a score or category, not a frequency. For this example, the mode is Luigi's, not $f = 42$.

Restaurant	f
College Grill	5
George & Harry's	16
Luigi's	42
Oasis Diner	18
Roxbury Inn	7
Sutter's Mill	12

town. The result is a sample of $n = 100$ scores with each score corresponding to the restaurant that the student named.

For these data, the mode is Luigi's, the restaurant (score) that was named most frequently as a favorite place. Although we can identify a modal response for these data, you should notice that it would be impossible to compute a mean or a median. For example, you cannot add the scores to determine a mean (How much is 5 College Grills plus 42 Luigi's?). Also, it is impossible to list the scores in order because the restaurants do not form any natural order. For example, the College Grill is not "more than" or "less than" the Oasis Diner, they are simply two different restaurants. Thus, it is impossible to obtain the median by finding the midpoint of the list. In general, the mode is the only measure of central tendency that can be used with data from a nominal scale of measurement.

The mode also can be useful because it is the only measure of central tendency that corresponds to an actual score in the data; by definition, the mode is the most frequently occurring score. The mean and the median, on the other hand, are both calculated values and often produce an answer that does not equal any score in the distribution. For example, in Figure 3.6 (p. 86) we presented a distribution with a mean of 4 and a median of 2.5. Note that none of the scores is equal to 4 and none of the scores is equal to 2.5. However, the mode for this distribution is $X = 2$; there are three individuals who actually have scores of $X = 2$.

In a frequency distribution graph, the greatest frequency appears as the tallest part of the figure. To find the mode, you simply identify the score located directly beneath the highest point in the distribution.

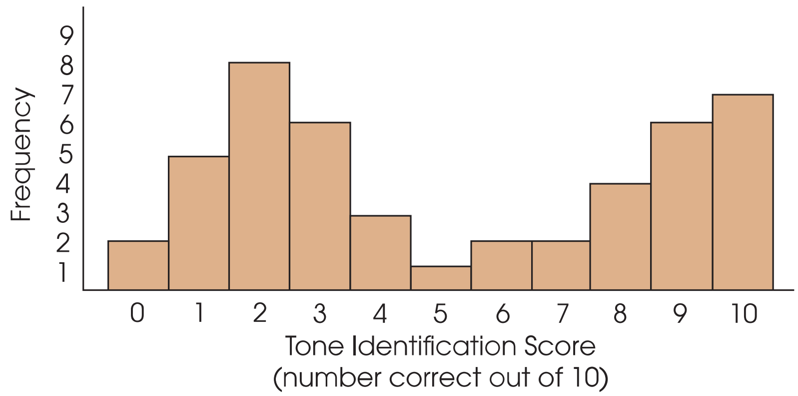
Although a distribution has only one mean and only one median, it is possible to have more than one mode. Specifically, it is possible to have two or more scores that have the same highest frequency. In a frequency distribution graph, the different modes correspond to distinct, equally high peaks. A distribution with two modes is said to be *bimodal,* and a distribution with more than two modes is called *multimodal*. Occasionally, a distribution with several equally high points is said to have no mode.

Incidentally, a bimodal distribution is often an indication that two separate and distinct groups of individuals exist within the same population (or sample). For example, if you measured height for each person in a set of 100 college students, the resulting distribution would probably have two modes, one corresponding primarily to the males in the group and one corresponding primarily to the females.

Technically, the mode is the score with the absolute highest frequency. However, the term *mode* is often used more casually to refer to scores with relatively high frequencies—that is, scores that correspond to peaks in a distribution even though the peaks are not the absolute highest points. For example, Athos, et al. (2007) asked people to identify the pitch for both pure tones and piano tones. Participants were presented with a series of tones and had to name the note corresponding to each tone. Nearly half the participants (44%) had extraordinary pitch-naming ability (absolute pitch), and were able to identify most of the tones correctly. Most of the other participants performed around chance level, apparently guessing the pitch names randomly. Figure 3.7 shows a distribution of scores that is consistent with the results of the study. There are two distinct peaks in the distribution, one located at $X = 2$ (chance performance) and the other located at $X = 10$ (perfect performance). Each of these values is a mode in the distribution. Note, however, that the two modes do not have identical frequencies. Eight people scored at $X = 2$ and only seven had scores of $X = 10$. Nonetheless, both of these points are called modes. When two modes have unequal frequencies, researchers occasionally differentiate the two values by calling the taller peak the *major mode,* and the shorter one the *minor mode*.

FIGURE 3.7

A frequency distribution for tone identification scores. An example of bimodal distributions.



LEARNING CHECK

1. During the month of October, an instructor recorded the number of absences for each student in a class of $n = 20$ and obtained the following distribution.

Number of Absences	*f*
5	1
4	2
3	7
2	5
1	3
0	2

a. Using the mean, what is the average number of absences for the class?

b. Using the median, what is the average number of absences for the class?

c. Using the mode, what is the average number of absences for the class?

ANSWERS **1.**

a. The mean is 47/20 = 2.35.

b. The median is 2.5.

c. The mode is 3.

3.5 SELECTING A MEASURE OF CENTRAL TENDENCY

How do you decide which measure of central tendency to use? The answer to this question depends on several factors. Before we discuss these factors, however, note that you usually can compute two or even three measures of central tendency for the same set of data. Although the three measures often produce similar results, there are situations in which they are very different (see Section 3.6). Also note that the mean is usually the preferred measure of central tendency. Because the mean uses every score in the distribution, it typically produces a good representative value. Remember that the goal of central tendency is to find the single value that best represents the entire distribution. Besides being a good representative, the mean has the added advantage of being closely related to variance and standard deviation, the most common measures of variability (Chapter 4). This relationship makes the mean a valuable measure for purposes of inferential statistics. For these reasons, and others, the mean generally is considered to be the best of the three measures of central tendency. But there are specific situations in which it is impossible to compute a mean or in which the mean is not particularly representative. It is in these situations that the mode and the median are used.

WHEN TO USE THE MEDIAN

We consider four situations in which the median serves as a valuable alternative to the mean. In the first three cases, the data consist of numerical values (interval or ratio scales) for which you would normally compute the mean. However, each case also involves a special problem so that it is either impossible to compute the mean, or the calculation of the mean produces a value that is not central or not representative of the distribution. The fourth situation involves measuring central tendency for ordinal data.

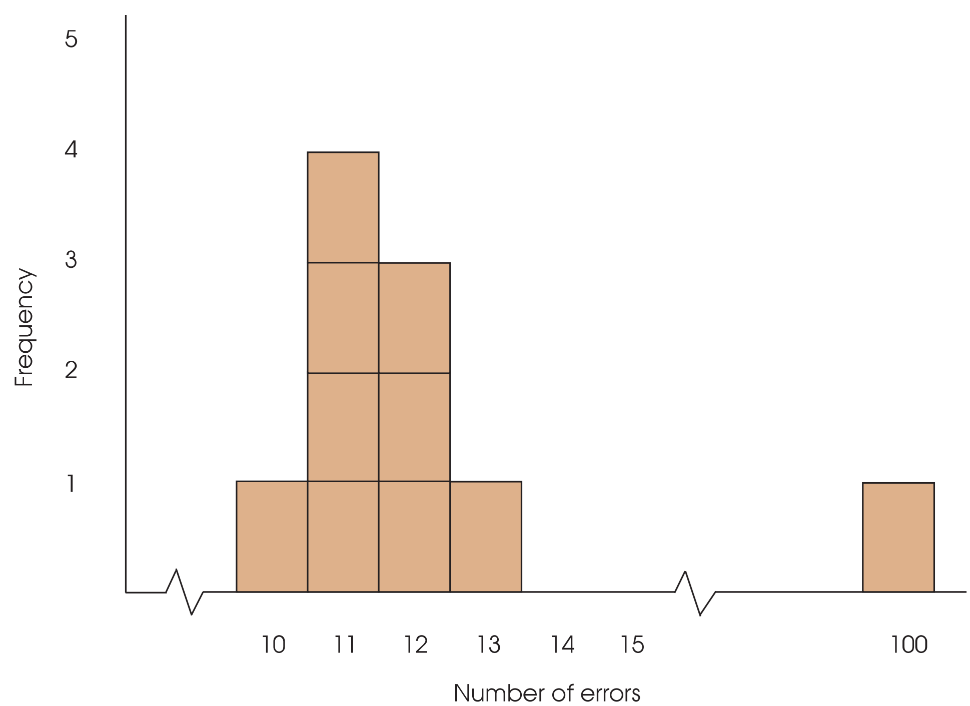
Extreme scores or skewed distributions When a distribution has a few extreme scores, scores that are very different in value from most of the others, then the mean may not be a good representative of the majority of the distribution. The problem comes from the fact that one or two extreme values can have a large influence and cause the mean to be displaced. In this situation, the fact that the mean uses all of the scores equally can be a disadvantage. Consider, for example, the distribution of $n = 10$ scores in Figure 3.8. For this sample, the mean is

$$M = \frac{\Sigma X}{n} = \frac{203}{10} = 20.3$$

Notice that the mean is not very representative of any score in this distribution. Although most of the scores are clustered between 10 and 13, the extreme score of $X = 100$ inflates the value of ΣX and distorts the mean.

FIGURE 3.8

Frequency distribution of errors committed before reaching learning criterion. Notice that the graph shows two *breaks* in the *X*-axis. Rather than listing all the scores from 0 to 100, the graph jumps directly to the first score, which is $X = 10$, and then jumps directly from $X = 15$ to $X = 100$. The breaks shown in the *X*-axis are the conventional way of notifying the reader that some values have been omitted.



The median, on the other hand, is not easily affected by extreme scores. For this sample, $n = 10$, so there should be five scores on either side of the median. The median is 11.50. Notice that this is a very representative value. Also note that the median would be unchanged even if the extreme score were 1000 instead of only 100. Because it is relatively unaffected by extreme scores, the median commonly is used when reporting the average value for a skewed distribution. For example, the distribution of personal incomes is very skewed, with a small segment of the population earning incomes that are astronomical. These extreme values distort the mean, so that it is not very representative of the salaries that most of us earn. The median is the preferred measure of central tendency when extreme scores exist.

Undetermined values Occasionally, you encounter a situation in which an individual has an unknown or undetermined score. In psychology, this often occurs in learning experiments in which you are measuring the number of errors (or amount of time) required for an individual to solve a particular problem. For example, suppose that participants are asked to assemble a wooden puzzle as quickly as possible. The experimenter records how long (in minutes) it takes each individual to arrange all of the pieces to complete the puzzle. Table 3.5 presents results for a sample of $n = 6$ people.

TABLE 3.5

Number of minutes needed to assemble a wooden puzzle.

Person	Time (Min.)
1	8
2	11
3	12
4	13
5	17
6	Never finished

Notice that person 6 never completed the puzzle. After an hour, this person still showed no sign of solving the puzzle, so the experimenter stopped him or her. This person has an undetermined score. (There are two important points to be noted. First, the experimenter should not throw out this individual's score. The whole purpose for using a sample is to gain a picture of the population, and this individual tells us that part of the population cannot solve the puzzle. Second, this person should not be given a score of $X = 60$ minutes. Even though the experimenter stopped the individual after 1 hour, the person did not finish the puzzle. The score that is recorded is the amount of time needed to finish. For this individual, we do not know how long this is.)

It is impossible to compute the mean for these data because of the undetermined value. We cannot calculate the ΣX part of the formula for the mean. However, it is possible to determine the median. For these data, the median is 12.5. Three scores are below the median, and three scores (including the undetermined value) are above the median.

Number of Pizzas (X)	f
5 or more	3
4	2
3	2
2	3
1	6
0	4

Open-ended distributions A distribution is said to be *open-ended* when there is no upper limit (or lower limit) for one of the categories. The table in the margin provides an example of an open-ended distribution, showing the number of pizzas eaten during a 1 month period for a sample of $n = 20$ high school students. The top category in this distribution shows that three of the students consumed "5 or more" pizzas. This is an open-ended category. Notice that it is impossible to compute a mean for these data because you cannot find ΣX (the total number of pizzas for all 20 students). However, you can find the median. Listing the 20 scores in order produces $X = 1$ and $X = 2$ as the middle two scores. For these data, the median is 1.5.

Ordinal scale Many researchers believe that it is not appropriate to use the mean to describe central tendency for ordinal data. When scores are measured on an ordinal scale, the median is always appropriate and is usually the preferred measure of central tendency.

You should recall that ordinal measurements allow you to determine direction (greater than or less than) but do not allow you to determine distance. The median is compatible with this type of measurement because it is defined by direction: half of the scores are above the median and half are below the median. The mean, on the other hand, defines central tendency in terms of distance. Remember that the mean is the balance point for the distribution, so that the distances above the mean are exactly balanced by the distances below the mean. Because the mean is defined in terms of distances, and because ordinal scales do not measure distance, it is not appropriate to compute a mean for scores from an ordinal scale.

WHEN TO USE THE MODE

We consider three situations in which the mode is commonly used as an alternative to the mean, or is used in conjunction with the mean to describe central tendency.

Nominal scales The primary advantage of the mode is that it can be used to measure and describe central tendency for data that are measured on a nominal scale. Recall that the categories that make up a nominal scale are differentiated only by name. Because nominal scales do not measure quantity (distance or direction), it is impossible to compute a mean or a median for data from a nominal scale. Therefore, the mode is the only option for describing central tendency for nominal data.

Discrete variables Recall that discrete variables are those that exist only in whole, indivisible categories. Often, discrete variables are numerical values, such as the number of children in a family or the number of rooms in a house. When these variables produce numerical scores, it is possible to calculate means. In this situation, the calculated means are usually fractional values that cannot actually exist. For example, computing means generates results such as “the average family has 2.4 children and a house with 5.33 rooms.” On the other hand, the mode always identifies the most typical case and, therefore, it produces more sensible measures of central tendency. Using the mode, our conclusion would be “the typical, or modal, family has 2 children and a house with 5 rooms.” In many situations, especially with discrete variables, people are more comfortable using the realistic, whole-number values produced by the mode.

Describing shape Because the mode requires little or no calculation, it is often included as a supplementary measure along with the mean or median as a no-cost extra. The value of the mode (or modes) in this situation is that it gives an indication of the shape of the distribution as well as a measure of central tendency. Remember that the mode identifies the location of the peak (or peaks) in the frequency distribution graph. For example, if you are told that a set of exam scores has a mean of 72 and a mode of 80, you should have a better picture of the distribution than would be available from the mean alone (see Section 3.6).

IN THE LITERATURE

REPORTING MEASURES OF CENTRAL TENDENCY

Measures of central tendency are commonly used in the behavioral sciences to summarize and describe the results of a research study. For example, a researcher may report the sample means from two different treatments or the median score for a large sample. These values may be reported in verbal descriptions of the results, in tables, or in graphs.

In reporting results, many behavioral science journals use guidelines adopted by the American Psychological Association (APA), as outlined in the *Publication Manual of the American Psychological Association* (2010). We refer to the APA manual from time to time in describing how data and research results are reported in the scientific literature. The APA style uses the letter *M* as the symbol for the sample mean. Thus, a study might state:

> The treatment group showed fewer errors ($M = 2.56$) on the task than the control group ($M = 11.76$).

When there are many means to report, tables with headings provide an organized and more easily understood presentation. Table 3.6 illustrates this point.

The median can be reported using the abbreviation *Mdn,* as in "Mdn = 8.5 errors," or it can simply be reported in narrative text, as follows:

> The median number of errors for the treatment group was 8.5, compared to a median of 13 for the control group.

There is no special symbol or convention for reporting the mode. If mentioned at all, the mode is usually just reported in narrative text.

PRESENTING MEANS AND MEDIANS IN GRAPHS

Graphs also can be used to report and compare measures of central tendency. Usually, graphs are used to display values obtained for sample means, but occasionally sample medians are reported in graphs (modes are rarely, if ever, shown in a graph). The value of a graph is that it allows several means (or medians) to be shown simultaneously, so it is possible to make quick comparisons between groups or treatment conditions. When preparing a graph, it is customary to list the different groups or treatment conditions on the horizontal axis. Typically, these are the different values that make up the independent variable or the quasi-independent variable. Values for the dependent variable (the scores) are listed on the vertical axis. The means (or medians) are then displayed using a *line graph,* a *histogram,* or a *bar graph,* depending on the scale of measurement used for the independent variable.

Figure 3.9 shows an example of a *line graph* displaying the relationship between drug dose (the independent variable) and food consumption (the dependent variable). In this study, there were five different drug doses (treatment conditions) and they are listed along the horizontal axis. The five means appear as points in the graph. To construct this graph, a point was placed above each treatment condition so that the vertical position of the point corresponds to the mean score for the treatment condition. The points are then connected with straight lines. A line graph is used when the values on the horizontal axis are measured on an interval or a ratio scale. An alternative to the line graph is a *histogram*. For this example, the histogram would show a bar above each drug dose so that the height of each bar corresponds to the mean food consumption for that group, with no space between adjacent bars.

TABLE 3.6

The mean number of errors made on the task for treatment and control groups, divided by gender.

	Treatment	Control
Females	1.45	8.36
Males	3.83	14.77

FIGURE 3.9

The relationship between an independent variable (drug dose) and a dependent variable (food consumption). Because drug dose is a continuous variable, a continuous line is used to connect the different dose levels.

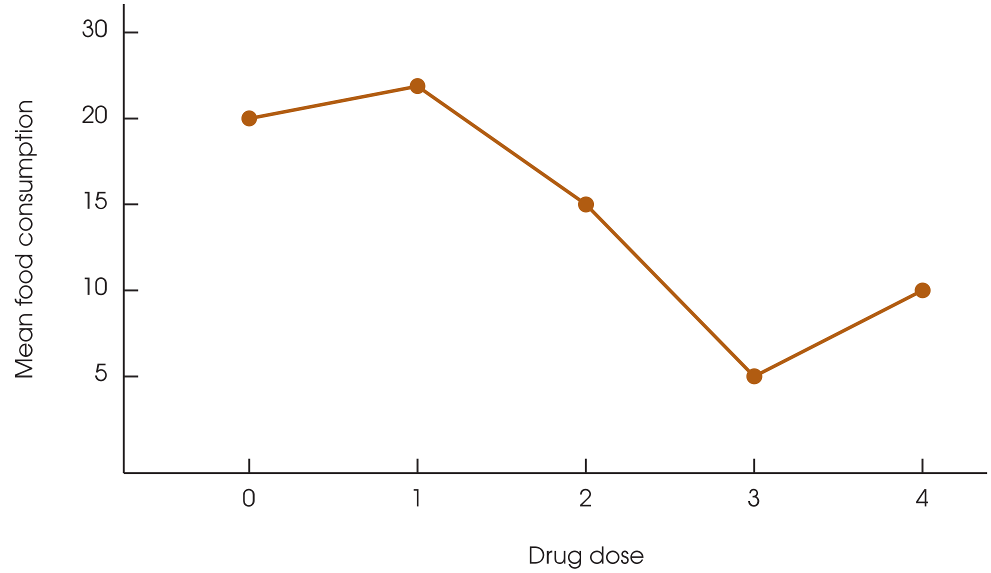


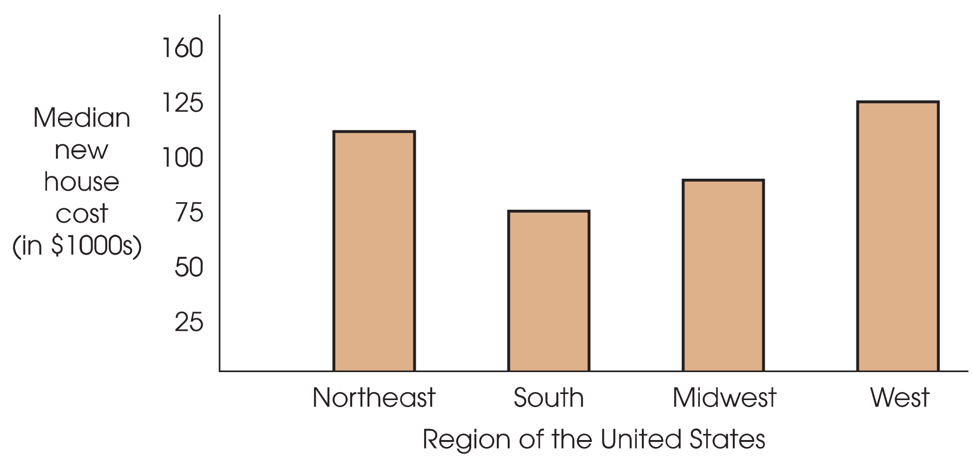
Figure 3.10 shows a bar graph displaying the median selling price for single-family homes in different regions of the United States. Bar graphs are used to present means (or medians) when the groups or treatments shown on the horizontal axis are measured on a nominal or an ordinal scale. To construct a bar graph, you simply draw a bar directly above each group or treatment so that the height of the bar corresponds to the mean (or median) for that group or treatment. For a bar graph, a space is left between adjacent bars to indicate that the scale of measurement is nominal or ordinal.

When constructing graphs of any type, you should recall the basic rules that we introduced in Chapter 2:

1. The height of a graph should be approximately two-thirds to three-quarters of its length.
2. Normally, you start numbering both the *X*-axis and the *Y*-axis with zero at the point where the two axes intersect. However, when a value of zero is part of the

FIGURE 3.10

Median cost of a new, single-family home by region.



data, it is common to move the zero point away from the intersection so that the graph does not overlap the axes (see Figure 3.9).

Following these rules helps to produce a graph that provides an accurate presentation of the information in a set of data. Although it is possible to construct graphs that distort the results of a study (see Box 2.1), researchers have an ethical responsibility to present an honest and accurate report of their research results. ❑

3.6 CENTRAL TENDENCY AND THE SHAPE OF THE DISTRIBUTION

We have identified three different measures of central tendency, and often a researcher calculates all three for a single set of data. Because the mean, the median, and the mode are all trying to measure the same thing, it is reasonable to expect that these three values should be related. In fact, there are some consistent and predictable relationships among the three measures of central tendency. Specifically, there are situations in which all three measures have exactly the same value. On the other hand, there are situations in which the three measures are guaranteed to be different. In part, the relationships among the mean, median, and mode are determined by the shape of the distribution. We consider two general types of distributions.

SYMMETRICAL DISTRIBUTIONS

For a *symmetrical distribution,* the right-hand side of the graph is a mirror image of the left-hand side. If a distribution is perfectly symmetrical, the median is exactly at the center because exactly half of the area in the graph is on either side of the center. The mean also is exactly at the center of a perfectly symmetrical distribution because each score on the left side of the distribution is balanced by a corresponding score (the mirror image) on the right side. As a result, the mean (the balance point) is located at the center of the distribution. Thus, for a perfectly symmetrical distribution, the mean and the median are the same (Figure 3.11). If a distribution is roughly symmetrical, but not perfect, the mean and median are close together in the center of the distribution.

If a symmetrical distribution has only one mode, it is also in the center of the distribution. Thus, for a perfectly symmetrical distribution with one mode, all three measures of central tendency, the mean, the median, and the mode, have the same value. For a roughly symmetrical distribution, the three measures are clustered together in the center of the distribution. On the other hand, a bimodal distribution that is symmetrical [see Figure 3.11(b)] has the mean and median together in the center with the modes on each side. A rectangular distribution [see Figure 3.11(c)] has no mode because all X values occur with the same frequency. Still, the mean and the median are in the center of the distribution.

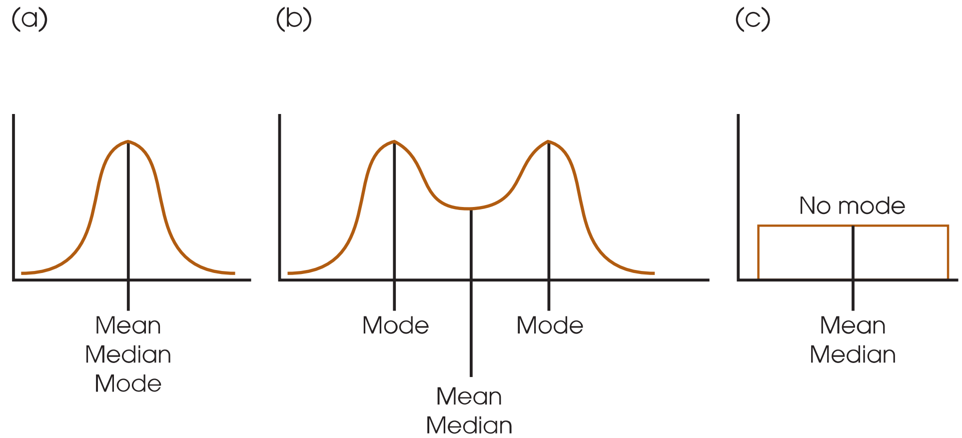
SKEWED DISTRIBUTIONS

The positions of the mean, median, and mode are not as consistently predictable in distributions of discrete variables (see Von Hippel, 2005).

In *skewed distributions,* especially distributions for continuous variables, there is a strong tendency for the mean, median, and mode to be located in predictably different positions. Figure 3.12(a), for example, shows a positively skewed distribution with the peak (highest frequency) on the left-hand side. This is the position of the mode. However, it should be clear that the vertical line drawn at the mode does not divide the distribution into two equal parts. To have exactly 50% of the distribution

FIGURE 3.11

Measures of central tendency for three symmetrical distributions: normal, bimodal, and rectangular.



on each side, the median must be located to the right of the mode. Finally, the mean is located to the right of the median because it is the measure influenced most by the extreme scores in the tail and is displaced farthest to the right toward the tail of the distribution. Therefore, in a positively skewed distribution, the order of the three measures of central tendency from smallest to largest (left to right) is the mode, the median, and the mean.

Negatively skewed distributions are lopsided in the opposite direction, with the scores piling up on the right-hand side and the tail tapering off to the left. The grades on an easy exam, for example, tend to form a negatively skewed distribution [see Figure 3.12(b)]. For a distribution with negative skew, the mode is on the right-hand side (with the peak), whereas the mean is displaced toward the left by the extreme scores in the tail. As before, the median is located between the mean and the mode. In order from smallest value to largest value (left to right), the three measures of central tendency for a negatively skewed distribution are the mean, the median, and the mode.

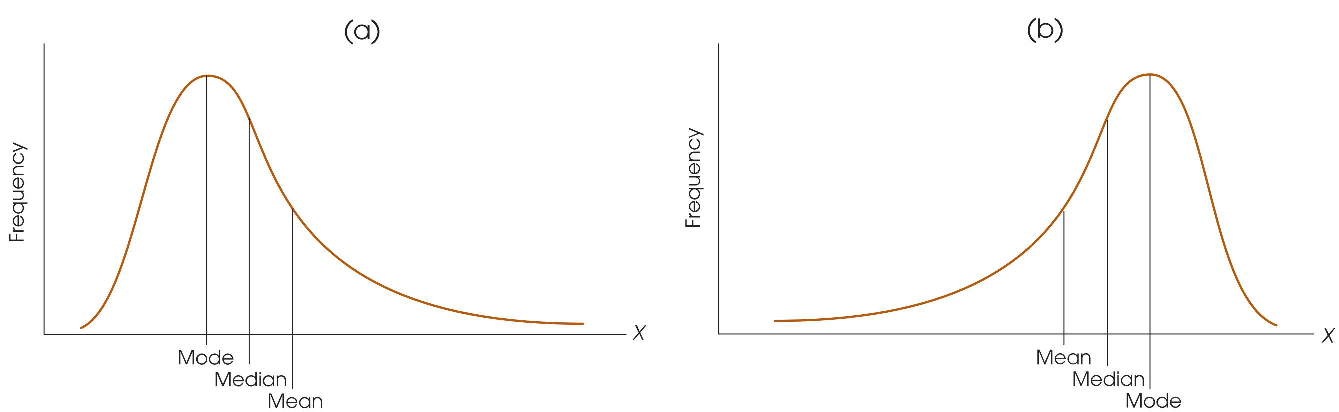


FIGURE 3.12

Measures of central tendency for skewed distributions.

LEARNING CHECK

1. Which measure of central tendency is most affected if one extremely large score is added to a distribution? (mean, median, mode)
2. Why is it usually considered inappropriate to compute a mean for scores measured on an ordinal scale?
3. In a perfectly symmetrical distribution, the mean, the median, and the mode will all have the same value. (True or false?)
4. A distribution with a mean of 70 and a median of 75 is probably positively skewed. (True or false?)

ANSWERS

1. mean
2. The definition of the mean is based on distances (the mean balances the distances) and ordinal scales do not measure distance.
3. False, if the distribution is bimodal.
4. False. The mean is displaced toward the tail on the left-hand side.

SUMMARY

1. The purpose of central tendency is to determine the single value that identifies the center of the distribution and best represents the entire set of scores. The three standard measures of central tendency are the mode, the median, and the mean.
2. The mean is the arithmetic average. It is computed by adding all of the scores and then dividing by the number of scores. Conceptually, the mean is obtained by dividing the total (ΣX) equally among the number of individuals (N or n). The mean can also be defined as the balance point for the distribution. The distances above the mean are exactly balanced by the distances below the mean. Although the calculation is the same for a population or a sample mean, a population mean is identified by the symbol μ, and a sample mean is identified by M. In most situations with numerical scores from an interval or a ratio scale, the mean is the preferred measure of central tendency.
3. Changing any score in the distribution causes the mean to be changed. When a constant value is added to (or subtracted from) every score in a distribution, the same constant value is added to (or subtracted from) the mean. If every score is multiplied by a constant, the mean is multiplied by the same constant.
4. The median is the midpoint of a distribution of scores. The median is the preferred measure of central tendency when a distribution has a few extreme scores that displace the value of the mean. The median also is used for open-ended distributions and when there are undetermined (infinite) scores that make it impossible to compute a mean. Finally, the median is the preferred measure of central tendency for data from an ordinal scale.
5. The mode is the most frequently occurring score in a distribution. It is easily located by finding the peak in a frequency distribution graph. For data measured on a nominal scale, the mode is the appropriate measure of central tendency. It is possible for a distribution to have more than one mode.
6. For symmetrical distributions, the mean is equal to the median. If there is only one mode, then it has the same value, too.
7. For skewed distributions, the mode is located toward the side where the scores pile up, and the mean is pulled toward the extreme scores in the tail. The median is usually located between these two values.

KEY TERMS

central tendency (73)
population mean (μ) (75)
sample mean (*M*) (75)
weighted mean (77)
median (83)
mode (87)
bimodal (88)
multimodal (88)
major mode (88)
minor mode (88)
line graph (93)
symmetrical distribution (95)
skewed distribution (95)
positive skew (95)
negative skew (96)

RESOURCES

Book Companion Website: www.cengage.com/psychology/gravetter

You can find a tutorial quiz and other learning exercises for Chapter 3 on the book companion website. The website also includes a workshop entitled *Central Tendency and Variability* that reviews the basic concept of the mean and introduces the concept of variability that is presented in Chapter 4.

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SPSS

General instructions for using SPSS are presented in Appendix D. Following are detailed instructions for using SPSS to compute the **Mean** and ΣX for a set of scores.

Data Entry

1. Enter all of the scores in one column of the data editor, probably VAR00001.

Data Analysis

1. Click **Analyze** on the tool bar, select **Descriptive Statistics,** and click on **Descriptives.**
2. Highlight the column label for the set of scores (VAR00001) in the left box and click the arrow to move it into the **Variable** box.
3. If you want ΣX as well as the mean, click on the **Options** box, select **Sum,** then click **Continue.**
4. Click **OK.**

SPSS Output

SPSS produces a summary table listing the number of scores (N), the maximum and minimum scores, the sum of the scores (if you selected this option), the mean, and the standard deviation. Note: The standard deviation is a measure of variability that is presented in Chapter 4.

FOCUS ON PROBLEM SOLVING

X	f
4	1
3	4
2	3
1	2

1. Although the three measures of central tendency appear to be very simple to calculate, there is always a chance for errors. The most common sources of error are listed next.
   a. Many students find it very difficult to compute the mean for data presented in a frequency distribution table. They tend to ignore the frequencies in the table and simply average the score values listed in the X column. You must use the frequencies *and* the scores! Remember that the number of scores is found by $N = \Sigma f$, and the sum of all N scores is found by ΣfX. For the distribution shown in the margin, the mean is $\frac{24}{10} = 2.40$.
   b. The median is the midpoint of the distribution of scores, not the midpoint of the scale of measurement. For a 100-point test, for example, many students incorrectly assume that the median must be $X = 50$. To find the median, you must have the *complete set* of individual scores. The median separates the individuals into two equal-sized groups.
   c. The most common error with the mode is for students to report the highest frequency in a distribution rather than the *score* with the highest frequency. Remember that the purpose of central tendency is to find the most representative score. For the distribution in the margin, the mode is $X = 3$, not $f = 4$.

DEMONSTRATION 3.1

COMPUTING MEASURES OF CENTRAL TENDENCY

For the following sample, find the mean, the median, and the mode. The scores are:

5, 6, 9, 11, 5, 11, 8, 14, 2, 11

Compute the mean The calculation of the mean requires two pieces of information: the sum of the scores, ΣX; and the number of scores, n. For this sample, $n = 10$ and

$$\Sigma X = 5 + 6 + 9 + 11 + 5 + 11 + 8 + 14 + 2 + 11 = 82$$

Therefore, the sample mean is

$$M = \frac{\Sigma X}{n} = \frac{82}{10} = 8.2$$

Find the median To find the median, first list the scores in order from smallest to largest. With an even number of scores, the median is the average of the middle two scores in the list. Listed in order, the scores are:

2, 5, 5, 6, 8, 9, 11, 11, 11, 14

The middle two scores are 8 and 9, and the median is 8.5.

Find the mode For this sample, $X = 11$ is the score that occurs most frequently. The mode is $X = 11$.

PROBLEMS

1. What general purpose is served by a good measure of central tendency?
2. Why is it necessary to have more than one method for measuring central tendency?
3. Find the mean, median, and mode for the following sample of scores:

   6, 2, 4, 1, 2, 2, 3, 4, 3, 2
4. Find the mean, median, and mode for the following sample of scores:

   8, 7, 8, 8, 4, 9, 10, 7, 8, 8, 9, 8
5. Find the mean, median, and mode for the scores in the following frequency distribution table:

X	f
8	1
7	4
6	2
5	2
4	2
3	1

6. Find the mean, median, and mode for the scores in the following frequency distribution table:

X	f
10	1
9	2
8	3
7	3
6	4
5	2

7. For the following sample
   a. Assume that the scores are measurements of a continuous variable and find the median by locating the precise midpoint of the distribution.
   b. Assume that the scores are measurements of a discrete variable and find the median.

   Scores: 1, 2, 3, 3, 3, 4
8. A sample of $n = 7$ scores has a mean of $M = 9$. What is the value of ΣX for this sample?
9. A population with a mean of $\mu = 10$ has $\Sigma X = 250$. How many scores are in the population?

10. A sample of $n = 8$ scores has a mean of $M = 10$. If one new person with a score of $X = 1$ is added to the sample, what is the value for the new mean?

11. A sample of $n = 5$ scores has a mean of $M = 12$. If one person with a score of $X = 8$ is removed from the sample, what is the value for the new mean?

12. A sample of $n = 11$ scores has a mean of $M = 4$. One person with a score of $X = 16$ is added to the sample. What is the value for the new sample mean?

13. A sample of $n = 9$ scores has a mean of $M = 10$. One person with a score of $X = 2$ is removed from the sample. What is the value for the new sample mean?

14. A population of $N = 20$ scores has a mean of $\mu = 15$. One score in the population is changed from $X = 8$ to $X = 28$. What is the value for the new population mean?

15. A sample of $n = 7$ scores has a mean of $M = 9$. One score in the sample is changed from $X = 19$ to $X = 5$. What is the value for the new sample mean?

16. A sample of $n = 7$ scores has a mean of $M = 5$. After one new score is added to the sample, the new mean is found to be $M = 6$. What is the value of the new score? (Hint: Compare the values for ΣX before and after the score was added.)

17. A population of $N = 16$ scores has a mean of $\mu = 20$. After one score is removed from the population, the new mean is found to be $\mu = 19$. What is the value of the score that was removed? (Hint: Compare the values for ΣX before and after the score was removed.)

18. One sample has a mean of $M = 4$ and a second sample has a mean of $M = 8$. The two samples are combined into a single set of scores.
- **a.** What is the mean for the combined set if both of the original samples have $n = 7$ scores?
- **b.** What is the mean for the combined set if the first sample has $n = 3$ and the second sample has $n = 7$?
- **c.** What is the mean for the combined set if the first sample has $n = 7$ and the second sample has $n = 3$?

19. One sample has a mean of $M = 5$ and a second sample has a mean of $M = 10$. The two samples are combined into a single set of scores.
- **a.** What is the mean for the combined set if both of the original samples have $n = 5$ scores?
- **b.** What is the mean for the combined set if the first sample has $n = 4$ scores and the second sample has $n = 6$?
- **c.** What is the mean for the combined set if the first sample has $n = 6$ scores and the second sample has $n = 4$?

20. Explain why the mean is often not a good measure of central tendency for a skewed distribution.

21. Identify the circumstances in which the median rather than the mean is the preferred measure of central tendency.

22. For each of the following situations, identify the measure of central tendency (mean, median, or mode) that would provide the best description of the average score:
- **a.** A news reporter interviewed people shopping in a local mall and asked how much they spent on summer vacations. Most people traveled locally and reported modest amounts but one couple had flown to Paris for a month and paid a small fortune.
- **b.** A marketing researcher asked consumers to select their favorite from a set of four designs for a new product logo.
- **c.** A driving instructor recorded the number of orange cones that each student ran over during the first attempt at parallel parking.

23. One question on a student survey asks: In a typical week, how many times do you eat at a fast-food restaurant? The following frequency distribution table summarizes the results for a sample of $n = 20$ students.

Number of times per week	f
5 or more	2
4	2
3	3
2	6
1	4
0	3

- **a.** Find the mode for this distribution.
- **b.** Find the median for the distribution.
- **c.** Explain why you cannot compute the mean using the data in the table.

24. A nutritionist studying weight gain for college freshmen obtains a sample of $n = 20$ first-year students at the state college. Each student is weighed on the first day of school and again on the last day of the semester. The following scores measure the change in weight, in pounds, for each student. A positive score indicates a weight gain during the semester.

+5, +6, +3, +1, +8, +5, +4, +4, +3, –1
+2, +7, +1, +5, +8, 0, +4, +6, +5, +3

a. Sketch a histogram showing the distribution of weight-change scores.
b. Calculate the mean weight-change score for this sample.
c. Does there appear to be a consistent trend in weight change during the semester?

25. Does it ever seem to you that the weather is nice during the work week, but lousy on the weekend? Cerveny and Balling (1998) have confirmed that this is not your imagination—pollution accumulating during the work week most likely spoils the weekend weather for people on the Atlantic coast. Consider the following hypothetical data showing the daily amount of rainfall for 10 weeks during the summer.

Week	Average Daily Rainfall on Weekdays (Mon.–Fri.)	Average Daily Rainfall on Weekends (Sat.–Sun.)
1	1.2	1.5
2	0.6	2.0
3	0.0	1.8
4	1.6	1.5
5	0.8	2.2
6	2.1	2.4
7	0.2	0.8
8	0.9	1.6
9	1.1	1.2
10	1.4	1.7

a. Calculate the average daily rainfall (the mean) during the week, and the average daily rainfall for weekends.
b. Based on the two means, does there appear to be a pattern in the data?