10

Market Structure: Duopoly and Oligopoly

Despite its shortcomings, the analysis of monopolistically competitive industries provides valuable insights into the operations of markets in general. We will next examine the cases of duopoly and oligopoly. An oligopoly is an industry comprising "a few" firms. What constitutes "a few" in this context, however, is somewhat debatable. A duopoly, which is a special case of oligopoly, is an industry comprising two firms.

The distinguishing feature of oligopolistic or duopolistic market structures, especially compared with perfect competition or monopoly, is not simply a matter of the number of firms in the industry. Rather, it is the degree to which the output, pricing, and other decisions of one firm affect, and are affected by, similar decisions made by other firms in the industry. What is important is the interdependence of the managerial decisions among the various firms in the industry.

The interdependence of firm behavior in duopolistic or oligopolistic industries contrasts with market structures encountered in earlier chapters. There was previously no need to consider the strategic behavior of rival firms, either because the output of each firm was very small relative to industry output (perfect competition) or because the firm had no competitors (monopoly) or because of some combination of the two (monopolistic competition). In the United States, where collusion between and among firms is illegal, oligopolistic behavior may be modeled analytically as a noncooperative game in which the actions of one firm to increase market share will, unless countered, result in a reduction of the market share of other firms in the industry. Thus, action will be followed by reaction. This interdependence is the essence of an analysis of duopolistic or oligopolistic market structures.

CHARACTERISTICS OF DUOPOLY AND OLIGOPOLY

There are a number of approaches to the analysis of duopolistic and oligopolistic markets. Each of the models we discuss is developed for the duopolistic market but can easily be generalized to the case of oligopolies. Before examining these analytical approaches, we make some general statements about the basic characteristics of duopolies and oligopolies.

NUMBER AND SIZE DISTRIBUTION OF SELLERS

"Oligopoly" refers to the condition in which industry output is dominated by relatively few large firms. Although there is no precise definition attached to the word "few," two to eight firms controlling 75% or more of a market could be defined as an oligopoly. However an oligopolistic market structures is defined, its distinguishing characteristic is strategic interaction, which refers to the extent to which the pricing, output, and other decisions of one firm affect, and are affected by, the decisions of other firms.

The interdependence of firms in an industry is illustrated in Figure 10.1, which shows the demand curve faced by all firms in the industry, *DD*, and the demand curve faced by an individual firm, *dd*. The rationale behind the diagram is as follows. If all firms in the industry decide to lower their price, say from P_1 to P_2, then the quantity demanded by consumers will increase from Q_1 to Q_2.

Suppose, however, that a single firm in the industry decided to reduce price from P_1 to P_2 in the expectation that other firms would not respond in a similar manner. In this case, the firm could anticipate a substantial increase in its sales, say from Q_1 to Q_3. This implies that over this price range, the demand curve facing the individual firm is more price elastic than the

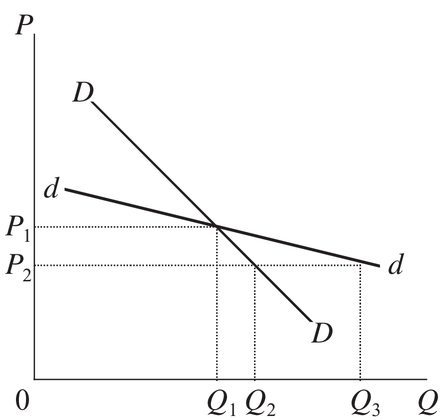

FIGURE 10.1 Oligopolistic industry and individual demand curves.

demand curve faced by the entire industry. The decision by one firm to unilaterally lower its selling price will result in a substantially larger market share, provided this price reduction is not matched by the firm's rivals—a dubious assumption, indeed.

NUMBER AND SIZE DISTRIBUTION OF BUYERS

The number and size distribution of buyers in duopolistic and oligopolistic is usually unspecified, but generally is assumed to involve a large number of buyers.

PRODUCT DIFFERENTIATION

Products sold by duopolies and oligopolies may be either homogeneous or differentiated. If the product is homogeneous, the industry is said to be purely duopolistic or purely oligopolistic. Examples of pure oligopolies are the steel and copper industries. Examples of industries producing differentiated products are the automobile and television industries.

CONDITIONS OF ENTRY AND EXIT

For either duopolies or oligopolies to persist in the long run, there must exist conditions that prevent the entrance of new firms into the industry. There is disagreement among economists over just what these conditions are. Bain (1956) has argued that these conditions should be defined as any advantage that existing firms hold over potential competitors, while Stigler (1968) argues that these barriers to entry comprise any costs that must be paid by potential competitors that are not borne by existing firms in the industry. Many of the barriers to entry erected by oligopolists are the same as those used by monopolists (see Chapter 8). Oligopolist also can control the industry supply of a product and enhance its market power through the control of distribution outlets, such as by persuading retail chains to carry only its product. Persuasion may take the form of selective discounts, long-term supply contracts, or gifts to management. Devices such as product warranties also serve as an effective barrier to entry. New car warranties, for example, typically require the exclusive use of authorized parts and service. Such warranties limit the ability of potential competitors from offering better or less-expensive products.

Definition: A duopoly is an industry comprising two firms producing homogeneous or differentiated products; it is difficult to enter or leave the industry.

Definition: A oligopoly is an industry comprising a few firms producing homogeneous or differentiated products; it is difficult to enter or leave the industry.

MEASURING INDUSTRIAL CONCENTRATION

It was demonstrated in Chapter 8 that perfect competition results in an efficient allocation of resources. Perfect competition results in the production of goods and services that consumers want at least cost. As we move further away from the assumptions underlying the paradigm of perfect competition, with monopoly being the extreme case, firms acquire increasing levels of market power, which usually results in prices that are higher and output levels that are lower than socially optimal levels.

Oligopolies are characterized by a "few" firms dominating the output of an industry. In many respects, an oligopolistic industry is like art—you know it when you see it. But is it possible to measure the extent to which production is attributable to a select number of firms? Is it possible to gauge the degree of industrial concentration? To illustrate the concerns associated with industrial concentration, it is useful to review the historical development of antitrust legislation in the United States, where the federal government has attempted to remedy the socially nonoptimal outcomes of imperfect competition either by enacting regulations to encourage competition and limit market power, or by regulating industries to encourage socially desirable outcomes.

In 1887 Congress created the Interstate Commerce Commission to correct abuses in the railroad industry, and in 1890 it passed the landmark Sherman Antitrust Act, which asserted that monopolies and restraints of trade were illegal. Unfortunately, the Sherman Act was deficient in that its provisions were subject to alternative interpretations. Although the Sherman Act banned monopolies, and certain kinds of monopolistic behavior were illegal, it was unclear what constituted a restraint of trade.

Not surprisingly, actions brought by the U.S. Department of Justice against firms believed to be in violation of the Sherman Act ended up in the courts. Two of the most significant court challenges to prosecution under the Sherman Act involved Standard Oil and American Tobacco. While the U.S. Supreme Court in 1911 found both companies in violation of provisions of the Sherman Act, the Court also made it clear that not every action that seemed to restrain trade was illegal. The Justices ruled that market structure alone was not a sufficient reason for prosecution under the Sherman Act, indicating that only "unreasonable" actions to restrain trade violated the terms of the law. As a result of this "rule of reason," between 1911 and 1920 actions by the Justice Department against Eastman Kodak, International Harvester, United Shoe Machinery, and United States Steel were dismissed. Federal courts ruled that although each of these companies controlled an overwhelming share of its respective market, there was no evidence that these companies engaged in "unreasonable conduct."

In an effort to strengthen the Sherman Act and clarify the *rule of reason*, in 1914 Congress passed the Clayton Act, which made illegal certain specific practices. In general, the Clayton Act limited mergers that lessened competition or tended to create monopolies. In that same year, the Federal Trade Commission was created to investigate "the organization, business conduct, practices, and management of companies" engaged in interstate commerce. Although the Clayton Act clarified many of the provisions of the Sherman Act, the focus remained on the "rule of reason." This changed in 1945, when the Aluminum Corporation of America (Alcoa) was prosecuted for violating the Sherman Act by monopolizing the raw aluminum market.

In a landmark case, *United States* versus *Aluminum Company of America*, the Court ruled that while Alcoa engaged in "normal, prudent, but not predatory business practices" it was the structure of the market *per se* that constituted restraint of trade. On the basis of the *per se rule*, the Court ordered the dissolution of Alcoa. The following year, other court cases resulted in an extension of the Clayton Act that made illegal both tacit and explicit acts of *collusion*. In its extreme form, collusion results in pricing and output results that reflect monopolistic behavior. The implications of collusive behavior by firms in an industry will be discussed at greater length later in this chapter.

In the years to follow, Congress enacted several additional pieces of legislation to deal with the problems associated with monopolistic behavior and restraint of trade. In 1950, for example, the Celler–Kefauver Act, gave the Justice Department the power to monitor and enforce the provisions of the Clayton Act. Nevertheless, there remained considerable uncertainty about what constituted an unacceptable merger. In response, the Justice Department promulgated guidelines for identifying mergers that were deemed to be unacceptable. These guidelines were initially based on the notion of a *concentration ratio*. It was determined, for example, that if the four largest firms in an industry controlled 75% or more of a market, any firm with a 15% market share attempting to acquire another firm in the industry would be challenged under the terms of the Clayton Act.

CONCENTRATION RATIO

The *concentration ratio* compares the dollar value of total shipments in an industry accounted for by a given number of firms in an industry. The U.S. Census Bureau, for example, calculates concentration ratios for the 4, 8, 20, and 50 largest companies, which are grouped according to a standardized industrial classification.[1] (See later: Table 10.1.)

[1] In 1997 the U.S. Census Bureau replaced the U.S. Standard Industrial Classification (SIC) system with the North American Industry Classification System (NAICS). NAICS industries are identified with a 6-digit code, which accommodates a larger number of sectors and provides more flexibility in designating subsectors. The new system also provides for greater detail for the three NAICS countries (the United States, Canada, and Mexico). The international NAICS agreement fixes only the first five digits of the code. Thus, the sixth digit in any given item in the U.S. code may differ from that of the Canadian of Mexican code. The SIC system had a 4-digit code.

TABLE 10.1 Concentration Ratios and Herfindahl–Hirschman Indices in Manufacturing, 1997

NAICS	Industry	Number of companies	Value of shipments ($ millions)	Largest 4 companies	Largest 8 companies[a]	HHI for largest 50 companies[a]
312221	Cigarettes	9	29,253	98.9	(D)	(D)
331411	Primary copper	9	6,128	94.5	(D)	2,392.2
327213	Glass containers	11	4,198	91.1	98.0	2,959.9
312120	Breweries	494	18,203	89.7	93.4	(D)
311230	Breakfast cereals	48	6,556	86.7	94.7	2,772.7
336411	Aircraft	172	57,893	84.8	96.0	(D)
336111	Automobiles	173	95,366	79.5	96.3	2,349.7
325611	Soap and detergents	738	17,773	65.6	77.9	1,618.6
325110	Petrochemicals	42	19,468	59.8	83.3	1,187.0
331312	Primary aluminum	13	6,225	59.2	81.7	1,230.6
334413	Semiconductors	993	78,479	52.5	64.0	1,080.1
334111	Electronic computers	531	66,302	45.4	68.5	727.9
337111	Iron and steel	191	56,994	32.7	52.7	445.3
324110	Petroleum refineries	122	158,668	28.5	48.6	422.1
322121	Paper mills	121	42,966	37.6	59.2	541.7
325412	Pharmaceuticals	707	66,735	35.6	50.1	462.4
323117	Book printing	690	5,517	31.9	45.1	363.7
332510	Hardware	906	11,061	17.4	27.7	154.6
321113	Sawmills	4,024	24,632	16.8	23.2	112.3

[a] (D), data omitted because of possible disclosure; data are included in higher level totals.
Source: U.S. Census Bureau, 1997 Economic Census.

Definition: Concentration ratios measure the percentage of the total industry revenue or market share that is accounted for by the largest firms in an industry.

Although the concentration ratios in Table 10.1 will provide useful insights into the degree of industrial concentration, it is important not to read too much into the statistics. To begin with, standard industrial classifications are based on the similarity of production processes but ignore substitutability across products, such as glass versus plastic containers. U.S. Census data describe domestically produced goods and do not include import competing products. Table 10.1 indicates, for example, that the eight largest U.S. makers of motor vehicles and bodies account for 91% of industry output. By omitting data from foreign competitors, especially from Japanese automobile manufacturers, this statistic clearly overstates the actual market share of U.S. automakers.

Another weakness of concentration ratios is that they are not sensitive to differences within categories. The concentration ratio, for example, makes no distinction between industry *A*, in which the top four companies have 24% of the market, and industry *B*, in which the largest firm has 90%

of the market, while the next three companies account for an additional 6%. In both industries the concentration ratio for the largest four companies is 96%.

HERFINDAHL–HIRSCHMAN INDEX

In 1982 and 1984, guidelines of the U.S. Department Justice for identifying unacceptable mergers were modified with the development of the *Herfindahl–Hirschman Index* (HHI). The HHI is calculated as

$$\text{HHI} = \sum_{i=1 \rightarrow n} S_i^{\,2} \tag{10.1}$$

where n is the number of companies in the industry and S_i is the ith company's market share expressed in percentage points. The Herfindahl–Hirschman Index ranges in value from zero to 10,000. According to the modified guidelines, the Justice Department views any industry with an HHI of 1,000 or less as unconcentrated. Mergers in unconcentrated industries will go unchallenged. If the index is between 1,000 and 1,800, a proposed merger will be challenged by the Justice Department if, as a result of the merger, the index rises by more than 100 points. Finally, if the HHI is greater than 1,800, proposed mergers will be challenged if the index increases by more than 50 points. Table 10.1 summarizes concentration ratios for the largest four and eight companies and the Herfindahl–Hirschman Index for the 19 industries listed.

Definition: The Herfindahl–Hirschman Index is a measure of the size distribution of firms in an industry that considers the market share of all firms and gives a disproportionately large weight to larger firms.

The HHI is superior to the concentration ratio in that it not only uses the market share information of all firms in the industry, but by squaring individual market shares, gives greater weight to larger firms. Thus the HHI for industry A in our earlier example is 2,304, while the HHI for the more concentrated industry B is 8,112. According to the Department of Justice guidelines, both markets are concentrated.

MODELS OF DUOPOLY AND OLIGOPOLY

As mentioned earlier, the distinctive characteristic of duopolies and oligopolies is the interdependence of firms. It is difficult to formulate models of duopoly and oligopoly because of the many ways in which firms deal with this interdependence. Thus, there is no general theory to explain this interdependence. The models presented next are based on specific assumptions regarding the nature of this interaction.

SWEEZY ("KINKED" DEMAND CURVE) MODEL

Although managers of oligopolistic firms are aware of the law of demand, they are also aware that their pricing and output decisions depend on the pricing and output decisions of their competitors. More specifically, such firms know that their pricing and output decisions will provoke pricing and output adjustments by their competitors. Another notable characteristic of oligopolistic industries is the relative infrequency of price changes. Paul Sweezy (1939) attempted to explain this price rigidity by suggesting that oligopolists face a "kinked" demand curve, as illustrated in Figure 10.2.

Definition: Price rigidity is characterized by the tendency of product prices to change infrequently in oligopolistic industries.

Definition: The "kinked" demand curve is a model of firm behavior that seeks to explain price rigidities in oligopolistic industries.

Figure 10.2 depicts the situation of a typical firm operating in an oligopolistic industry. The demand curve for the product of the firm really comprises two demand curves, D_1 and D_2. Unlike a monopoly or monopolistically competitive firm that has a degree of market power along the length of a single demand curve, the oligopolist faces a demand curve characterized by a "kink," illustrated in Figure 10.2 as the heavily darkened portions of demand curves D_1 and D_2.

Suppose initially that the price of the oligopoly's product is P^*. If the firm raises the price of its product above P^* and its competitors do not follow the price increase, it will lose some market share. The firm realizes this and is reluctant to sacrifice its market position to its competitors. On the other hand, if the firm attempts to capture market share by lowering price, the price decrease will be matched by its rivals, who are not willing to cede their market share. The firm whose experience is depicted in Figure

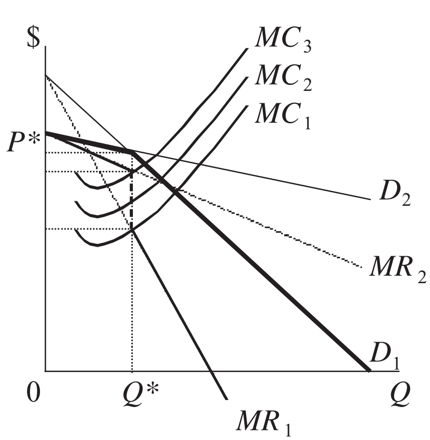

FIGURE 10.2 The "kinked" demand curve (Sweezy) model.

10.2 posts a small increase in sales as the inflation-adjusted purchasing power by consumers increases following an industry-wide decrease in prices, but the increase in sales is considerably less than the loss of sales from a comparable increase in prices. In other words, demand for the oligopolist's product is relatively more elastic for price increases than for price decreases.

Figure 10.2 also illustrates why prices in oligopolistic industries change more infrequently than in market structures characterized by more robust competition. Assume that the few firms in this oligopolistic industry are of comparable size. The marginal revenue curve associated with the "kinked" demand curve is illustrated by the heavy dashed line in Figure 10.2. Because of the "kink" at output level Q^*, the marginal revenue curve is discontinuous.

Figure 10.2 also assumes the usual U-shaped marginal cost curves. As always, the firm maximizes its profit at the output level at which $MR = MC$. This occurs at Q^*. Note, however, that because of the discontinuity of the marginal revenue curve, marginal cost can fluctuate from MC_1 to MC_3 without a corresponding change in the profit-maximizing price or output level. This result differs from cases considered thus for, in which an increase (decrease) in marginal cost will be matched by an increase (decrease) in price and a decrease (increase) in output. The importance of this result is that the adoption of more efficient production technologies, which results in lower marginal costs, may not result in significant reductions in the market price of the product. Conversely, an increase in marginal costs may not be immediately passed along to the consumer.

The "kinked" demand curve analysis has been criticized on two important points. While the analysis offers some explanation for price stability in oligopolistic industries, it offers no insights with respect to how prices are originally determined. Moreover, empirical research generally has failed to verify predictions of the model. Stigler (1947), for example, found that in oligopolistic industries price increases were just as likely to be matched as were price cuts.

Problem 10.1. Lightning Company is a firm in an oligopolistic industry. Lightning faces a "kinked" demand curve for its product, which is characterized by the following equations:

$$Q_1 = 82 - 8P$$

$$Q_2 = 44 - 3P$$

Suppose further that the firm's total cost equation is

$$TC = 8 + Q + 0.05Q^2$$

a. Give the price and output level for Lightning's product.
b. Based on your answer to part a, what is the firm's profit?

c. Determine the range of values within which Lightning's marginal cost may vary without affecting the prevailing price and output level.
d. Based on your answer to part a, what is the firm's marginal cost? Is it consistent with your answer to part c?
e. Suppose that Lightning's total cost equation changed to $TC = 12 + 5Q + 0.1Q^2$. Will the firm continue to operate at the same price and output level? If not, what price will the firm charge and how many units will it produce?
f. Based on your answer to part e, what is the Lightning's profit?

Solution

a. $82 - 8P = 44 - 3P$
$P^* = 7.6$
$Q^* = 82 - 8(7.6) = 21.2$

b. $\pi = TR - TC = 7.6(21.2) - 8 - 21.2 - 0.05(21.1)^2$
$= 161.21 - 8 - 21.2 - 22.47 = 109.45$

c. $P = 10.25 - \frac{1}{8}Q_1$

$TR_1 = 10.25Q_1 - \frac{1}{8}Q_1^2$

$MR_1 = 10.25 - \frac{1}{4}Q_1 = 10.25 - \frac{1}{4}(21.2) = 4.95$

$P = 14.67 - \frac{1}{3}Q_2$

$TR_2 = 14.67Q_2 - \frac{1}{3}Q_2^2$

$MR_2 = 14.67 - \frac{2}{3}Q_2 = 14.67 - \frac{2}{3}(21.2) = 0.54$

Marginal cost may vary between 0.54 and 4.95 without affecting the prevailing (profit-maximizing) price and output level.

d. $MC = \frac{dTC}{dQ} = 1 + 0.1Q = 1 + 0.1(21.2) = 3.12$

This result is consistent with the answer to part c, since it lies between 0.54 and 4.95.

e. The firm will maximize its profit where $MC = MR$. Marginal cost is

$$MC = \frac{dTC}{dQ} = 5 + 0.2Q$$

The relevant portion of the marginal revenue curve is

$$MR_1 = 10.25 - \frac{1}{4}Q_1$$

Equating marginal cost with marginal revenue yields

$$5 + 0.2Q_1 = 10.25 - 0.25Q_1$$

$$Q^* = 11.67$$

$$P^* = 10.25 - \frac{1}{8}(11.67) = 8.79$$

In other words, Lightning will not continue to produce at the original price and output level. Note that at the new output level Lightning's marginal cost is

$$MC = 5 + 0.2(11.67) = 7.32$$

which falls outside the range of values calculated in part c.

f. $\pi = TR - TC = 8.79(11.67) - 12 - 5(11.67) - 0.1(11.67)^2$
$= 102.58 - 12 - 58.35 - 13.62 = 18.61$

Problem 10.2. Suppose that International Dynamo is a contractor in the oligopolistic aerospace industry. International Dynamo faces a "kinked" demand curve for its product, which is defined by the equations

$$Q_1 = 200 - 2P$$

$$Q_2 = 60 - 0.4P$$

Suppose further that International Dynamo has a constant marginal cost MC = \$50.

a. Give the price and output level for International Dynamo's product.
b. Based on your answer to part a, what is International Dynamo's profit?
c. Determine the range of values within which marginal cost may vary without affecting the prevailing market price and output level.
d. Diagram your answers to parts a, b, and c.

Solution

a. We determine the price and output level for International Dynamo's product at the "kink" of the "kinked" demand curve, which occurs at the intersection of the two demand curves. Solving the two demand curves simultaneously yields

$$200 - 2P = 60 - 0.4P$$

$$P^* = \$87.50$$

At P^* = \$87.50, International's total output is

$$Q^* = 200 - 2(87.50) = 200 - 175 = 25 \text{ units}$$

b. Since MC is constant, $MC = ATC$. By the definition of ATC

$$ATC = \frac{TC}{Q}$$

$$TC = ATC \times Q = MC \times Q = 50(25) = \$1,250$$

Total revenue is

$$TR = PQ = 87.50(25) = \$2{,}187.50$$

Total profit is, therefore,

$$\pi = TR - TC = 2{,}187.50 - 1{,}250 = \$937.50$$

c. To determine the range of values within which marginal cost may vary without affecting the price and output level, first derive the marginal revenue function for International Dynamo. Solving the demand equations for P yields

$$P = 100 - 0.5Q_1$$

$$P = 150 = 2.5Q_2$$

Total revenue is defined as

$$TR = PQ$$

Applying this definition to the demand functions yields

$$TR_1 = 100Q - 0.5Q^2$$

$$TR_2 = 150Q - 2.5Q^2$$

The corresponding marginal revenue functions are

$$MR_1 = \frac{dTR_1}{dQ} = 100 - Q$$

$$MR_2 = \frac{dTR_2}{dQ} = 150 - 5Q$$

These marginal revenue functions, however, are not relevant for all positive values of Q; MR_1 is relevant only for values $0 \leq Q \leq 25$; MR_2 is relevant for values $Q \geq 25$. For the firm to maximize profit, MC must equal MR. At $Q = 25$,

$$MR_1 = 100 - 25 = 75$$

$$MR_2 = 150 - 5(25) = 25$$

Thus, marginal cost may vary between 25 and 75 without affecting the prevailing (profit-maximizing) price and output level.

d. Consider Figure 10.3.

COURNOT MODEL

A classic treatment of duopolies (and oligopolies) was first formulated by the French economist Augustin Cournot in the early nineteenth century. (see Cournot, 1897). Cournot began by assuming that duopolies produce a homogeneous product. The critical assumption of the model deals with the firms' output decision-making process. In the Cournot model, each firm

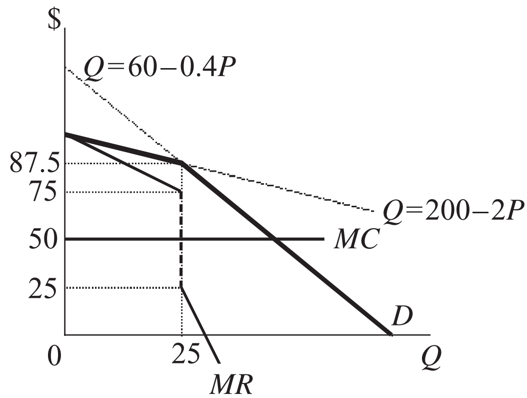

FIGURE 10.3 Diagrammatic solution to problem 10.2.

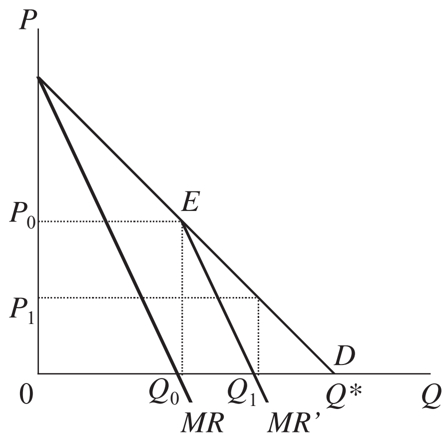

FIGURE 10.4 The Cournot model.

decides how much to produce and assumes that its rival will not alter its level of production in response. Additionally, total output of both firms equals the output for the industry. The process whereby equilibrium is established in the Cournot model may be illustrated by considering Figure 10.4.

Definition: The Cournot model is a theory of strategic interaction in which each firm decides how much to produce by assuming that its rivals will not alter their level of production in response.

To simplify matters, assume that the demand curve for the product is linear and that the marginal cost of production for each firm is zero. Cournot's example was that of a monopolist selling spring water produced at zero cost. Assume that firm A is the first to enter the industry. Thus, to maximize its profits ($MC = MR$), firm A will produce $Q_0 = 1/2Q^*$ units of output and charge a price of P_0. With a linear demand curve and zero marginal cost of production, Q_0 is half the output, where $P = 0$, or Q^*. The latter condition also assumes a perfectly competitive industry, where individual

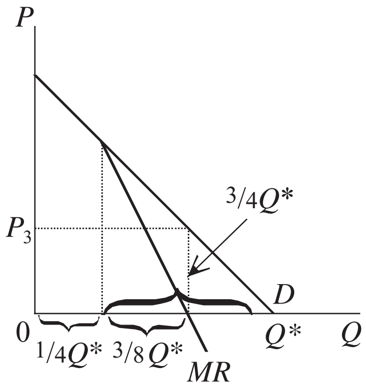

FIGURE 10.5 Determination of market shares in the Cournot model.

firms take the selling price as constant. Since $MC = 0$, maximizing profits is equivalent to maximizing total revenue, since $P = MR = MC = 0$.

Since the barriers to entry into this industry are low, the existence of economic profit attracts firm *B* into production. firm *B* also sells spring water that is produced at zero cost. In the Cournot model, firm *B* takes the output of firm *A* (Q_0) as given. Thus, from the point of view of firm *B* the vertical axis has been shifted to Q_0. The demand curve relevant to firm *B* is the line segment *ED*. To maximize its profits, firm *B* will produce such that marginal revenue is equal to zero marginal cost, which occurs at an output level of $Q_1–Q_0$, which is $\frac{1}{2}Q_0$ or $\frac{1}{4}Q^*$. The combined output of the industry is now $\frac{1}{2}Q^* + \frac{1}{4}Q^* = \frac{3}{4}Q^*$.

This, of course, is not the end of the story. Since total industry output is now Q_1, the market price of the product must fall to P_1. If firm *A* attempts to maintain a price of P_0, it will lose part of its market share to firm *B*. In the Cournot model, firm *A* will assume that firm *B* will continue to produce $\frac{1}{4}Q^*$. firm *A* will subsequently adjust its output to maximize its profit based on the remaining $\frac{3}{4}Q^*$ of the market. This situation is depicted in Figure 10.5.

It can be seen in Figure 10.5 that firm *A* can maximize its profits by producing half of the remaining three-quarters of the market, or $\frac{3}{8}Q^*$. Combined industry output is now $\frac{1}{4}Q^* + \frac{3}{8}Q^* = \frac{5}{8}Q^*$. Firm *B* will, of course react by taking firm *A*'s output of $\frac{3}{8}Q^*$ units as given and adjusting output to maximize its profit based on the remaining $\frac{5}{8}Q^*$ of the market. Extending the analysis, this means that firm *B* will increase its output to $\frac{5}{16}Q^*$. This process of action and reaction, which is summarized in Table 10.2, will come to an end when both firms have a market share equal to $\frac{1}{3}Q^*$. When firm *A* produces $\frac{1}{3}Q^*$, this leaves $\frac{2}{3}Q^*$ remaining for firm *B* to maximize its profits. Since half of the remaining market is $\frac{1}{3}Q^*$, the process now comes to a halt. The Cournot model can be generalized to include industries comprising of more than two firms. Cournot demonstrated that when the marginal cost of production is zero ($MC = 0$), then total industry output is given as

TABLE 10.2 Firm and Industry Output

Iteration	Q^A	Q^B	$Q^A + Q^B$
1	$\frac{1}{2}Q^*$	0	$\frac{1}{2}Q^*$
2	$\frac{1}{2}Q^*$	$\frac{1}{4}Q^*$	$\frac{3}{4}Q^*$
3	$\frac{3}{8}Q^*$	$\frac{1}{4}Q^*$	$\frac{5}{8}Q^*$
4	$\frac{3}{8}Q^*$	$\frac{5}{16}Q^*$	$\frac{11}{16}Q^*$
5	$\frac{11}{32}Q^*$	$\frac{5}{16}Q^*$	$\frac{21}{32}Q^*$
⋮	⋮	⋮	⋮
i	$\frac{1}{3}Q^*$	$\frac{1}{3}Q^*$	$\frac{2}{3}Q^*$

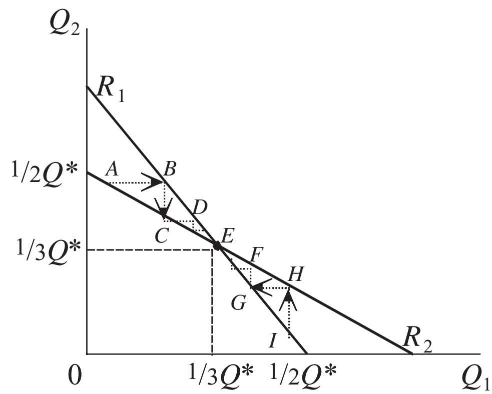

FIGURE 10.6 Reaction functions and the adjustment to a Cournot equilibrium in a duopolistic industry.

$$Q = \frac{nQ^*}{n+1} \tag{10.2}$$

where n is the number of firms in the industry.

From Equation (10.2) it is clearly recognized that as $n \to \infty$, then $Q \to Q^*$. This is the situation of perfect competition described earlier, where $MC = 0$ and $MR = P$. It may also be seen that the average market share of each firm in the industry is

$$Q_i = \frac{Q}{n} = \frac{Q^*}{n+1} \tag{10.3}$$

where i represents the ith firm in the industry, Clearly, as the number of firms in the industry increases, the market share of each individual firm will decrease. Recall that for the two-firm case, the average market share of each was $1/(2+1)Q^* = \frac{1}{3}Q^*$, where Q^* is the total output of a perfectly competitive industry.

The adjustment process just described may be illustrated with the use of the reaction functions (to be discussed in greater detail shortly) illustrated in Figure 10.6, where R_1 is the reaction function for firm 1 and R_2 is the reaction function for firm 2. Equilibrium at output levels Q_1* and Q_2* will be stable provided the reaction curve of firm 1 is steeper than that of firm 2.

Starting at point *I* in Figure 10.6, the output of firm 1 is greater than its equilibrium level of output Q_1* and the output of firm 2 is lower than its equilibrium level of output Q_2*. Given firm 1's output, firm 2 will increase its output to point *H*, as was the case, for example, in the move from iteration 3 to iteration 4 in Table 10.2. Firm 1 will react by reducing its output to point *G* (iteration 5). Continuing in this manner will eventually lead to the equilibrium output level at point *E*. Analogous reasoning would produce the same result if the process were to begin at point *A* with firm 2 producing "too much" and firm 1 producing "too little."

The analysis thus far assumes that the demand functions that confront the two firms are identical and that production occurs at zero marginal cost ($MC = 0$). Of course, neither of these assumptions will necessarily be valid. To see this, consider the following, more general, description of the Cournot duopoly model. Since the sum of the output of two firms equals the industry output $Q = Q_1 + Q_2$, the market demand function may be written

$$P = f(Q_1 + Q_2) \tag{10.4}$$

where Q_1 and Q_2 represent the outputs of firm 1 and firm 2, respectively. The total revenue of each duopolist may be written as

$$TR_1 = Q_1 f(Q_1 + Q_2) \tag{10.5a}$$

$$TR_2 = Q_2 f(Q_1 + Q_2) \tag{10.5b}$$

The profits of the firm are

$$\pi_1 = Q_1 f(Q_1 + Q_2) - TC_1(Q_1) \tag{10.6}$$

$$\pi_2 = Q_2 f(Q_1 + Q_2) - TC_2(Q_2) \tag{10.7}$$

The basic behavioral assumption underlying the Cournot model is that each duopolist will maximize its profit without regard to the actions of its rival. In other words, the firm assumes that its rival's output is invariant with respect to its own output decision. Thus, each duopolist maximizes profit holding output of its rival constant. Taking the appropriate first partial derivative, setting the results equal to zero we find

$$\frac{\partial \pi_1}{\partial Q_1} = \frac{\partial TR}{\partial Q_1} - \frac{\partial TC_1}{\partial Q_1} = 0 \tag{10.8}$$

or

$$MR_1 = MC_1 \tag{10.9}$$

Similarly for firm 2,

$$\frac{\partial \pi_2}{\partial Q_2} = \frac{\partial TR}{\partial Q_2} - \frac{\partial TC_2}{\partial Q_2} = 0 \tag{10.10}$$

or

$$MR_2 = MC_2 \tag{10.11}$$

The marginal revenue of the duopolists is not necessarily equal. Bearing in mind that $Q = Q_1 + Q_2$, then $\partial Q/\partial Q_1 = \partial Q/\partial Q_2 = 1$. The marginal revenues of the duopolists are, therefore

$$\frac{\partial TR_i}{\partial Q} = P + Q_i\left(\frac{dP}{dQ}\right), i = 1,2 \tag{10.12}$$

Clearly, since $dP/dQ < 0$, the duopolist with the largest output will have the smallest marginal revenue. That is, an increase in the output by either firm will result in a reduction in price, while the marginal revenue of both firms will be affected. The second-order condition for profit maximization is

$$\frac{\partial^2 \pi i}{\partial Q_2^2} = \frac{\partial^2 TR_i}{\partial Q_i^2} - \frac{\partial^2 TC_i}{\partial Q_2^2} < 0, i = 1,2 \tag{10.13}$$

or

$$\frac{\partial^2 TR_i}{\partial Q_i^2} < \frac{\partial^2 TC_i}{\partial Q_2^2}, i = 1,2 \tag{10.14}$$

This result simply says that the firm's marginal revenue must be increasing less rapidly than marginal cost.

Thus, the Cournot solution asserts that each duopolist (oligopolist) will be in equilibrium if Q_1 and Q_2 maximize each firm's profits and each firm's output remains unchanged. This process may be described more fully by introducing an additional step before solving for the equilibrium output levels. Reaction functions express the output of each firm as a function of its rival's output. Solving the first-order conditions, these reaction functions may be written as

$$Q_1 = R_1(Q_2) \tag{10.15}$$

$$Q_2 = R_2(Q_1) \tag{10.16}$$

In the case of firm 1, the expression states that for any specified value of Q_2 the corresponding value of Q_1 maximizes π_1, and similarly for firm 2. The solution values are illustrated in Figure 10.7.

Problem 10.3. Suppose that an industry comprising two firms produces a homogeneous product. Consider the following demand and individual firm's cost function:

$$P = 200 - 2(Q_1 + Q_2)$$

$$TC_1 = 4Q_1$$

$$TC_2 = 4Q_2$$

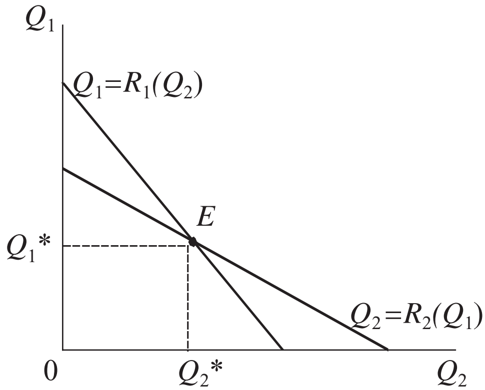

FIGURE 10.7 Cournot equilibrium.

a. Calculate each firm's reaction function.
b. Calculate the equilibrium price, profit-maximizing output levels, and profits for each firm. Assume that each duopolist maximizes its profit and that each firm's output decision is invariant with respect to the output decision of its rival.

Solution

a. The total revenue function for firm 1 is

$$TR_1 = 200Q_1 - 2Q_1^2 - 2Q_1Q_2$$

therefore, the total profit function for firm 1 is

$$\pi_1 = 200Q_1 - 2Q_1^2 - 2Q_1Q_2 - 4Q_1 = 196Q_1 - 2Q_1^2 - 2Q_1Q_2$$

For firm 1, taking the first partial derivative with respect to Q_1, setting equal to zero and solving yields

$$\frac{\partial \pi}{\partial Q_1} = 196 - 4Q_1 - 2Q_2 = 0$$

For firm 2,

$$TR_2 = 200Q_2 - 2Q_1Q_2 + 2Q_2^2$$

$$\pi_2 = 200Q_2 - 2Q_1Q_2 - 2Q_2^2 - 4Q_2 = 196Q_2 - 2Q_1Q_2 - 2Q_2^2$$

Taking the first partial derivative with respect to Q_1, setting equal to zero and solving

$$\frac{\partial \pi_2}{\partial Q_2} = 196 - 2Q_1 - 4Q_2 = 0$$

These first-order conditions yield the reactions functions

$$Q_1 = 49 - 0.5Q_2$$

$$Q_2 = 49 - 0.5Q_1$$

b. These reaction functions may be solved simultaneously to yield the equilibrium output levels

$$Q_1^* = Q_2^* = 32.67$$

Thus, total industry output is

$$Q_1^* = Q_2^* = 65.34$$

Substituting these results into the profit functions yields

$$\pi_1^* = 196(32.67) - 2(32.67)^2 - 2(32.67)(32.67) = \$2,134$$

$$\pi_2^* = 196(32.67) - 2(32.67)^2 - 2(32.67)(32.67) = \$2,134$$

The equilibrium price can be found by using the demand equation

$$P^* = 200 - 2(32.67 + 32.67) = \$69.32$$

BERTRAND MODEL

Cournot's constant-output assumption was criticized by the nineteenth-century French mathematician and economist Joseph Bertrand in 1881.[2] Bertrand argued that each firm sets the price of its product to maximize profits and ignores the price charged by its rival. This assumption is analogous to that adopted by Cournot in that both duopolists expect their rival to keep price, rather than output, constant. The demand curve facing each firm in the Bertrand model is

$$Q_1 = f_1(P_1, P_2) \tag{10.17a}$$

$$Q_2 = f_2(P_1, P_2) \tag{10.17b}$$

Once again, for simplicity, assume that each firm has constant and equal marginal cost. The total revenue and profit functions for each firm are

$$TR_1 = P_1 f_1(P_1, P_2) \tag{10.18a}$$

$$TR_2 = P_2 f_2(P_1, P_2) \tag{10.18b}$$

and

$$\pi_1 = P_1 f_1(P_1, P_2) - TC_1(P_1, P_2) \tag{10.19a}$$

$$\pi_2 = P_2 f_2(P_1, P_2) - TC_2(P_1, P_2) \tag{10.19b}$$

In the Bertrand model, the objective of each firm in the industry is to maximize Equations (10.19) with respect to its selling price, and assuming

[2] *Journal des Savants*, September, 1883.

that the price charged by its rival remains unchanged. As Problem 10.4 illustrates, it is easily seen that both firms will charge the same price when $MC_1 = MC_2$.

Definition: The Bertrand model is a theory of strategic interaction in which a firm sets the price of its product to maximize profits and ignores the prices charged by its rivals. The Bertrand model is analogous to Cournot model in that the firms expect their rivals to keep prices, rather than output, constant.

Problem 10.4. Suppose that an industry comprising two firms producing a homogeneous product. Suppose that the demand functions for two profit-maximizing firms in a duopolistic industry are

$$Q_1 = 50 - 0.5P_1 + 0.25P_2$$
$$Q_2 = 50 - 0.5P_2 + 0.25P_1$$

Suppose, further, that the firms' total cost functions are

$$TC_1 = 4Q_1$$
$$TC_2 = 4Q_2$$

where P_1 and P_2 represent the prices charged by each firm producing Q_1 and Q_2 units of output.

a. What is the inverse demand equation for this product?
b. What are the equilibrium price, profit-maximizing output levels, and profits for each firm?

Solution

a. Total industry output is given as

$$Q = Q_1 + Q_2$$
$$= (50 - 0.5P_1 + 0.25P_2) + (50 - 0.5P_2 + 0.25P_1) = 100 - 0.25(P_1 + P_2)$$
$$0.25(P_1 + P_2) = 100 - (Q_1 + Q_2)$$

In equilibrium $P = P_1 = P_2$. Thus

$$0.25(2P) = 100 - (Q_1 + Q_2)$$
$$0.5P = 100 - (Q_1 + Q_2)$$
$$P = 200 - 2(Q_1 + Q_2$$

which is the demand equation in Problem 10.3.

b. The total revenue function for firm 1 is

$$TR_1 = P_1Q_1 = P_1(50 - 0.5P_1 + 0.25P_2) = 50P_1 - 0.5P_1^2 + 0.25P_1P_2$$

Similarly, the total revenue function for firm 2 is

$$TR_2 = P_2Q_2 = 50P_2 - 0.5P_2^2 + 0.25P_1P_2$$

The total cost functions for the two firms are

$$TC_1 = 4Q_1 = 4(50 - 0.5P_1 + 0.25P_2) = 200 - 2P_1 + P_2$$

$$TC_2 = 4Q_2 = 200 - 2P_2 + P_1$$

The firms' profit functions are

$$\pi_1 = TR_1 - TC_1 = -200 + 52P_1 - 0.5P_1^2 - P_2 + 0.25P_1P_2$$

$$\pi_2 = TR_2 - TC_2 = -200 + 52P_2 - 0.5P_2^2 - P_1 + 0.25P_1P_2$$

For firm 1, taking the first partial derivative with respect to P_1 and setting the result equal to zero yields

$$\frac{\partial \pi_1}{\partial P_1} = 52 - P_1 + 0.25P_2 = 0$$

For firm 2,

$$\frac{\partial \pi_2}{\partial P_2} = 52 - P_2 + 0.25P_1 = 0$$

These first-order conditions yield the reaction functions

$$P_1 = 52 + 0.25P_2$$

$$P_2 = 52 + 0.25P_1$$

Solving the reaction functions for the equilibrium price yields

$$P_1{}^* = 52 + 0.25(52 + 0.25P_1) = \$69.33$$

$$P_2{}^* = 52 + 0.25(69.33) = \$69.33$$

The profit-maximizing output levels are

$$Q_1{}^* = 50 - 0.5(69.33) + 0.25(69.33) = 32.67$$

$$Q_2{}^* = 50 - 0.5(69.33) + 0.25(69.33) = 32.67$$

Thus, total industry output is

$$Q_1^* + Q_2^* = 65.34$$

Finally, each firm's profits are

$$\pi_1{}^* = -200 + 52(69.33) - 0.5(69.33)^2 - 69.33 + 0.25(69.33)(69.33)$$
$$= \$2,134.17$$

$$\pi_2{}^* = -200 + 52(69.33) - 0.5(69.33)^2 - 69.33 + 0.25(69.33)(69.33)$$
$$= \$2,134.17$$

The reader should note from Problems 10.3 and 10.4 that for identical demand and cost functions, except for rounding the Cournot and Bertrand results, duopoly models are the same. The reader should verify that for firms producing a homogeneous product, the solution to the Bertrand model will be quite different from the solution to the Cournot model if the firms in the industry do not have identical marginal costs. To see this, suppose, initially, that each firm charges a price greater than MC_2. If $MC_1 > MC_2$, then firm 1 will be able to capture the entire market by charging a price that is only slightly below MC_1.

STACKELBERG MODEL

A variation on the Cournot model, the *Stackelberg model* posits two firms. Firm 2, which is referred to as the "Stackelberg leader," believes that firm 1 will behave as in the Cournot model by taking the output of firm 2 as constant. Firm 2 will then attempt to exploit the behavior of firm 1, called the "Stackelberg follower," by incorporating the known reaction of the follower into its production decisions. Depending on the total cost functions of the two firms, different solutions may emerge. But, if the two firms have identical total cost equations, the first mover will capture a larger share of the market and earn greater profits. This is illustrated in Problem 10.5.

Definition: The Stackelberg model is a theory of strategic interaction in which one firm, the "Stackelberg leader," believes that its rival, the "Stackelberg follower," will not alter its level of output. The production decisions of the Stackelberg leader will exploit the anticipated behavior of the Stackelberg follower.

Problem 10.5. Consider once again the situation described in Problems 10.3 and 10.4, where the demand equation for two profit-maximizing firms in a duopolistic industry is

$$P = 200 - 2(Q_1 + Q_2)$$

and the firm's total cost functions are

$$TC_1 = 4Q_1$$

$$TC_2 = 4Q_2$$

where Q_1 and Q_2 represent the output levels of firm 1 and firm 2, respectively. Assume that firm 2 is a Stackelberg leader and firm 1 is a Stackelberg follower. What are the equilibrium price, profit-maximizing output levels, and profits for each firm?

Solution. From the solution to the Cournot duopoly problem, the reaction function of firm 1 is

$$Q_1 = 49 - 0.5Q_2$$

The profit function of firm 2 is

$$\pi_2 = 196Q_2 - 2Q_2^2 - 2Q_1Q_2$$

Substituting firm 1's reaction function into firm 2's profit function yields

$$\pi_2 = 196Q_2 - 2Q_2^2 - 2Q_2(49 - 0.5Q_2) = 98_2 - Q_2^2$$

The first-order condition is

$$\frac{\partial \pi_2}{\partial Q_2} = 98 - 2Q_2 = 0$$

$$Q_2{}^* = 49$$

Substituting into the reaction function, the output of firm 1 is

$$Q_1^* = 49 - 0.5(49) = 24.5$$

Thus, total industry output is

$$Q_1{}^* + Q_2{}^* = 73.5$$

The equilibrium price in this market is

$$P^* = 200 - 2(49 + 24.5) = \$53$$

The profits of the two firms are

$$\pi_1{}^* = 98(24.5) - (24.5)^2 = \$1,800.75$$

$$\pi_2{}^* = 98(49) - (49)^2 = \$2,401$$

Compare these answers with the results obtained in Problems 10.3 and 10.4. Note that in the Stackelberg solution total industry output is higher (\$73.5 > \$65.34) and the product price is lower (\$53 < \$69.3) than in both the Cournot and Bertrand solutions. Moreover, in both the Cournot and Bertrand solutions both firms' profits were \$2,134, which is less than the profit of the Stackelberg follower, but greater than the Stackelberg leader. Finally, in the Stackelberg solution firm 1's profit of \$1,800.75 is three-fourths that of firm 2.

The duopoly models discussed have been criticized for the simplicity of their underlying assumptions. As E. H. Chamberlin (1933) noted: "When a move by one seller evidently forces the other to make a countermove, he is very stupidly refusing to look further than his nose if he proceeds on the assumption that it will not" (p. 46). Chamberlin was quick to realize that mutual interdependence would lead oligopolistic firms to explicitly or tacitly agree to charge monopoly prices and divide the profits. Chamberlin's contribution to the analysis of oligopolies was to recognize that the price and output of one firm will affect, and be affected by, the price and output decisions of other firms in the industry.

On the other hand, we can use game theory (discussed briefly in the next section and in more detail in Chapter 13), to illustrate the Cournot and Bertrand models as static games, in which in equilibrium the underlying assumptions are fulfilled. The Stackelberg model can be shown to be a dynamic game and, once again, the equilibrium assumptions are satisfied (see, e.g., Bierman and Fernandez, 1998, Chapters 2 and 6).

Definition: Mutual interdependence in pricing occurs when firms in an oligopolistic industry recognize that their pricing policies depend on the pricing policies of other firms in the industry.

COLLUSION

When duopolists or oligopolists recognize their mutual interdependence, they might agree to coordinate their output decisions to maximize the output of the entire industry. Collusion may take the form of explicit price-fixing agreements, through so-called price leadership, or by means of other practices that lessen competitive pressures. The exact nature of the collusive practices will depend on the particular characteristics of the industry. The implementation of such practices will, however, be constrained by antitrust regulation.

Definition: Collusion represents a formal agreement among firms in an oligopolistic industry to restrict competition to increase industry profits.

Definition: Price fixing is a form of collusion in which firms in an oligopolistic industry conspire to set product prices.

Definition: Price leadership is a form of price collusion in which a firm in an oligopolistic industry initiates a price change that is matched by other firms in the industry.

Perhaps the most well-known manifestation of collusive behavior is the *cartel*. A cartel is a formal agreement among firms in an oligopolistic industry to allocate market share and/or industry profits. The Organization of Petroleum Exporting Countries (OPEC) is probably the most famous of all cartels. In the mid-1970s, OPEC began to restrict the quantity of oil produced, which resulted in a dramatic increase in oil and gasoline prices. Many economists have attributed global recession and inflation to these output restrictions.

Definition: A cartel is an explicit agreement among firms in an oligopolistic industry to allocate market share and/or industry profits.

Many people believe that cartels are organized for the purpose of increasing product prices by restricting output, but in fact the opposite might occur. In the mid-1980s an international coffee cartel attempted to lower prices by increasing output! Why? The answer can be found in the price elasticity of demand, which was discussed in Chapter 4. If the demand for a product is price inelastic, as was the case of petroleum in the 1970s, producers will be able to increase total revenues by lowering output. On the other hand, if demand is price elastic, as was the case of coffee in the 1980s, producers should be able to earn higher revenues by increasing output and lowering prices. Of course, the actions by coffee producers received very little press coverage. After all, consumers rarely complain about lower prices.

When firms in an industry agree to coordinate their output decisions, the profit-maximizing behavior of the cartel is analytically identical to that of a multiplant monopolist in which the profit function is the difference between the total revenue and total costs functions of each firm.

Problem 10.6. Consider, again, the demand and cost equations given in Problem 10.5. Suppose that the two firms in the industry decide to jointly determine output levels for the purpose of maximizing industry profit. Determine the profit-maximizing levels of output, the equilibrium price, and total industry profit.

Solution. The industry profit function is given as

$$\pi = \pi_1 + \pi_2 = 196Q_1 + 196Q_2 - 2Q_1^2 - 2Q_2^2 - 4Q_1Q_2$$

Taking the first partial derivatives, setting the result equal to zero, and solving yields

$$\frac{\partial \pi}{\partial Q_1} = 196 - 4Q_2 - 4Q_1 = 0$$

$$\frac{\partial \pi}{\partial Q_2} = 195 - 4Q_1 - 4Q_2 = 0$$

The firms' reaction functions are

$$Q_1 = 49 - Q_2$$

$$Q_2 = 49 - Q_1$$

This system of linear equations yields the profit maximizing output levels for Q_1 and Q_2 of

$$Q_1{*} = Q_2{*} = 24.5$$

The product's price is given as

$$P{*} = 200 - 2(24.5 + 24.5) = \$102$$

with industry profit given as \$4,802. In the example of the Cournot solution to Problem 10.3, on the other hand, total industry profit was \$4,268 (\$2,134 + \$2,134).

GAME THEORY

Today, *game theory* is perhaps the most important tool in the economist's analytical kit for studying *strategic behavior*. Strategic behavior is concerned with how individuals and groups make decisions when they recognize that their actions affect, and are affected by, the actions of other individuals or groups. In other words, the decision-making process is mutually interdependent.

Definition: Strategic behavior recognizes that decisions of competing individuals and groups are mutually interdependent.

In each of the models thus far discussed, strategic behavior was central to an understanding of how equilibrium prices and quantities were established in oligopolistic industries. We saw in the discussion of the "kinked" demand curve, for example, that the decision by one firm in an oligopolistic industry to lower its product price to capture increased market share is likely to be countered by lower prices from rivals. On the other hand, unless justified by a mutual increase in the marginal cost of production, a price increase by one firm is likely to go unchallenged by other firms in the industry. Similar considerations of move and countermove were also explicit recognized in the form of reaction functions in our discussions of the Cournot, Bertrand, and Stackelberg models.

Game theory represents an improvement over earlier models discussed in this chapter in that it attempts to analyze the strategic interaction of firms in any competitive environment. Although more exhaustive discussion of game theory will be deferred to Chapter 13, a brief introduction is presented here to highlight the potential usefulness of this methodology in the analysis of the interdependency of pricing decisions by firms in oligopolistic industries.

Definition: Game theory is the study of how rivals make decisions in situations involving strategic interaction (i.e., move and countermove) to achieve an optimal outcome.

What is a game? There are a number of elements that are common to all games. To begin with, all games have rules. These rules define the order of play, that is, the sequence of moves by each player. The moves of each player in a game are based on strategies. A *strategy* is a sort of game plan. It is a decision rule that the player will apply when decisions about the next move need to be made. Knowledge of that player's strategy allows us to predict what course of action that player will take when confronted with choices. The collection of strategies for each player is called a *strategy profile*. Strategy profiles are often depicted within curly braces {}. Each strategy profile defines the outcome of the game and the payoffs to each player.

Definition: A strategy is a decision rule that indicates what action a player will take when confronted with a decision.

Definition: A strategy profile represents the collection of strategies adopted by a player.

Definition: A payoff represents the gain or loss to each player in a game.

Each of these basic elements of games is illustrated in what is perhaps the best-known of all game theoretic scenarios—the *Prisoners' Dilemma*. The Prisoners' Dilemma is an example of a two-person, noncooperative, simultaneous-move, one-shot game in which both players have a strictly dominant strategy, that is, one that results in the largest payoff regardless of the strategies adopted by any other player. The Prisoners' Dilemma is an example of a noncooperative game in the sense that the players are unable or unwilling to collude to achieve an outcome that is optimal for both.

In the Prisoners' Dilemma the players are required to move simultaneously. Simultaneous-move games are sometimes referred to as *static games*. An example of a simultaneous-move game would be the children's game rock–paper–scissors. In this game, both players are required to recite in unison the words "rock, paper, scissors." When they say the word "scissors" both are required to simultaneously show a rock (fist), a paper (open hand), or a scissors (index and middle finger separated). The winner of the game depends on what each player shows. If one player shows rock and the other player shows scissors, then rock wins because rock breaks scissors. If one player shows rock and the other player shows paper, then paper wins because paper covers rock. If one player shows scissors and the other player shows paper, then scissors wins because scissors cut paper. Strictly speaking, it is not absolutely necessary that both players actually move at the same time. The important thing is that neither player knows what the other player plans to show until both have moved. It is only necessary that neither player be aware of the decision of the other player until after both have moved.

Finally, the Prisoners' Dilemma is an example of a one-shot game. In a one-shot game, both players have one, and only one, move. In fact, most games, such as chess or checkers, involve multiple moves in which the players "take turns" (i.e., move sequentially). *Sequential-move games* are sometimes referred to as *dynamic games*. Except for the first move, the move of each player will depend on the moves made by the other player.

Definition: In a simultaneous-move game neither player is aware of the decision of the other player until after a pair of moves has been made.

Definition: A strictly dominant strategy results in the largest payoff to a player regardless of the strategy adopted by any other player.

Definition: The Prisoners' Dilemma is a two-person, simultaneous-move, noncooperative, one-shot game in which each player adopts the strategy that yields the largest payoff, regardless of the strategy adopted by the other player.

		Suspect *B*	
		Do not confess	Confess
Suspect *A*	Do not confess	(6 months in jail, 6 months in jail)	(10 years in jail, 0 years in jail)
	Confess	(0 years in jail, 10 years in jail)	(5 years in jail, 5 years in jail)

Payoffs: (Suspect *A*, Suspect *B*)

FIGURE 10.8 Payoff matrix for the Prisoners' Dilemma.

To illustrate the Prisoners' Dilemma, consider the following situation, which is described in Schotter (1985) (see also Luce and Raiffa, 1957, Chapter 5). Two individuals are taken into custody by the police following the robbery of a store, but after of the booty has been disposed of. Although the police believe the suspects to be guilty, they do not have enough evidence to convict them. In an effort to extract a confession, the suspects are taken to separate rooms and interrogated. If neither individual confesses, the most that either one can be convicted of is loitering at the scene of the crime, which carries a penalty of 6 months in jail. On the other hand, if one confesses and turns state's evidence against the other, the person who talks will go free by a grant of immunity, while the other will receive 10 years in prison. Finally, if both suspects confess, both will be convicted, but because of a lack of evidence (the stolen items having been disposed of prior to their arrest) the penalty is 5 years on the lesser charge of breaking and entering. The decision problem and outcomes facing each suspect are illustrated in Figure 10.8.

The entries in the cells of the *payoff matrix* refer to the gain or loss to each player from each combination of strategies. The payoffs are often depicted in parentheses. The first entry in parentheses in each cell refers to the payoff to suspect *A*, while the second entry refers to the payoff to suspect *B*. We will adopt the convention that the first entry refers to the payoff to the player indicated on the left of the payoff matrix, while the second entry refers in each cell refers to the payoff to the player indicated at the top. The situation depicted in Figure 10.8 is sometimes referred to as a *normal-form game*.

Definition: A normal-form game summarizes the players, possible strategies, and payoffs from alternative strategies in a simultaneous-move game.

In the situation depicted in Figure 10.8, the worst outcome is reserved for the suspect who does not confess if the other suspect does confess. To see this, consider the lower left-hand cell of the payoff matrix, which represents the decision by suspect *A* to confess and the decision by suspect *B* not to confess. The result of the strategy profile *{Confess, Do not confess}* is that suspect *A* is set free, while suspect *B* goes to prison for 10 years. Since the payoff matrix is symmetric, the strategy profile *{Do not confess, Confess}* results in the opposite outcome.

It should be remembered that the Prisoners' Dilemma is a noncooperative game. Neither suspect has any idea what the other plans to do before making his or her own move. The key element is strategic uncertainty. Since both suspects are being held incommunicado, they are unable to cooperate. Under the circumstances, if both suspects are rational, the decision of each suspect (i.e., the move that will result in the largest payoff), will be to confess. Why? Consider the problem from suspect *A*'s perspective. If suspect *B* does not confess, the more advantageous response is to confess, since this will result in no prison time as opposed to 6 months by not confessing. On the other hand, if suspect *B* confesses, suspect *A* would be well advised to confess because this would result in 5 years in prison, compared with 10 years by not confessing. In other words, suspect *A*'s best strategy is to confess, regardless of the strategy adopted by suspect *B*. Since the payoff matrix is symmetric, the same thing is true for suspect *B*. In this case, both suspects' strictly dominant strategy is to confess. The strictly dominant strategy equilibrium for this game is *{Confess, Confess}*. In this case, both suspects will receive 5 years in prison.

The foregoing solution is called a *Nash equilibrium*, in honor of John Forbes Nash Jr. who, along with John Harsanyi and Reinhard Selten, received the 1994 Nobel Prize in economic science for pioneering work in game theory. A noncooperative game has a Nash equilibrium when neither player can improve the payoff by unilaterally changing strategies. Nash created quite a stir in the economics profession in 1950, when he first proposed his now famous solution to noncooperative games, which he called a "fixed-point equilibrium." The reason was that his result seemed to contradict Adam Smith's famous metaphor of the invisible hand, which asserts that the welfare of society as a whole is maximized when each individual pursues his or her own private interests. According to the situation depicted in Figure 10.8, it is clearly in the best interest of both suspects to adopt the joint strategy of not confessing. This would result in an optimal solution, at least for the suspects, of only 6 months in prison.

Definition: A Nash equilibrium occurs in a noncooperative game when each player adopts a strategy that is the best response to what is believed to be the strategy adopted by the other players. When a game is in Nash equilibrium, neither player can improve the payoff by unilaterally changing strategies.

The Prisoners' Dilemma provides some very important insights into the strategic behavior of oligopolists. To see this, consider the situation of a duopolistic industry. Suppose that firm *A* and firm *B* are confronted with the decision to charge a "high" price or a "low" price for their product.

In the case of the "kinked" demand curve model discussed earlier, for example, each firm recognizes that a unilateral change in price is likely to precipitate a response from the rival firm. More specifically, if firm *A* charges a "high" price, but firm *B* charges a "low" price, then firm *B* will gain market share at firm *A*'s expense, and *vice versa*. On the other hand, if the two firms were to collude, they could act as a profit-maximizing monopolist and both would benefit. But collusion, at least in the United States, is illegal, so it may be possible to model the strategic behavior of the two firms as a game similar to the Prisoners' Dilemma (i.e., a two-person, noncooperative, simultaneous-move, one-shot game). To see this, suppose that the alternatives facing each firm in the present situation are as summarized in Figure 10.9. The numbers in each cell represent the expected profit that can be earned by each firm given any combination of a high price and a low price strategy.

Does either firm have a strictly dominant strategy in this scenario? To answer this question, consider the problem from the perspective of firm *B*. If firm *A* charges a "high" price, it will be in firm *B*'s interest to charge a "low" price. Why? If firm *B* adopts a *high-price* strategy, it will earn a profit of $1,000,000 compared with a profit of $5,000,000 adopts a low-price strategy. On the other hand, if firm *A* charges a "low" price, then firm *B* will earn a profit of $100,000 if it charges a "high" price and $250,000 if it charges a "low" price. In this case, regardless of the strategy adopted by firm *A*, it will be in firm *B*'s best interest to charge a "low" price. Thus, firm *B*'s dominant strategy is to charge a "low" price. What about firm *A*? Since the entries in the payoff matrix are symmetric, the outcome will be identical. If firm *B* charges a "high" price, it will be in firm *A*'s best interest to adopt a *low-price* strategy, since it will earn a profit of $5,000,000, compared with a profit of only $1,000,000 by adopting a high-price strategy. If firm *B* charges a "low" price, it will again be firm *A*'s best interest to charge a "low" price and earn a profit of $250,000 as opposed to earning a profit of only $100,000 by charging a "high" price. Thus, firm *A*'s dominant strategy is also to charge a "low" price. Thus, in this noncooperative game, where the pricing decision of one firm is independent of the pricing decision of the other firm, it pays for both firms to charge a "low" price, with each firm earning a profit of $250,000. In other words, the strictly dominant strategy equilibrium for this game is *{Low price, Low price}*.

The reader should note that the solution to the game depicted in Figure 10.9 is a Nash equilibrium because neither firm can improve its payoff by unilaterally switching to another strategy. On the other hand, if both firms were to cooperate and charge a "high" price, each firm could earn profits of $1,000,000. Note, however, that a *{High price, High price}* strategy profile is not a Nash equilibrium, since either player could improve its payoff by switching strategies. That is, firm *A* could earn profits of $5,000,000 by charg-

		Firm *B*: High price	Firm *B*: Low price
Firm *A*	High price	($1,000,000, $1,000,000)	($100,000, $5,000,000)
	Low price	($5,000,000, $100,000)	($250,000, $250,000)

Payoffs: (Firm *A*, Firm *B*)

FIGURE 10.9 Game theory and interdependent pricing behavior.

ing "low" price, provided firm *B* continues to charge a "high" price. Of course, if firm *A* were to charge a "low" price, firm *B* would respond by lowering its price as well.

Historically, such cartels have proven to be highly unstable. Even if both firms were legally permitted to collude, distrust of the other firm's motives and intentions might compel each to charge a lower price anyway. Whether the collusive arrangement is legal or illegal, the incentive for cartel members to cheat is strong. Economic history is replete with examples of cartels that have collapsed because of the promise of gain at the expense of other members of the cartel. For such cartel arrangements to be maintained, it must be possible to enforce the agreement by effectively penalizing cheaters. The conditions under which this is likely to occur will be discussed in Chapter 13.

Problem 10.7. Why do fast-food restaurants tend to cluster in the same immediate vicinity? Consider the following situation concerning the owners of two hamburger franchises, Burger Queen and Wally's. Route 795 was recently extended from Baconsville to Hashbrowntown. Both franchise owners currently operate profitable restaurants in Hashbrowntown, a small town of about 25,000 residents. The exit off Route 795 is 5 miles from Hashbrowntown. Both franchise owners are considering moving their restaurants from the center of town to a location near the exit ramp. Regardless of location, we will assume that there is only enough business for two fast-food franchises to operate profitably. The franchise owners calculate that by relocating they will continue to receive some in-town business, but will also gain customers who use the exit as a rest stop. The payoff matrix for either strategy in this game is illustrated in Figure 10.10. The first entry in each cell of the payoff matrix refers to the payoff to Wally's and the second entry refers to the payoff to Burger Queen.

a. Does either franchise owner have a strictly dominant strategy?
b. Is the solution to this game a Nash equilibrium?

		Burger Queen: Exit ramp	Burger Queen: Hashbrowntown
Wally's	Exit ramp	($150,000, $150,000)	($1,000,000, $100,000)
	Hashbrowntown	($100,000, $1,000,000)	($250,000, $250,000)

Payoffs: (Wally s, Burger Queen)

FIGURE 10.10 Payoff matrix for problem 10.7.

Solution

a. Both franchise owners have a dominant strategy to relocate to the exit ramp. Consider the problem from Burger Queen's perspective. If Wally's relocates near the exit ramp, it will be in Burger Queen's best interest to relocate there as well, since the payoff of $150,000 is greater than the alternative of $100,000 by remaining in Hashbrowntown. If Wally's decides to remain in Hashbrowntown, then, once again, it will be in Burger Queen's best interest to relocate, since the payoff of $1,000,000 is greater than $250,000. Thus, Burger Queen's dominant strategy is to locate near the exit ramp. Since the entries in the payoff matrix are symmetrical, the same must be true for Wally's. Thus, the dominant-strategy equilibrium for this game is *{Exit ramp, Exit ramp}*.
b. Note that the optimal solution for both franchise owners is to agree to remain in Hashbrowntown, since the payoff to both fast-food restaurants will be greater. But, this would require cooperation between Burger Queen and Wally's. If collusive behavior is ruled out, the dominant-strategy equilibrium *{Exit ramp, exit Ramp}* is also a Nash equilibrium, since neither franchise can unilaterally improve its payoff by choosing a different strategy.

CHAPTER REVIEW

The characteristics of oligopoly are relatively few sellers, either standardized or differentiated products, price interdependence, and relatively difficult entry into and exit from the industry. A duopoly is an industry comprising two firms producing homogeneous or differentiated products in which entry and exit into and from the industry is difficult.

Two common measures for determining the degree of industrial concentration are the *concentration ratio* and the *Herfindahl–Hirschman Index*. Concentration ratios measure the percentage of total industry revenue or market share accounted for by the industry's largest firms. The Herfindahl–Hirschman Index is a measure of the size distribution of firms in an industry but assigns greater weight to larger firms.

Mutual interdependence in pricing decisions, which is characteristic of industries with high concentration ratios, makes it difficult to determine the optimal price for a firm's product. *Collusion* occurs when firms coordinate their output and pricing decisions to maximize the output of the entire industry. Collusion may take the form of explicit price-fixing agreements, through so-called price leadership, or by means of other practices that lessen competitive pressures. Perhaps the best-known example of collusive behavior is the *cartel*, which is a formal agreement among producers to allocate market share and/or industry profits.

Four popular models of firm behavior in oligopolistic industries are the *Sweezy ("kinked" demand curve) model*, the *Cournot model*, the *Bertrand model*, and the *Stackelberg model*. The Sweezy model, which provides insights into the pricing dynamics of oligopolistic firms, assumes that firms will follow a price decrease by other firms in the industry but will not follow a price increase. In the Cournot model, each firm decides how much to produce and assumes that its rival will not alter its level of production in response. The Bertrand model argues that each firm sets the price of its product to maximize profits and ignores the price charged by its rival. Finally, the Stackelberg model assumes that one firm will behave as in the Cournot model by taking the output of its rival as constant, but the rival will incorporate this behavior into its production decisions.

Game theory is perhaps the most important tool in the economist's analytical kit for analyzing *strategic behavior*. Strategic behavior is concerned with how individuals make decisions when they recognize that their actions affect, and are affected by, the actions of other individuals or groups. The Prisoners' Dilemma is an example of a two-person, noncooperative, simultaneous-move, one-shot game in which both players have a strictly dominant strategy (i.e., one that results in the largest payoff regardless of the strategy adopted by any other players). A Nash equilibrium occurs in a noncooperative game when each player adopts a strategy that is the best response to what is believed to be the strategy adopted by any other player. When a two-person game is in Nash equilibrium, neither player can improve the payoff by unilaterally changing strategies.

KEY TERMS AND CONCEPTS

Bertrand model A theory of strategic interaction in which a firm sets the price of its product to maximize profits and ignores the prices charged by its rivals.

Cartel An agreement among firms in an oligopolistic industry to allocate market share and/or industry profits.

Collusion A formal agreement among firms in an oligopolistic industry to restrict competition to increase industry profits. Collusion may occur when firms in an oligopolistic industry recognize that their pricing policies are mutually interdependent. Collusion may take the form of explicit price-fixing agreements, so-called price leadership, or other practices that ameliorate competitive pressures.

Concentration ratios One way to distinguish an oligopoly from other market structures is through the use of *concentration ratios*, which measure the percentage of the total industry revenue or market share that is accounted for by the largest firms in an industry.

Cournot model The theory of strategic interaction according to which each firm decides how much to produce by assuming that its rivals will not alter their level of production in response.

Duopoly An industry comprising two firms producing homogeneous or differentiated products; it is very difficult to enter the industry and to leave it.

Game theory Game theory is the study of how rivals make decisions in situations involving strategic interaction (i.e., move and countermove) to achieve some optimal outcome. The best-known of game theoretic scenarios is the Prisoners' Dilemma, which is a two-person, noncooperative, simultaneous-move, one-shot game.

Herfindahl–Hirschman Index A measure of the size distribution of firms in an industry that considers the market share of all firms and gives disproportionate weight to larger firms.

"Kinked" demand curve A model of firm behavior that seeks to explain price rigidities in oligopolistic industries. The model postulates that a firm will not raise its price because the increase will not be matched by its competitors, which would result in a loss of market share. The firm realizes this and is reluctant to sacrifice its market position to its competitors. On the other side, a firm will not lower its price, since the reduction will be matched by its competitors who themselves are not willing to cede market share.

Mutual interdependence in pricing Exists when firms in an oligopolistic industry recognize that their pricing policies are mutually interdependent. When mutual interdependence in pricing is recognized, firms might agree to coordinate their output decisions to maximize industry profits.

Nash equilibrium Occurs in a noncooperative game when each player adopts a strategy that is the best response to what is believed to be the strategy adopted by any other player. When a two-person game is in Nash equilibrium, neither player can improve the payoff by unilaterally changing strategies.

Normal-form game Summarizes the players, possible strategies, and payoffs from alternative strategies in a simultaneous-move game.

Oligopoly An industry comprising a few firms producing homogeneous or differentiated products, it is very difficult to enter the industry and to leave it.

Payoff The gain or loss to each player in a game.

Price fixing A form of collusion in which firms in an oligopolistic industry conspire to set product prices. Price leadership is a form of price fixing.

Price leadership A form of price collusion in which a firm in an oligopolistic industry initiates a price change that is matched by other firms in the industry.

Price rigidities The result of the tendency of product prices to change infrequently in oligopolistic industries.

Prisoners' Dilemma A two-person, simultaneous-move, noncooperative, one-shot game in which each player adopts the strategy that yields the largest payoff, regardless of the strategy adopted by the other player.

Product differentiation Goods or services that are in fact somewhat different or are perceived to be so by the consumer but nonetheless perform the same basic function are said to exemplify product differentiation.

Reaction function In the Cournot duopoly model, a firm's reaction function indicates a profit-maximizing firm's output level given the output level of its rival. In The Bertrand duopoly model, a firm's reaction function indicates a profit-maximizing firm's price given the price changed by its rival.

Simultaneous-move game A game in which neither player is aware of the decision of the other player until after the moves have been made.

Stackelberg model The theory of strategic interaction in which one firm, the "Stackelberg leader," believes that its rival, the "Stackelberg follower," will not alter its level of output. The production decisions of the Stackelberg leader will exploit the anticipated behavior of the Stackelberg follower.

Strategic behavior Actions reflecting the recognition that the behavior of an individual or group affects, and is affected by the actions of other individuals or groups.

Strategy A decision rule that indicates what action a player will take when confronted with the need to make a decision.

Strategy profile The collection of strategies adopted by a player.

Strictly dominant strategy A strategy that results in the largest payoff to a player regardless of the strategy adopted by other players.

CHAPTER QUESTIONS

10.1 In contrast to perfect and monopolistic competition, oligopolistic market structures are characterized by interdependence in pricing and output decisions. Explain.

10.2 Oligopolies are characterized by "a few" firms in the industry. What is meant by "a few firms," and when does "a few" become "too many"?

10.3 Product differentiation is an essential characteristic of oligopolistic market structures. Do you agree? Explain.

10.4 What is the concentration ratio? What are the weaknesses of concentration ratios as measures of oligopolistic market structures?

10.5 Explain why the Herfindahl–Hirschman Index is superior to the concentration ratio.

10.6 Bertrand criticized Cournot's duopoly model for its assumption of constant prices. Do you agree with this statement? If not, then why not?

10.7 What is a reaction function?

10.8 How does the Stackelberg duopoly model modify the Cournot duopoly model?

10.9 E. H. Chamberlin criticized the Cournot, Bertrand, and Stackelberg duopoly models for the naivete of their underlying assumptions. To what, specifically, was Chamberlin referring?

10.10 What is a cartel? In what way is an analysis of a cartel similar to an analysis of a monopoly?

10.11 The "kinked" demand curve model suffers from the same weakness as the Cournot, Bertrand, and Stackelberg models in that it fails to consider the interdependence of pricing and output decisions of rival firms in oligopolistic industries. Do you agree? Explain.

10.12 The "kinked" demand curve model has been criticized on two important points. What are these points?

10.13 In what way does the application of game theory as an explanation of interdependent behavior among firms in oligopolistic industries represent an improvement over earlier models?

10.14 What is a Nash equilibrium?

10.15 The Prisoners' Dilemma is an example of a one-shot, two-player, simultaneous-move, noncooperative game. If the players are allowed to cooperate, a Nash equilibrium is no longer possible. Do you agree with this statement? If not, then why not?

CHAPTER EXERCISES

10.1 Suppose that the demand function for an industry's output is $P = 55 - Q$. Suppose, further, that the industry comprises two firms with constant average total and marginal cost, $ATC = MC = 5$. Finally, assume that each firm in the industry believes that its rival will not alter its output when determining how much to produce.

a. Give the equilibrium price, quantity, and profit of each firm in the industry. (*Hint*: Use the Cournot duopoly model to analyze the situation.)

b. Assuming that this is a perfectly competitive industry, give the price and output level.
c. Suppose that there are 10 firms in this industry. What is output of the industry? What is the output level of each firm?
d. Suppose that the industry is dominated by a single profit-maximizing firm. What is the firm's output? How much will the firm charge for its product? What is the firm's profit?

10.2 Consider the following market demand and cost equations for two firms in a duopolistic industry.

$$P = 100 - 5(Q_1 + Q_2)$$
$$TC_1 = 5Q_1$$
$$TC_2 = 5Q_2$$

a. Determine each firm's reaction function.
b. Give the equilibrium price and profit-maximizing output for each firm, and each firm's maximum profit.

10.3 Suppose that the inverse market demand equation for the homogeneous output of a duopolistic industry is

$$P = A - (Q_1 + Q_2)$$

and that the two firms' cost equations are

$$TC_1 = B$$
$$TC_2 = C$$

where A, B, and C are positive constants. What is the profit-maximizing level of output for each firm?

10.4 Suppose that firm 2 in Exercise 10.2 is a Stackelberg leader and that firm 1 is a Stackelberg follower. What is the profit-maximizing output level for each firm?

10.5 Suppose that the demand functions for the product of two profit-maximizing firms in a duopolistic industry are

$$Q_1 = 50 - 5P_1 + 2.5P_2$$
$$Q_2 = 20 - 2.5P_2 + 5P_1$$

Total cost functions for the two firms are

$$TC_1 = 25Q_1$$
$$TC_2 = 50Q_2$$

a. What are the reaction functions for each firm?
b. Give the equilibrium price, profit-maximizing output, and profits for each firm.

		Cord: 750 cars a month	Cord: 500 cars a month
Auburn	750 cars a month	($5,000,000, $5,000,000)	($3,000,000, $6,000,000)
	500 cars a month	($6,000,000, $3,000,000)	($4,000,000, $4,000,000)

Payoffs: (Auburn, Cord)

FIGURE E10.8 Payoff matrix for chapter exercise 10.8.

10.6 Suppose that an oligopolist is charging a price of $500 and is selling 200 units of output per day. If the oligopolist were to increase price above $500, quantity demanded would decline by 4 units for every $1 increase in price. On the other hand, if the oligopolist were to lower the price below $500, quantity demanded would increase by only 1 unit for every $1 decrease in price. If the marginal cost of producing the output is constant, within what range may marginal cost vary without causing the profit-maximizing oligopolist to change either the price of the product or the level of output?

10.7 Thunder Corporation is an oligopolistic firm that faces a "kinked" demand curve for its product. If Thunder charges more than the prevailing market price, the demand curve for its product may be described by the demand equation

$$Q_1 = 40 - 2P$$

On the other hand, if Thunder charges less than the prevailing market price, it faces the demand curve

$$Q_2 = 12 - 0.4P$$

a. What is the prevailing market price for Thunder's product?
b. At the prevailing market price, what is Thunder's total output?
c. What is Thunder's marginal revenue function?
d. Assuming that Thunder Corporation is a profit maximizer, at the prevailing market price what is the possible range of values for marginal cost?
e. Diagram your answers to parts a, b, and c.

10.8 In the country of Arcadia there are two equal-sized automobile manufacturers that share the domestic market: Auburn Motorcar Company and Cord Automobile Corporation. Each company can produce 500 or 750 midsized automobiles a month. The payoff matrix for either strategy in this game is illustrated in Figure E10.8. The first entry in each cell of the payoff matrix refers to the payoff to Auburn and the second entry refers to the payoff to Cord.

a. Does either firm have a dominant strategy?
b. What is the Nash equilibrium for this game?

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11

Pricing Practices

We have thus far discussed output and pricing decisions under some very simplistic assumptions. We have assumed, for example, that a firm is a profit maximizer, that it produces and sells a single good or service, that all production takes place in a single location, that the firm operates within a well-defined market structure, and that management has precise knowledge about the firm's production, revenue, and cost functions. In addition, we assumed that the firm sells its output at the same price to all consumers in all markets. These conditions, however, are rarely observed in reality. These in the next two chapters we apply the tools of economic analysis developed earlier to more specific real-world situations, including multiplant and multiproduct operations, differential pricing, and non-profit-maximizing behavior.

PRICE DISCRIMINATION

For firms with market power, price discrimination refers to the practice of tailoring a firm's pricing practices to fit specific situations for the purpose of extracting maximum profit. Price discrimination may involve charging different buyers different prices for the same product or charging the same consumer different prices for different quantities of the same product. Price discrimination may involve pricing practices that limit the consumers' ability to exercise discretion in the amounts or types of goods and services purchased. In whatever guise price discrimination is practiced, it is often viewed by the consumer, when the consumer understands what is going on, as somehow nefarious, or at the very least "unfair."

Definition: Price discrimination occurs when profit-maximizing firms charge different individuals or groups different prices for the same good or service.

The literature generally discusses three degrees of price discrimination. First-degree price discrimination, which involves charging each individual a different price for each unit of a given product, is potentially the most profitable of the three types of price discrimination. First-degree price discrimination is the least often observed because of very difficult informational requirements. Second-degree price discrimination differs from first-degree price discrimination in that the firm attempts to maximize profits by "packaging" its products, rather than selling each good or service one unit at a time. Finally, third-degree price discrimination occurs when firms charge different groups different prices for the same good or service. While not as profitable as first-degree and second-degree price discrimination, third-degree price discrimination is the most commonly observed type of differential pricing. A recurring theme in most, but not all, price discriminatory behavior is the attempt by the firm to extract all or some consumer surplus.

FIRST-DEGREE PRICE DISCRIMINATION

We have noted that price discrimination occurs when different groups are charged different prices for the same product subject to certain conditions. Theoretically, price discrimination could take place at any level of group aggregation. Price discrimination at its most disaggregated level occurs when each "group" consists a single individual. First-degree price discrimination occurs when firms charge each individual a different price for each unit purchased. The price charged for each unit purchased is based on the seller's knowledge of each individual's demand curve. Because it is virtually impossible to satisfy this informational requirement, first-degree price discrimination is extremely rare. Nevertheless, an analysis of first-degree price discrimination is important because it underscores the rationale underlying differential pricing.

Definition: First-degree price discrimination occurs when a seller charges each individual a different price for each unit purchased.

The purpose of first-degree price discrimination is to extract the total amount of *consumer surplus* from each individual customer. The concept of consumer surplus was introduced in Chapter 8. Consumer surplus represents the dollar value of benefits received from purchasing an amount of a good or service in excess of benefits actually paid for. In Figure 11.1, which illustrates an individual's demand (marginal benefit) curve for a particular product, the market price of the product is \$3. At that price, the consumer

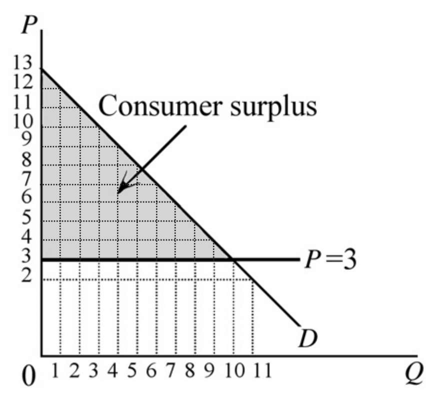

FIGURE 11.1 Consumer surplus.

purchases 10 units of the product. The total expenditure by the consumer, and therefore the total revenues to the firm, is $\$3 \times 10 = \$30$. It is clear from Figure 11.1, however, that the individual would have been willing to pay much more for the 10 units purchased at \$3. In fact, as we will see, only the tenth unit was worth \$3 to the consumer. Each preceding unit was worth more than \$3.

Suppose that we lived in a world of truth tellers. The consumer whose behavior is represented in Figure 11.1 enters a shop to purchase some amount of a particular product. The consumer is completely knowledgeable of his or her preferences and the value (to the consumer) of each additional unit. The process begins when the shopkeeper inquires how much the consumer is willing to pay for the first unit of the good. The consumer truthfully states a willingness to pay \$12. A deal is struck, the sale is made, and the consumer expends \$12, which becomes \$12 in revenue to the shopkeeper. The process continues. The shopkeeper then inquires how much the consumer is willing to pay for the second unit. By the law of diminishing marginal utility, the consumer truthfully acknowledges a willingness to pay \$11. Once again, a deal is struck, the sale is made, and the consumer expends an additional \$11, which becomes an additional \$11 in revenue to the shopkeeper.

This process continues until the tenth unit is purchased for \$3. The consumer will not purchase an eleventh unit, since the amount paid (\$3) will exceed the dollar value of the marginal benefits received (\$2). By proceeding in this manner, the consumer has paid for each item purchased an amount equivalent to the marginal benefit received, or a total expenditure of \$75. This amount is \$45 greater than would have been paid in a conventional market transaction. In other words, the shopkeeper was able extract \$45 in consumer surplus.

Definition: Consumer surplus is the value of benefits received per unit of output consumed minus the product's selling price.

Of course, this mind experiment is unrealistic in the extreme. Moreover, the amount of consumer surplus we calculated is only a rough approximation. With the price variations made arbitrarily small, the actual value of consumer surplus is the value of the shaded area in Figure 11.1. Our scenario, however, underscores the benefits to the firm being able to engage in first-degree price discrimination.

Alas, we do not live in a world of truth tellers. Even if we were completely cognizant of our individual utility functions, we would more than likely understate the true value of the next additional unit offered for sale. Moreover, even if the firm knew each consumer's demand equation, the realities of actual market transactions make it extremely unlikely that the firm would be able to extract the full amount of consumer surplus. Transactions are seldom, if ever, conducted in such a piecemeal fashion.

More formally, for discrete changes in sales (Q), consumer surplus may be approximated as

$$CS = \sum_{i=1 \to n} (P_i \times \Delta Q) - P_n Q_n \tag{11.1}$$

where Q_n is the quantity demanded by individual i at the market price, P_n. If we assume that the individual's demand function is linear, that is,

$$P_i = b_0 + b_1 Q_i \tag{11.2}$$

then consumer surplus is approximated as

$$CS = \sum_{i=1 \to n} (b_0 + b_1 Q_i)\Delta Q - P_n Q_n \tag{11.3}$$

Examination of Equation (11.3) suggests that the smaller ΔQ, the better the approximation of the shaded area in Figure 11.1. It can be easily demonstrated, and can be seen by inspection, that for a linear demand equation, as $\Delta Q \to 0$ the value of the shaded area in Figure 11.1 may be calculated as

$$CS = 0.5(b_0 - P_n)Q_n \tag{11.4}$$

In Chapter 2 we introduced the concept of the integral as accurately representing the area under a curve. The concept of the integral can be applied in this instance to calculate the value of consumer surplus. Defining the demand curve as $P = f(Q)$, consumer surplus may be defined as

$$CS = \int f(Q)dQ - P^*Q^*$$

where P_n and Q_n are the equilibrium price and quantity, respectively. Substituting Equation (11.2) into the integral equation yields

$$
\begin{aligned}
CS &= \int_0^n (b_0 + b_1 Q_i) dQ - P_n Q_n \\
&= [b_0 Q_i + 0.5 b_1 Q_i^2]_0^n - P_n Q_n \\
&= [b_0 Q_n + 0.5 b_1 Q_n^2] - \left[b_0(0) + 0.5 b_1 (0)^2\right] - P_n Q_n \\
&= [b_0 Q_n + 0.5 b_1 Q_n^2] - P_n Q_n
\end{aligned}
$$

If we assume that the demand equation is linear and that the firm is able to extract consumer surplus, how can we find the profit-maximizing price and output level? If the firm is able to extract consumer surplus, total revenue is

$$TR = PQ + 0.5(b_0 - P)Q \tag{11.5}$$

If we assume that total cost as an increasing function of output, then the total profit function is

$$\pi(Q) = TR(Q) - TC(Q) \tag{11.6}$$

Substituting Equations (11.4) and (11.5) into Equation (11.6) yields

$$
\begin{aligned}
\pi &= (b_0 - b_1 Q)Q + 0.5[b_0 - (b_0 + b_1 Q)Q] - TC \\
&= b_0 Q + 0.5 b_1 Q^2 - TC
\end{aligned} \tag{11.7}
$$

The first- and second-order conditions for profit maximization are

$$\frac{d\pi}{dQ} = b_0 + b_1 Q - MC = 0 \tag{11.8a}$$

$$\frac{d\pi^2}{dQ^2} = \frac{b_1 - dMC}{dQ} < 0 \tag{11.8b}$$

Solving Equation (11.8a) for output yields

$$Q^* = \frac{MC - b_0}{b_1} \tag{11.9}$$

Substituting Equation (11.9) into Equation (11.2) yields

$$P^* = b_0 + b_1 \left(\frac{MC - b_0}{b_1} \right) = b_0 + (MC - b_0) = MC \tag{11.10}$$

Under the circumstances, the firm attempting to extract consumer surplus does not actually charge a price equal to marginal cost. Instead, the firm will calculate consumer surplus by substituting Equation (11.10) into Equation (11.4). It should be noted that Equation (11.10) looks similar to the one the profit-maximizing firm operating in a perfectly competitive industry. Of course, the crucial difference is that $P > MC$ for a profit-maximizing firm facing a downward-sloping demand curve for its product.

Problem 11.1. Assume that an individual's demand equation is

$$P_i = 20 - 2Q_i$$

Suppose that the market price of the product is $P_n = \$6$.
a. Approximate the value of this individual's consumer surplus for $\Delta Q = 1$.
b. What is value of consumer surplus as $\Delta Q \to 0$?

Solution

a. The equation for approximating the value of consumer surplus for discrete changes in Q when the demand function is linear is

$$CS = \sum_{i=1 \to n} (b_0 + b_1 Q_i)\Delta Q - P_n Q_n$$

For $P_n = \$6$ and $\Delta Q = 1$ this equation becomes

$$CS = \sum_{i=1 \to n} (20 - 2Q_i) - 42$$

For values of Q_i from 0 to 7 this becomes

$$\begin{aligned} CS &= [20-2(1)]+[20-2(2)]+[20-2(3)]+[20-2(4)] \\ &\quad +[20-2(5)]+[20-2(6)]+[20-2(7)]-42 \\ &= 18+16+14+12+10+8+6-42 = \$42 \end{aligned}$$

The approximate value of consuming 7 units of this good is approximately \$84 dollars. If the consumer pays \$6 for 7 units of the good, then the individual's total expenditure is \$42. The approximate dollar value of benefits received, but not paid for, is \$42.

b. The value of the individual's consumer surplus as $\Delta Q \to 0$ is given by the expression

$$CS = 0.5(b_0 - P_n)Q_n$$

Substituting into this expression we obtain

$$CS = 0.5(20-6)7 = 0.5(14)7 = \$49$$

The actual value of consumer surplus is \$49, compared with the approximated value of \$42 calculated in part a.

SECOND-DEGREE PRICE DISCRIMINATION

Sometimes referred to as *volume discounting*, *second-degree price discrimination* differs from first-degree price discrimination in the manner in which the firm attempts to extract consumer surplus. In the case of second-

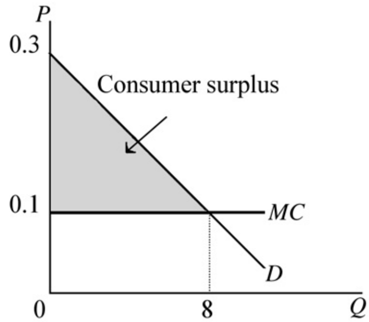

FIGURE 11.2 Block pricing.

degree price discrimination, sellers attempt to maximize profits by selling product in "blocks" or "bundles" rather than one unit at a time. There are two common types of second-degree price discrimination: *block pricing* and *commodity bundling*.

Definition: Second-degree price discrimination occurs when firms sell their product in "blocks" or "bundles" rather than one unit at a time.

Block Pricing

Block pricing, or selling a product in fixed quantities, is similar to first-degree price discrimination in that the seller is trying to maximize profits by extracting all or part of the buyer's consumer surplus. Eight frankfurter rolls in a package and a six-pack of beer are examples of block pricing.

The rationale behind block pricing is to charge a price for the package that approximates, but does not exceed, the total benefits obtained by the consumer. Suppose, for example, that the estimated demand equation of the average consumer for frankfurter rolls is given as $Q = 24 - 80P$. Solving this equation for P yields $P = 0.3 - 0.0125Q$. Suppose, further, that the marginal cost of producing a frankfurter roll is constant at \$0.10. This situation is illustrated in Figure 11.2.

With block pricing the firm will attempt to get the consumer to pay for the full value received for the eight frankfurter rolls by charging a single price for the package. If frankfurter rolls were sold for \$0.10 each, the total expenditure by the typical consumer would be \$0.80. The firm will add the value of consumer surplus to the package of eight frankfurter rolls, as follows:

$$\begin{aligned}\text{Block price} &= TR = PQ + CS = PQ + 0.5(b_0 - P)Q \\ &= 0.1(8) + 0.5(0.3 - 0.1)8 = \$1.60\end{aligned}$$

The profit earned by the firm is

$$\pi = TR - TC = PQ + 0.5(b_0 - P)Q - (MC \times Q) = \$1.60 - \$0.80 = \$0.80$$

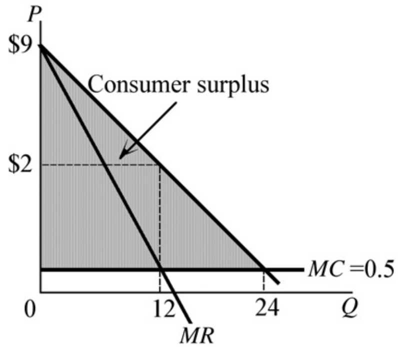

FIGURE 11.3 Amusement park pricing.

If this firm operated in a perfectly competitive industry and frankfurter rolls were sold individually, the selling price would be $0.10 per roll and the firm would break even. In other words, the firm would earn only normal profits, since $TR = TC$.

One interesting variation of block pricing is amusement park pricing. While it is not possible for the management of an amusement park to know the demand equation for each individual entering the park, and therefore first-degree price discrimination is out of the question, suppose that management had estimated the demand equation of the average park visitor. Figure 11.3 illustrates such a demand relationship.

In Figure 11.3 the marginal cost to the amusement park of providing a ride is assumed to be $0.50. If the amusement park is a profit maximizer, it will set the average price of a ride at $2 per ride (i.e., where $MR = MC$). At $2 per ride, the average park visitor will ride 12 times for an average total expenditure of $24 per park visitor. The total profit per visitor is

$$\pi = TR - TC = PQ - (MC \times Q) = 2(12) - 0.5(12) = \$18$$

At the profit-maximizing price, however, the average park visitor will enjoy a consumer surplus on the first 11 rides. The challenge confronting the managers of the amusement park is to extract this consumer surplus.

Rather than charging on a per-ride basis, many amusement parks charge a one-time admission fee, which allows park visitors to ride as often as they like. What admission fee should the amusement park charge? The park will calculate consumer surplus as if the price per ride is equal to the marginal cost to the amusement park of providing a single ride. Substituting Equation (11.22) into Equation (11.16), the amount of consumer surplus is

$$CS = 0.5(9 - 0.5)24 = \$102$$

The one-time admission fee charged by the amusement park should equal the marginal cost of providing a ride multiplied by the number of rides, plus the amount of consumer surplus. On average, the amusement park expects each guest to ride approximately 24 times. Thus, the amusement park should charge a one-time admission of \$114 [$(MC \times Q) + CS =$ \$0.5(24) + \$102].

The main difference between the block pricing of frankfurter rolls and admission to an amusement park is that while frankfurter rolls are very much a private good, amusement park rides take on the characteristics of a *public good*. The distinction between private and public goods will be discussed in greater detail in Chapter 15. For now, it is enough to say that the ownership rights of private goods are well defined. The owner of the private property rights to a good or service is able to exclude all other individuals from consuming that particular product. Moreover, once the product has been consumed, as in this case frankfurter rolls, there is no more of the good available for anyone else to consume. In other words, private goods have the properties of *excludability* and *depletability*.

The situation is quite different with public goods. For one thing, use by one person of a public good such as commercial radio programming or television broadcasts does not decrease its availability to others. Another important characteristic of a public good is unlimited access by individuals who have not paid for the good. This is the characteristic of nonexcludability. While cable television broadcasts possess the characteristic of nondepletability, they are not public goods because nonpayers can be excluded from their use.

In the case of public goods, private markets often fail because consumers are unwilling to reveal their true preferences for the good or service, which makes it difficult, if not impossible, to correctly price the good. This phenomenon is often referred to as the *free-rider problem*. In the case of pure public goods, the government is often obliged to step in to provide the good or service. The most commonly cited examples of public goods are national defense and police and fire protection. The provision of public goods is financed through tax levies.

Block pricing by amusement parks is similar to block pricing by cable television companies in that the success of this pricing policy depends crucially on management's ability to deny access to nonpayers. This is usually accomplished by controlling access to the park. It is not unusual for large amusement parks, such as the Six Flags, Busch Gardens, or Disney World theme parks, to be isolated from densely populated areas. Access to the park is typically limited to one or a few points, and the perimeter of the park is characterized by high walls, fences, or *a* natural obstacle, such as a lake, constantly guarded by security personnel. It is much more difficult for older amusement parks, which are usually located in densely populated metropolitan areas, to engage in a one-time admission fee pricing policy because of the difficulty associated with controlling access to park grounds. In such cases, an alternative pricing policy to extract consumer surplus is necessary. One such technique is to sell identifying bracelets that enable park visitors to ride as often as they like for a limited period of time, say, two hours. This approach is often advertised as a POP (pay-one-price) plan. Thus, access to rides is not controlled at the park entrance, but at the entrance to individual rides.

Ironically, whatever technique is used to extract consumer surplus by amusement parks, it is good public relations. Park visitors like the convenience of not having to pay per ride. What is more, most park visitors believe that this pricing practice is a by-product of the management's concern for the comfort and convenience of guests, which is probably true. Finally, and most important, many amusement park visitors believe that they are getting their money's worth by being able to ride as many times as they like, which is, of course, true. But do they get more than their money's worth? This may also be true, but it should not be forgotten that the purpose of this type of pricing is to maximize amusement park profits by extracting as much consumer surplus as possible.

Problem 11.2. Seven Banners High Adventure has estimated the following demand equation for the average summer visitor to its theme park

$$Q = 27 - 3P$$

where Q represents the number or rides by each guest and P the price per ride in U.S. dollars. The total cost of providing a ride is characterized by the equation

$$TC = 1 + Q$$

Seven Banners is a profit maximizer considering two different pricing schemes: charging on a per-ride basis or charging a one-time admission fee and allowing park visitors to ride as often as they like.

a. How much should the park charge on a per-ride basis, and what is the total profit to Seven Banners per customer?
b. Suppose that Seven Banners decides to charge a one-time admission fee to extract the consumer surplus of the average park guest. What is the estimated average profit per park guest? How much should Seven Banners charge as a one-time admission fee? What is the amount of consumer surplus of the average park guest?

Solution

a. Solving the demand equation for P yields

$$P = 9 - \frac{Q}{3}$$

The per-customer total revenue equation is

$$TR = PQ = \left(9 - \frac{Q}{3}\right)Q = 9Q - \frac{Q^2}{3}$$

The per-customer total profit equation is

$$\pi = TR - TC = 9Q - \frac{Q^2}{3} - (1 + Q) = -1 + 8Q - \frac{Q^2}{3}$$

The first- and second-order conditions for profit maximization are $d\pi/dQ = 0$ and $d^2\pi/dQ^2 < 0$, respectively. The profit-maximizing output level is

$$\frac{d\pi}{dQ} = 8 - \frac{2Q}{3} = 0$$

$$Q^* = 12$$

To verify that this is a local maximum, we write the second derivative of the profit function

$$\frac{d^2\pi}{dQ^2} = \frac{-2}{3} < 0$$

which satisfies the second-order condition for a local maximum. The profit-maximizing price per ride is, therefore,

$$P^* = 9 - \frac{12}{3} = 5$$

The estimated average profit per Seven Banners guest with per-ride pricing is

$$\pi = -1 + 8(12) - \frac{(12)^2}{3} = \$47$$

b. If Seven Banners charges a one-time admission fee, it will attempt to extract the total amount of consumer surplus. Since the demand equation is linear, the estimated consumer surplus per average rider is given by the equation

$$CS = 0.5(b_0 - P)Q$$ 1

From Equation (11.7) the profit equation for Seven Banners is

$$\pi = TR - TC = (b_0 + b_1Q)Q + 0.5[b_0 - (b_0 + b_1Q)]Q - TC$$

$$= \left(9 - \frac{Q}{3}\right)Q + 0.5\left[9 - \left(9 - \frac{Q}{3}\right)\right]Q - (1 + Q)$$

$$= 9Q - \frac{Q^2}{3} + \frac{0.5Q^2}{3} - 1 - Q = 8Q - \frac{Q^2}{6} - 1$$

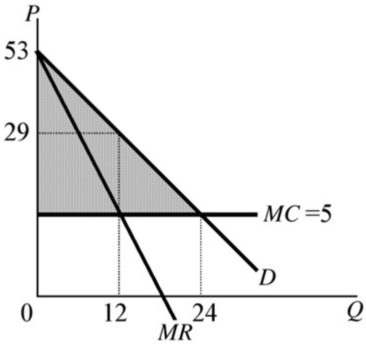

FIGURE 11.4 Two-part pricing.

The first-order condition for profit maximization is

$$\frac{d\pi}{dQ} = 8 - \frac{Q}{3} = 0$$

$$Q^* = 24$$

After substituting this value into the demand equation we get

$$P^* = 9 - \frac{24}{3} = 1 = MC$$

Total profit is, therefore,

$$\pi = 8Q - \frac{Q^2}{6} - 1 = 8(24) - \frac{(24)^2}{6} - 1 = 192 - 96 - 1 = \$95$$

The one-time admission fee should equal the total cost per guest of providing 24 rides plus the total amount of consumer surplus, that is,

$$\text{Admission fee} = TR = (MC \times Q) + CS = (MC \times Q) + 0.5(b_0 - MC)Q$$
$$= 1(24) + 0.5(9 - 1)24 = 24 + 96 = \$120$$

Thus the estimated consumer surplus of the average park guest is $96.

Two-Part Pricing

A variation of block pricing is *two-part pricing*. Two-part pricing is used to enhance a firm's profits by first charging a fixed fee for the right to purchase or use the good or service, then adding a per-unit charge. As in the case of block pricing, two-part pricing is often used by clubs to extract consumer surplus. To see how two-part pricing works, consider Figure 11.4, which illustrates the demand for country club membership.

In Figure 11.4 the per-visit demand to the country club is

$$Q = 26.5 - 0.5P$$

The club's total cost equation is

$$TC = 15 + 5Q$$

If the management of the country club were to charge its members a single price, the profit-maximizing price and output level would be 12 and \$29, respectively. The country club's profit would be (\$24 × 12) – (\$5 × 12) = \$288. At this price–quantity combination, each member of the club would receive consumer surplus (value received but not paid for) of 0.5[(53 – 29) × 12] = \$144.

If, on the other hand, the country club were to use two-part pricing, it could extract the maximum amount of consumer surplus, which is the shaded area in Figure 11.4. In this case, the club would charge an initiation fee of 0.5[(\$53 – \$5) × \$24] = \$576 and impose a per-visit charge of \$5 to cover the cost of services. It is clear that the initiation fee is pure profit and is a substantial improvement over the profit of \$288 earned by charging a single price per visit.

Commodity Bundling

Another form of second-degree price discrimination is *commodity bundling*. Commodity bundling involves combining two or more different products into a single package, which is sold at a single price. Like block pricing, commodity bundling is an attempt to enhance the firm's profits by extracting at least some consumer surplus.

A vacation package offered by a travel agent that includes airfare, hotel accommodations, meals, entertainment, ground transportation, and so on is an example of commodity bundling. Another example of commodity bundling, and one that has elicited considerable attention from the U.S. Department of Justice, is Microsoft's bundling of its Internet Explorer internet web browser with its Windows 98 software package. The federal government's interest stemmed not so much from Microsoft's ability to enhance profits by bundling its products, but from a near monopoly in the market for web browsers. Microsoft was able to ochieve because economies of scale.

To understand how commodity bundling enhances a company's profits, consider the case of a resort hotel that sells weekly vacation packages. Suppose that the package includes room, board, and entertainment. Let us further suppose that the marginal cost to the resort hotel of providing the package is \$1,000.

Management has identified two groups of individuals that would be interested in the vacation package. Although the hotel is not able to identify members of either group, it does know that each group values the components of the package differently. To keep the example simple, assume that

TABLE 11.1 Commodity bundling and vacation packages.

Group	Room and board	Entertainment
1	$2,500	$500
2	$1,800	$750

there are an equal number of members in each group. To further simplify the example, assume that total membership in each group is a single individual. Table 11.1 illustrates the maximum amount that each group will pay for the components of the package.

If the resort hotel could identify the members of each group, it might engage in first-degree price discrimination and charge members of the first group $3,000 and members of the second group $2,550 for the vacation package. Since the marginal cost of providing the service to each group is $1,000, the hotel's profit would be $3,550 per group. Since the hotel is not able to identify members of each group, what price should the hotel charge for the package?

Suppose the hotel decides to price each component of the package separately. If it charges $2,500 for room and board, it would sell only to the first group, and its total revenue would be $2,500. Members of the second group will not be interested because the price is above what the value they attach to room and board. If, on the other hand, the hotel were to charge $1,800 for room and board, it would sell to both groups for a total revenue of $3,600. Clearly, then, the hotel will charge $1,800.

The same scenario holds true for entertainment. If the hotel charges $750, then only members of the second group will purchase entertainment and the hotel will generate revenues of only $750. On the other hand, if the hotel charges $500, both groups will purchase entertainment and generate revenues of $1,000. Thus, whether the hotel charges per item or charges a package price of $1,800 + $500 = $2,300, the profit from each group will be $1,300. Since we have assumed that there is only one individual in each group, the hotel's total profit is $2,600.

Now, although a package price of $2,300 appears to be reasonable from the point of view of the profit-conscious hotel, the story does not end there. As it turns out, the hotel can do even better if it charges a package price of $1,800 + $750 = $2,550. The reason is simple. Management knows that the value of the package to the first group is $2,500 + $500 = $3,000. It also knows that the value to the second group is $1,800 + $750 = $2,550. By bundling room, board, and entertainment and selling the package for $2,550, the hotel will sell both components of the package to members of both groups. At a package price of $2,550, the hotel earns a profit of $1,550, instead of $1,300, from each group. Again, since we have assumed that there is only one person in each group, the hotel's total profit is now $3,100.

TABLE 11.2 Commodity bunding and new car options I.

Group	Power steering	CD stereo system
1	$1,700	$300
2	$1,600	$320
3	$1,500	$340

In the foregoing example, by bundling room, board, and entertainment and charging a single package price, the hotel has enhanced its profits by $250 per group member. The hotel has extracted the entire amount of consumer surplus from members of the second group and some consumer surplus from members of the first group.

Problem 11.3. A car dealership offers power steering and a compact disc stereo system as options in all new models. Suppose that the dealership sells to members of three different groups of new car buyers and that there are five individuals in each group. Table 11.2 illustrates how the members of each group value power steering and a compact disc stereo sound system.

Suppose that the per-unit cost of providing power steering and a CD stereo system is $1,200 and $250, respectively.

a. If the dealership sold each option separately, how much profit would it earn from each group member?
b. If the dealership cannot easily identify the members of each group, how should it price a package consisting of power steering and a CD stereo system? What will be the dealership's profit on each package sold?

Solution

a. If the dealership sells each item separately, it would change $1,500 for power steering, for a profit of $300 per sale. Given that there are five members in each group, the dealership has generated total profits of $4,500. By contrast, if the dealership sells power steering for $1,600, it will earn a profit of $400 per sale. But since only members of the second and third groups will purchase power steering, the dealership's total profit will only be $4,000.

   Similarly, the dealership will sell compact disc stereo systems for $300, for a profit of $50 per sale. Again, since there are five members in each group, the dealership's total profit will be $750. By contrast, if the dealership sells the option for $320 it will earn a profit of $70 per sale. Since, however, only members of the first and second group will opt for the CD stereo system at this price, the dealership's total profit will be $700.
b. If the dealership sells power steering and a CD stereo system at a package price of $1,800, as suggested in the answer to part a, the total

TABLE 11.3 Commodity bundling and new car options II.

Group	Power steering	CD stereo system
1	$2,000	$300
2	$1,800	$350
3	$1,500	$400

profit will be $4,700. However, if the dealership sells the package for $1,840, it will appeal to members of all three groups. In this way, the dealership will extract total consumer surplus from members of the third group, and at least some consumer surplus from the remaining two groups. The dealership's total profit will be $5,850.

Problem 11.4. Suppose that the members of each group in Problem 11.3 valued power steering and a compact disc stereo sound system as in Table 11.3.

The per-unit cost of providing power steering and a CD stereo system remains $1,200 and $250, respectively. How much will the dealership now change for power steering and a CD stereo system as a package? What will be the dealership's profit on each package sold? What is the dealership's total profit?

Solution. In Problem 11.3, we saw that the profit-maximizing price for the package was equivalent to the sum of the prices the third group was willing to pay for each option separately. If we were to follow that practice in this case, the profit on each package sold would be $1,900 – $1,450 = $450, for a total profit of $450 × 15 = $6,750. Suppose, however, that the dealership charged $2,150 for the package, which is the value placed on the package by the second group? The profit on each package sold would be $2,150 – $1,450 = $700, for a total profit of $700 × 10 = $7,000. Finally, if the dealership charged $2,300 for both options, which is the value placed on the package by the first group, the profit on each package would be $850, for a total profit of $850 × 5 = $4,250. Clearly, under the conditions specified in Table 11.3, the dealership will charge a package price of $2,150 and sell only to the first two groups.

THIRD-DEGREE PRICE DISCRIMINATION

In some cases, it is possible for the firm to charge different groups different prices for its goods or services. It is a common practice, for example, for theaters, restaurants, and amusement parks to offer senior citizen, student, and youth discounts. This kind of pricing strategy, which is perceived as altruistic or community spirited, has considerable public relations appeal. In reality, however, this *third-degree price discrimination* in fact results in increased company profits.

Definition: Third-degree price discrimination occurs when firms segment the market for a particular good or service into easily identifiable groups, then charge each group a different price.

For third-degree price discrimination to be effective, a number of conditions must be satisfied. First, the firm must be able to estimate each group's demand function. As we will see, the degree of price variation will depend of differences in each group's price elasticity of demand. In general, groups with higher price elasticities of demand will be charged a lower price.

A second condition that must be satisfied for a firm to engage in third-degree price discrimination is that members of each group must be easily identifiable by some distinguishable characteristic, such as age; or perhaps groups can be identified in terms of the time of the day in which the good or service, such as movie tickets, is purchased.

Finally, for third-degree price discrimination to be successful, it must not be possible for groups purchasing the good or service at a lower price to be able to resell that good or service to groups changed the higher price. If resales are possible, the firm would not be able to sell anything to the group paying the higher price because they would simply buy the good or service from the group eligible for the lower price.

The rationale behind third-degree price discrimination is straightforward. Different individuals or groups of individuals with different demand functions will have different marginal revenue functions. Since the marginal cost of producing the good is the same, regardless of which group purchases the good, the profit-maximizing condition must be $MC = MR_1 = MR_2 = \cdots = MR_n$, where n is the number of identifiable and separable groups. To see why this must be the case, suppose that $MR_1 > MC$. Clearly, in this case, it would pay for the firm to produce one more unit of the good or service and sell it to group 1, since the addition to total revenues would exceed the addition to total cost from producing the good. As more of the good or service is sold to group 1, marginal revenue will fall until $MR_1 = MC$ is established.

The mathematics of this third-degree price discrimination is fairly straightforward. Assume that a firm sells its product in two easily identifiable markets. The total output of the firm is, therefore,

$$Q = Q_1 + Q_2 \tag{11.11}$$

By the law of demand, the quantity sold in each market will vary inversely with the selling price. If the demand function of each group is known, the total revenue earned by the firm selling its product in each market will be

$$TR(Q) = TR_1(Q_1) + TR_2(Q_2) \tag{11.12}$$

where $TR_1 = P_1Q_1$ and $TR_2 = P_2Q_2$. The total cost of producing the good or service is a function of total output, or,

$$TC(Q) = TC(Q_1 + Q_2) \tag{11.13}$$

Note that the marginal cost of producing the good is the same for both markets. By the chain rule,

$$\frac{\partial TC(Q)}{\partial Q_1} = \left(\frac{dTC}{dQ}\right)\left(\frac{\partial Q}{\partial Q_1}\right) = \frac{\partial TC}{dQ} \tag{11.14}$$

since $\partial Q/\partial Q_1 = 1$. Likewise for Q_2,

$$\frac{\partial TC(Q)}{\partial Q_2} = \left(\frac{dTC}{dQ}\right)\left(\frac{\partial Q}{\partial Q_2}\right) = \frac{\partial TC}{dQ} \tag{11.15}$$

since $\partial Q/\partial Q_2 = 1$. Equations (11.14) and (11.15) simply affirm that the marginal cost of producing the good or service remains the same, regardless of the market in which it is sold.

Upon combining Equations (11.11) to (11.15), the firm's profit function may be written

$$\pi(Q_1, Q_2) = TR_1(Q_1) + TR_2(Q_2) - TC(Q_1 + Q_2) \tag{11.16}$$

Equation (11.16) indicates that profit is a function of both Q_1 and Q_2. The objective of the firm is to maximize profit with respect to both Q_1 and Q_2. Taking the first partial derivatives of the profit function with respect to Q_1 and Q_2, and setting the results equal to zero, we obtain

$$\frac{\partial \pi}{\partial Q_1} = \frac{\partial TR_1}{\partial Q_1} - \left(\frac{dTC}{dQ}\right)\left(\frac{\partial Q}{\partial Q_1}\right) = 0 \tag{11.17a}$$

$$\frac{\partial \pi}{\partial Q_2} = \frac{\partial TR_2}{\partial Q_2} - \left(\frac{dTC}{dQ}\right)\left(\frac{\partial Q}{\partial Q_2}\right) = 0 \tag{11.17b}$$

Solving Equations (11.17) simultaneously with respect to Q_1 and Q_2 yields the profit-maximizing unit sales in the two markets. Assuming that the second-order conditions are satisfied, the first-order conditions for profit maximization may be written as

$$MC = MR_1 = MR_2 \tag{11.18}$$

Finally, since $TR_1 = P_1Q_1$ and $TR_2 = P_2Q_2$, then

$$\begin{aligned} MR_1 &= P_1\left(\frac{dQ_1}{dQ_1}\right) + Q_1\left(\frac{dP_1}{dQ_1}\right) \\ &= P_1\left[1 + \left(\frac{dP_1}{dQ_1}\right)\left(\frac{Q_1}{P_1}\right)\right] = P_1\left(1 + \frac{1}{\varepsilon_1}\right) \end{aligned} \tag{11.19}$$

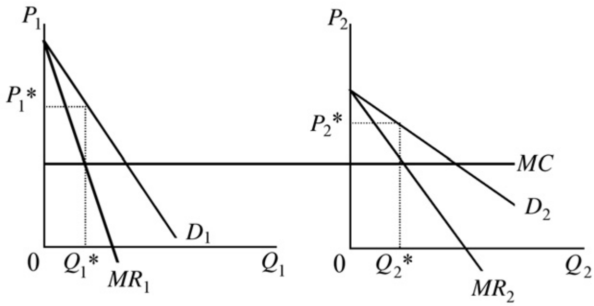

FIGURE 11.5 Third-degree price discrimination.

$$MR_2 = P_2\left(1+\frac{1}{\varepsilon_2}\right) \tag{11.20}$$

where ε_1 and ε_2 are the price elasticities of demand in the two markets. By the profit-maximizing condition in Equations (11.17), it is easy to see that the firm will charge the same price in the two markets only if $\varepsilon_1 = \varepsilon_2$. When $\varepsilon_1 \neq \varepsilon_2$, the prices in the two markets will not be the same. In fact, when $\varepsilon_1 > \varepsilon_2$, the price charged in the first market will be greater than the price charged in the second market. Figure 11.5 illustrates this solution for linear demand curves in the two markets and constant marginal cost.

Problem 11.5. Red Company sells its product in two separable and identifiable markets. The company's total cost equation is

$$TC = 6 + 10Q$$

The demand equations for its product in the two markets are

$$Q_1 = 10 - (0.2)P_1$$

$$Q_2 = 10 - (0.2)P_2$$

where $Q = Q_1 + Q_2$.

a. Assuming that the second-order conditions are satisfied, calculate the profit-maximizing price and output level in each market.
b. Verify that the demand for Red Company's product is less elastic in the market with the higher price.
c. Give the firm's total profit at the profit-maximizing prices and output levels.

Solution

a. This is an example of price discrimination. Solving the demand equations in both markets for price yields

$$P_1 = 50 - 5Q_1$$

$$P_2 = 30 - 2Q_2$$

The corresponding total revenue equations are

$$TR_1 = 50Q_1 - 5Q_1^2$$

$$TR_2 = 30Q_2 - 2Q_2^2$$

Red Company's total profit equation is

$$\pi = TR_1 + TR_2 - TC = 50Q_1 - 5Q_1^2 + 30Q_2 - 2Q_2^2 - 6 - 10(Q_1 + Q_2)$$

Maximizing this expression with respect to Q_1 and Q_2 yields

$$\frac{\partial \pi}{\partial Q_1} = 50 - 10Q_1 - 10 = 40 - 10Q_1 = 0$$

$$Q_1^* = 4$$

$$\frac{\partial \pi}{\partial Q_2} = 30 - 4Q_2 - 10 = 20 - 4Q_2 = 0$$

$$Q_2^* = 5$$

$$P_1^* = 50 - 5(4) = 50 - 20 = 30$$

$$P_2^* = 30 - 2(5) = 30 - 10 = 20$$

b. The relationships between the selling price and the price elasticity of demand in the two markets are

$$MR_1 = P_1\left(1 + \frac{1}{\varepsilon_1}\right)$$

$$MR_2 = P_2\left(1 + \frac{1}{\varepsilon_2}\right)$$

where

$$\varepsilon_1 = \left(\frac{dQ_1}{dP_1}\right)\left(\frac{P_1}{Q_1}\right)$$

$$\varepsilon_2 = \left(\frac{dQ_2}{dP_2}\right)\left(\frac{P_2}{Q_2}\right)$$

From the demand equations, $dQ_1/dP_1 = -0.2$ and $dQ_2/dP_2 = -0.5$. Substituting these results into preceding above relationships, we obtain

$$\varepsilon_1 = (-0.2)\left(\frac{30}{4}\right) = \frac{-6}{4} = -1.5$$

$$\varepsilon_2 = (-0.5)\left(\frac{20}{5}\right) = \frac{-10}{5} = -2$$

This verifies that the higher price is charged in the market where the price elasticity of demand is less elastic.

c. The firm's total profit at the profit-maximizing prices and output levels are

$$\pi^* = 50(4) - 5(4)^2 + 30(5) - 2(5)^2 - 6 - 10(4+5)$$
$$= 200 - 80 + 150 - 50 - 6 - 90 = 124$$

Problem 11.6. Copperline Mountain is a world-famous ski resort in Utah. Copperline Resorts operates the resort's ski-lift and grooming operations. When weather conditions are favorable, Copperline's total operating cost, which depends on the number of skiers who use the facilities each year, is given as

$$TC = 10S + 6$$

where S is the total number of skiers (in hundreds of thousands). The management of Copperline Resorts has determined that the demand for ski-lift tickets can be segmented into adult (S_A) and children 12 years old and under (S_C). The demand curve for each group is given as

$$S_A = 10 - 0.2P_A$$

$$S_C = 15 - 0.5P_C$$

where P_A and P_C are the prices charged for adults and children, respectively.

a. Assuming that Copperline Resorts is a profit maximizer, how many skiers will visit Copperline Mountain?
b. What prices should the company charge for adult and child's ski-lift tickets?
c. Assuming that the second-order conditions for profit maximization are satisfied, what is Copperline's total profit?

Solution

a. Total profit is given by the expression

$$\pi = TR - TC = (TR_A + TR_C) - TC$$
$$= P_A S_A + P_C S_C - TC$$
$$= (50 - 5S_A)S_A + (30 - 2S_C)S_C - [10(S_A + S_C) + 6]$$
$$= -6 + 40S_A + 20S_C - 5S_A^2 - 2S_C^2$$

Taking the first partial derivatives with respect to S_A and S_C, setting the results equal to zero, and solving, we write

$$\frac{\partial \pi}{\partial S_A} = 40 - 10S_A = 0$$

$$S_A = 4$$

$$\frac{\partial \pi}{\partial S_C} = 20 - 4S_C = 0$$

$$S_C = 5$$

The total number of skiers that will visit Copperline Mountain is

$$S = S_A = S_C = 4 + 5 = 9\ (\times 10^5)\ \text{skiers}$$

b. Substituting these results into the demand functions yields adult and child's, ski-lift ticket prices.

$$4 = 10 - 0.2P_A$$

$$P_A = \$30$$

$$5 = 15 - 0.5P_C$$

$$P_C = \$20$$

c. Substituting the results from part a into the total profit equation yields

$$\pi = -6 + 40(4) + 20(5) - 5(4)^2 - 2(5)^2$$
$$= -6 + 160 + 100 - 80 - 50 = \$124\ (\times 10^3)$$

Problem 11.7. Suppose that a firm sells its product in two separable markets. The demand equations are

$$Q_1 = 100 - P_1$$

$$Q_2 = 50 - 0.25P_2$$

The firm's total cost equation is

$$TC = 150 + 5Q + 0.5Q^2$$

a. If the firm engages in third-degree price discrimination, how much should it sell, and what price should it charge, in each market?
b. What is the firm's total profit?

Solution

a. Assuming that the firm is a profit maximizer, set $MR = MC$ in each market to determine the output sold and the price charged. Solving the demand equation for P in each market yields

$$P_1 = 100 - Q_1$$

$$P_2 = 200 - 4Q_2$$

The respective total and marginal revenue equations are

$$TR_1 = 100Q_1 - Q_1^2$$

$$TR_2 = 200Q_2 - Q_2^2$$

$$MR_1 = 100 - 2Q_1$$

$$MR_2 = 200 - 8Q_2$$

The firm's marginal cost equation is

$$MC = \frac{dTC}{dQ} = 5 + Q$$

Setting $MR = MC$ for each market yields

$$100 - 2Q_1 = 5 + Q_1$$

$$200 - 8Q_2 = 5 + Q_2$$

$$Q_1^* = 31.67$$

$$Q_2^* = 15$$

$$P_1^* = 100 - 31.67 = \$68.33$$

$$P_2^* = 200 - 4(15) = \$140.00$$

b. The firm's total profit is

$$\pi^* = P_1^*Q_1^* + P_2^*Q_2^* - \left[150 + 5\left(Q_1^* + Q_2^*\right) + 0.5\left(Q_1^* + Q_2^*\right)^2\right]$$
$$= 68.33(31.67) + 140(15) - (150 + 233.35 + 1{,}089.04) = \$2{,}791.62$$

Problem 11.8. Suppose that the firm in Problem 11.7 charges a uniform price in the two markets in which it sells its product.

a. Find the uniform price charged, and the quantity sold, in the two markets.
b. What is the firm's total profit?
c. Compare your answers to those obtained in Problem 11.7.

Solution

a. To determine the uniform price charged in each market, first add the two demand equations:

$$Q = Q_1 + Q_2 = 100 - P_1 + 50 - 0.25P_2 = 150 - 1.25P$$

Next, solve this equation for P:

$$P = 120 - 0.8Q$$

The total and marginal revenue equations are

$$TR = PQ = 120Q - 0.8Q^2$$

$$MR = 120 - 1.6Q$$

The profit-maximizing level of output is

$$MR = MC$$

$$120 - 1.6Q = 5 + Q$$

$$Q^* = 44.23$$

That is, the profit-maximizing output of the firm is 44.23 units. The uniform price is determined by substituting this result into the combined demand equation:

$$P^* = 120 - 0.8(44.23) = 120 - 35.38 = \$84.62$$

The amount of output sold in each market is

$$Q_1^* = 100 - 84.62 = 15.38$$

$$Q_2^* = 50 - 0.25(84.62) = 50 - 21.16 = 28.85$$

Note that the combined output of the two markets is equal to the total output Q^* already derived.

b. The firm's total profit is

$$\begin{aligned} \pi^* &= P^*Q^* - (150 + 5Q^* + 0.5Q^{*2}) \\ &= 84.62(44.23) - \left[150 + 5(44.23) + 0.5(44.23)^2\right] \\ &= 3{,}742.74 - (150 + 221.15 + 978.15) = \$2{,}393.44 \end{aligned}$$

c. The uniform price charged ($84.62) is between the prices charged in the two markets ($68.33 and $140.00) when the firm engaged in third-degree price discrimination. When the firm engaged in uniform pricing, the amount of output sold is lower in the first market (15.38 units compared with 31.67 units) and higher in the second market (28.85 units compared with 15 units). Finally, the firm's total profit with uniform pricing ($2,393.44) is lower than when the firm engaged in third-degree price discrimination ($2,791.62, from Problem 11.7).

When third-degree price discrimination is practiced in foreign trade it is sometimes referred to as *dumping*. This rather derogatory term is often used by domestic producers claiming unfair foreign competition. Defined by the U.S. Department of Commerce as selling at below fair market value, dumping results when a profit-maximizing exporter sells its product at a different, usually lower, price in the foreign market than it does in its home market. Recall that when resale between two markets is not possible, the monopolist will sell its product at a lower price in the market in which demand is more price elastic. In international trade theory, the difference between the home price and the foreign price is called the *dumping margin*.

NONMARGINAL PRICING

Most of the discussion of pricing practices thus far has assumed that management is attempting to optimize some corporate objective. For the most part, we have assumed that management attempts to maximize the firm's profits, but other optimizing behavior has been discussed, such as revenue maximization. In each case, we assumed that the firm was able to calculate its total cost and total revenue equations, and to systematically use that information to achieve the firm's objectives. If the firm's objective is to maximize profit, for example, then management will produce at an output level and charge a price at which marginal revenue equals marginal cost. This is the classic example of marginal pricing.

In reality, however, firms do not know their total revenue and total cost equations, nor are they ever likely to. In fact, because firms do not have this information, and in spite management's protestations to the contrary, most firms are (unwittingly) not profit maximizers. Moreover, even if this information were available, there are other corporate objectives, such as satisficing behavior, that do not readily lend themselves to marginal pricing strategies. Consequently, most firms engage in nonmarginal pricing. The most popular form of nonmarginal pricing is cost-plus pricing.

Definition: Firms determine the profit-maximizing price and output level by equating marginal revenue with marginal cost. When the firm's total revenue and total cost equations are unknown, however, management will often practice nonmarginal pricing. The most popular form of nonmarginal pricing is cost-plus pricing, also known as markup or full-cost pricing.

COST-PLUS PRICING

As we have seen, profit maximization occurs at the price–quantity combination at which where marginal cost equals marginal revenue. In reality, however, many firms are unable or unwilling to devote the resources necessary to accurately estimate the total revenue and total cost equations, or do not know enough about demand and cost conditions to determine the profit-maximizing price and output levels. Instead, many firms adopt rule-of-thumb methods for pricing their goods and services. Perhaps the most commonly used pricing practice is that of *cost-plus pricing*, also known as *mark up* or *full-cost pricing*. The rationale behind cost-plus pricing is straightforward: approximate the average cost of producing a unit of the good or service and then "mark up" the estimated cost per unit to arrive at a selling price.

Definition: Cost-plus pricing is the most popular form of nonmarginal pricing. It is the practice of adding a predetermined "markup" to a firm's estimated per-unit cost of production at the time of setting the selling price.

The firm begins by estimating the average variable cost (AVC) of producing a good or service. To this, the company adds a per-unit allocation for fixed cost. The result is sometimes referred to as the *fully allocated per-unit cost* of production. With the per-unit allocation for fixed cost denoted AFC and the fully allocated, average total cost ATC, the price a firm will charge for its product with the percentage mark up is

$$P = ATC(1+m) \tag{11.21}$$

where m is the percentage markup over the fully allocated per-unit cost of production. Solving Equation (11.21) for m reveals that the mark up may also be expressed as the difference between the selling price and the per-unit cost of production.

$$m = \frac{P - ATC}{ATC} \tag{11.22}$$

The numerator of Equation (11.22) can also be written as $P - AVC - AFC$. The expression $P - AVC$ is sometimes referred to as the *contribution margin per unit*. The marked-up selling price, therefore, may be referred to as the profit contribution per unit plus some allocation to defray overhead costs.

Problem 11.9. Suppose that the Nimrod Corporation has estimated the average variable cost of producing a spool of its best-selling brand of industrial wire, Mithril, at $20. The firm's total fixed cost is $20,000.

a. If Nimrod produces 500 spools of Mithril and its standard pricing practice is to add a 25% markup to its estimated per-spool cost of production, what price should Nimrod charge for its product?
b. Verify that the selling price calculated in part a represents a 25% markup over the estimated per-spool cost of production.

Solution

a. At a production level of 500 spools, Nimrod's per-unit fixed cost allocation is

$$AFC = \frac{20{,}000}{500} = 40$$

The cost-plus pricing equation is given as

$$P = ATC(1 + m)$$

where m is the percentage markup and ATC is the sum of the average variable cost of production (AVC) and the per-unit fixed cost allocation (AFC). Substituting, we write

$$P = (20 + 40)(1 + 0.25) = 60(1.25) = \$75$$

Nimrod should charge $75 per spool of Mithril. In other words, Nimrod should charge $15 over its estimated per-unit cost of production.

b. The percentage markup is given by the equation

$$m = \frac{(P - ATC)}{ATC}$$

Substituting the relevant data into this equation yields

$$m = \frac{75 - 60}{60} = \frac{15}{60} = 0.25$$

Of course, the advantage of cost-plus pricing is its simplicity. Cost-plus pricing requires less than complete information, and it is easy to use. Care must be exercised, however, when one is using this approach. The usefulness of cost-plus pricing will be significantly reduced unless the appropriate cost concepts are employed. As in the case of break-even analysis, care must be taken to include all relevant costs of production. Cost-plus pricing, which is based only on accounting (explicit) costs, will move the firm further away from an optimal (profit-maximizing) price and output level. Of course, the more appropriate approach would be to calculate total economic costs, which include both explicit and implicit costs of production.

There are two major criticisms of cost-plus pricing. The first criticism involves the assumption of fixed marginal cost, which at fixed input prices is in defiance of the law of diminishing marginal product. It is this assumption that allows us to further assume that marginal cost is approximately equal to the fully allocated per-unit cost of production. If it can be argued, however, that marginal cost is approximately constant over the firm's range of production, this criticism loses much of its sting.

A perhaps more serious criticism of cost-plus pricing is that it is insensitive to demand conditions. It should be noted that, in practice, the size of a firm's markup tends to reflect the price elasticity of demand for of goods of various types. Where the demand for a product is relatively less price elastic, because of, say, the paucity of close substitutes, the markup tends to be higher than when demand is relatively more price elastic. As will be presently demonstrated, to the extent that this observation is correct, the criticism of insensitivity loses some of its bite.

Recall from our discussion of the relationship between the price elasticity of demand and total revenue in Chapter 4, the relationship between marginal revenue, price, and the price elasticity of demand may be expressed as

$$MR = P\left(1 + \frac{1}{\varepsilon_p}\right) \tag{4.15}$$

The first-order condition for profit maximization is $MR = MC$. Replacing MR with MC in Equation (4.15) yields

$$MC = P\left(1 + \frac{1}{\varepsilon_p}\right) \tag{11.23}$$

Solving Equation (11.23) for P yields

$$P = \frac{MC}{1 + 1/\varepsilon_p} \tag{11.24}$$

If we assume that MC is approximately equal to the firm's fully allocated per-unit cost (ATC), Equation (11.24) becomes,

$$P = \frac{ATC}{1 + 1/\varepsilon_p} \tag{11.25}$$

Equating the right-hand side of this result to the right-hand side of Equation (11.21), we obtain

$$\frac{ATC}{1 + 1/\varepsilon_p} = ATC(1 + m)$$

where m is the percentage markup. Solving this expression for the markup yields

$$m = \frac{-1}{\varepsilon_p + 1} \tag{11.26}$$

Equation (11.26) suggests that when demand is price elastic, then the selling price should have a positive markup. Moreover, the greater the price elasticity of demand, the lower will be the markup. Suppose, for example, that $\varepsilon_p = -2.0$. Substituting this value into Equation (11.26), we find that the markup is $m = -1/(-2 + 1) = -1/-1 = 1$, or 100%. On the other hand, if $\varepsilon_p = -5.0$, then $m = -1/(-5 + 1) = -1/-4 = 0.25$, or a 25% markup.

What happens, however, if the demand for the good or service is price inelastic? Suppose, for example, that $\varepsilon_p = -0.8$. Substituting this into Equation (11.26) results in a markup of $m = -1/(-0.8 + 1) = -1/0.2 = -5$. This result suggests that the firm should mark down the price of its product by 500%! Equation (11.26) suggests that if the demand for a product is price inelastic, the firm should sell its output at below the fully allocated per-unit cost of production, a practice that is clearly not observed in the real world. Fortunately, this apparent paradox is easily resolved.

It will be recalled from Chapter 4, and is easily seen from Equation (4.15), that when the demand for a good or service is price inelastic, it marginal revenue must be negative. For the profit-maximizing firm, this suggests that marginal cost is negative, since the first-order condition for profit maximization is $MR = MC$, which is clearly impossible for positive input prices and positive marginal product of factors of production.

Problem 11.10. What is the estimated percentage markup over the fully allocated per-unit cost of production for the following price elasticities of demand?

a. $\varepsilon_p = -11$
b. $\varepsilon_p = -4$
c. $\varepsilon_p = -2.5$
d. $\varepsilon_p = -2.0$
e. $\varepsilon_p = -1.5$

Solution

a. $m = \dfrac{-1}{\varepsilon_p + 1} = \dfrac{-1}{-11 + 1} = 0.10$ or a 10% mark up

b. $m = \dfrac{-1}{\varepsilon_p + 1} = \dfrac{-1}{-4 + 1} = 0.333$ or a 33.3% mark up

c. $m = \dfrac{-1}{\varepsilon_p + 1} = \dfrac{-1}{-2.5 + 1} = 0.667$ or a 66.7% mark up

d. $m = \dfrac{-1}{\varepsilon_p + 1} = \dfrac{-1}{-2.0 + 1} = 1.0$ or a 100% mark up

e. $m = \dfrac{-1}{\varepsilon_p + 1} = \dfrac{-1}{-1.5 + 1} = 2.0$ or a 200% mark up

Problem 11.11. What is the percentage markup on the output of a firm operating in a perfectly competitive industry?

Solution. A firm operating in a perfectly competitive industry faces an infinitely elastic demand for its product. Substituting $\varepsilon_p = -\infty$ into Equation (11.26) yields

$$m = \frac{-1}{\varepsilon_p + 1} = \frac{-1}{-\infty + 1} = 0$$

A firm operating in a perfectly competitive industry cannot mark up the selling price of its product. This is as it should be, since such a firm has no market power; that is, the firm is a price taker. The firm must sell its product at the market-determined price.

Problem 11.12. Suppose that a firm's marginal cost of production is constant at \$25. Suppose further that the price elasticity of demand (ε_p) for the firm's product is +5.0.

a. Using cost-plus pricing, what price should the firm charge for its product?
b. Suppose that ε_p = –0.5. What price should the firm charge for its product?

Solution

a. The firm's profit-maximizing condition is

$$MR = MC$$

Recall from Chapter 4 that

$$MR = P\left(1 + \frac{1}{\varepsilon_p}\right)$$

Substituting this result into the profit-maximizing condition yields

$$MC = P\left(1 + \frac{1}{\varepsilon_p}\right)$$

Since MC is constant, then $MC = ATC$. After substituting, and rearranging, we obtain

$$P^* = ATC\frac{\varepsilon_p}{\varepsilon_p + 1} = 25\left(\frac{-5}{-5+1}\right) = 25\left(\frac{-5}{-4}\right) = \$31.25$$

b. If $\varepsilon_p = -0.5$, then

$$P^* = 25\left(\frac{-0.5}{-0.5+1}\right) = 25\left(\frac{-0.5}{0.5}\right) = -\$25.00$$

This result, however, is infeasible, since a firm would never charge a negative price for its product. Recall that a profit-maximizing firm will never produce along the inelastic portion of the demand curve.

MULTIPRODUCT PRICING

We have thus far considered primarily firms that produce and sell only one good or service at a single price. The only exception to this general statement was our discussion of commodity bundling, in which a firm sells a package of goods at a single price. We will now address the issue of pricing strategies of a single firm selling more than one product under alternative scenarios. These scenarios include the optimal pricing of two or more products with interdependent demands, optimal pricing of two or more products with independent demands that are jointly produced in variable proportions, and optimal pricing of two or more products with independent demands that are jointly produced in fixed proportions.

Definition: Multiproduct pricing involves optimal pricing strategies of firms producing and selling more than one good or service.

OPTIMAL PRICING OF TWO OR MORE PRODUCTS WITH INTERDEPENDENT DEMANDS AND INDEPENDENT PRODUCTION

Often a firm will produce two or more goods that are either complements or substitutes for each other. Dell Computer, for example, sells a number of different models of personal computers. These models are, to a degree, substitutes for each other. Personal computers also come with a variety of accessories (mouses, printers, modems, scanners, etc.). These options not only come in different models, and are, therefore, substitutes for each other, but they are also complements to the personal computers.

Because of the interrelationships inherent in the production of some goods and services, it stands to reason that an increase in the price of, say, a Dell personal computer model will lead to a reduction in the quantity demanded of that model and an increase in the demand for substitute models. Moreover, an increase in the price of the Dell personal computer model will lead to a reduction in the demand for complementary accessories. For this reason, a profit-maximizing firm must ascertain the optimal prices and output levels of each product manufactured jointly, rather than pricing each product independently.

The problem may be formally stated as follows. Consider the demand for two products produced by the same firm. If these two products are related, the demand functions may be expressed as

$$Q_1 = f_1(P_1, Q_2) \tag{11.27a}$$

$$Q_2 = f_2(P_2, Q_1) \tag{11.27b}$$

By the law of demand, $\partial Q_1/\partial P_1$ and $\partial Q_2/\partial P_2$ are negative. The signs of $\partial Q_1/\partial Q_2$ and $\partial Q_2/\partial Q_1$ depend on the relationship between Q_1 and Q_2. If the values of these first partial derivatives are positive, then Q_1 and Q_2 are complements. If the values of these first partials are negative, then Q_1 and Q_2 are substitutes.

Upon solving Equation (11.27a) for P_1 and Equation (11.27b) for P_2, and substituting these results into the total revenue equations, we write

$$TR_1(Q_1, Q_2) = P_1Q_1 = h_1(Q_1, Q_2)Q_1 \tag{11.28a}$$

$$TR_2(Q_1, Q_2) = P_2Q_2 = h_2(Q_1, Q_2)Q_2 \tag{11.28b}$$

Since the two goods are independently produced, the total cost functions are

$$TC_1 = TC_1(Q_1) \tag{11.29a}$$

$$TC_2 = TC_2(Q_2) \tag{11.29b}$$

The total profit equation for this firm is, therefore,

$$\begin{aligned}\pi &= TR_1(Q_1, Q_2) + TR_2(Q_1, Q_2) - TC_1(Q_1) - TC_2(Q_2)\\ &= P_1Q_1 + P_2Q_2 + TC_1(Q_1) - TC_2(Q_2)\\ &= h_1(Q_1, Q_2)Q_1 + h_2(Q_1, Q_2)Q_2 - TC_1(Q_1) - TC_2(Q_2)\end{aligned} \tag{11.30}$$

The first-order conditions for profit maximization are

$$\frac{\partial \pi}{\partial Q_1} = \frac{\partial TR_1}{\partial Q_1} + \frac{\partial TR_2}{\partial Q_1} - \frac{\partial TC_1}{\partial Q_1} = 0 \tag{11.31a}$$

$$\frac{\partial \pi}{\partial Q_2} = \frac{\partial TR_2}{\partial Q_2} + \frac{\partial TR_1}{\partial Q_2} - \frac{\partial TC_2}{\partial Q_2} = 0 \tag{11.31b}$$

which may be expressed as

$$MC_1 = \frac{\partial TR_1}{\partial Q_1} + \frac{\partial TR_2}{\partial Q_1} \tag{11.32a}$$

$$MC_2 = \frac{\partial TR_2}{\partial Q_2} + \frac{\partial TR_1}{\partial Q_2} \tag{11.32b}$$

We will assume that the second-order conditions for profit maximization are satisfied.

Equations (11.32) indicate that a firm producing two products with interrelated demands will maximize its profits by producing where marginal cost is equal to the change in total revenue derived from the sale of the product itself, plus the change in total revenue derived from the sale of the related product. If the second term on the right-hand side of Equation (11.31) is positive, then Q_1 and Q_2 are complements. If this term is negative, then Q_1 and Q_2 are substitutes.

Problem 11.13. Gizmo Brothers, Inc., manufactures two types of hi-tech yo-yo: the Exterminator and the Eliminator. Denoting Exterminator output as Q_1 and Eliminator output as Q_2, the company has estimated the following demand equations for its yo-yos:

$$Q_1 = 10 - 0.2P_1 - 0.4Q_2$$

$$Q_2 = 20 - 0.5P_2 - 2Q_1$$

The total cost equations for producing Exterminators and Eliminators are

$$TC_1 = 4 + 2Q_1^2$$

$$TC_2 = 8 + 6Q_2^2$$

a. If Gizmo Brothers is a profit-maximizing firm, how much should it charge for Exterminators and Eliminators? What is the profit-maximizing level of output for Exterminators and Eliminators?
b. What is Gizmo Brothers's profit?

Solution

a. Solving the demand equations for P_1 and P_2, respectively, yields

$$P_1 = 50 - 5Q_1 - 2Q_2$$

$$P_2 = 40 - 2Q_2 - 4Q_1$$

The profit equation is

$$\begin{aligned}\pi &= TR_1(Q_1, Q_2) + TR_2(Q_1, Q_2) - TC_1(Q_1) - TC_2(Q_2)\\ &= P_1Q_1 + P_2Q_2 - TC_1(Q_1) - TC_2(Q_2)\end{aligned}$$

Substitution yields

$$\begin{aligned}\pi &= (50 - 5Q_1 - 2Q_2)Q_1 + (40 - 2Q_2 - 4Q_1)Q_2 - (4 + 2Q_1^2) - (8 + 6Q_2^2)\\ &= 50Q_1 + 40Q_2 - 6Q_1Q_2 - 7Q_1^2 - 8Q_2^2 - 12\end{aligned}$$

The first-order conditions for profit maximization are

$$\frac{\partial \pi}{\partial Q_1} = 50 - 14Q_1 - 6Q_2 = 0$$

$$\frac{\partial \pi}{\partial Q_2} = 40 - 6Q_1 - 16Q_2 = 0$$

Recall from Chapter 2 that the second-order conditions for profit maximization are

$$\frac{\partial^2 \pi}{\partial Q_1^2} < 0$$

$$\frac{\partial^2 \pi}{\partial Q_2^2} < 0$$

$$\left(\frac{\partial^2 \pi}{\partial Q_1^2}\right)\left(\frac{\partial^2 \pi}{\partial Q_1^2}\right) - \left(\frac{\partial^2 \pi}{\partial Q_1 \partial Q_2}\right)^2 > 0$$

The appropriate second partial derivatives are

$$\frac{\partial^2 \pi}{\partial Q_1^2} = -14 < 0$$

$$\frac{\partial^2 \pi}{\partial Q_2^2} = -16 < 0$$

$$\frac{\partial^2 \pi}{\partial Q_1 \partial Q_2} = -6$$

$$(-14)(-16) - (6)^2 = 244 - 36 = 208 > 0$$

Thus, the second-order conditions for profit maximization are satisfied. Solving the first-order conditions for Q_1 and Q_2 we obtain

$$14Q_1 + 6Q_2 = 50$$

$$6Q_1 + 16Q_2 = 40$$

which may be solved simultaneously to yield

$$Q_1{}^* = 2.979$$

$$Q_2{}^* = 1.383$$

Upon substituting these results into the price equations, we have

$$P_1{}^* = 50 - 5(2.979) - 2(1.383) = \$32.34$$

$$P_2{}^* = 40 - 2(1.383) - 4(2.979) = \$25.32$$

b. Gizmo Brothers's profit is

$$\pi = 50(2.979) + 40(1.383) - 6(2.979)(1.383) - 7(2.979)^2 - 8(1.383)^2 - 12$$
$$= \$90.17$$

OPTIMAL PRICING OF TWO OR MORE PRODUCTS WITH INDEPENDENT DEMANDS JOINTLY PRODUCED IN VARIABLE PROPORTIONS

Let us now suppose that a firm sells two goods with independent demands that are jointly produced in variable proportions. An example of this might be a consumer electronics company that produces automobile taillight bulbs and flashlight bulbs on the same assembly line. In this case, the demand functions are given by the expressions

$$Q_1 = f_1(P_1) \tag{11.33a}$$

$$Q_2 = f_2(P_2) \tag{11.33b}$$

where $\partial Q_1/\partial P_1$ and $\partial Q_2/\partial P_2$ are negative. The total cost function is given by the expression

$$TC = TC(Q_1, Q_2) \tag{11.34}$$

The firm's total profit function is

$$\pi = TR_1(Q_1) + TR_2(Q_2) - TC(Q_1, Q_2) \tag{11.35}$$

Solving the demand equations for P_1 and P_2 and substituting the results into Equation (11.35) yields

$$\begin{aligned} \pi &= P_1Q_1 + P_2Q_2 - TC(Q_1, Q_2) \\ &= h_1(Q_1)Q_1 + h_2(Q_2)Q_2 - TC(Q_1, Q_2) \end{aligned} \tag{11.36}$$

The first-order conditions for profit maximization are

$$\frac{\partial \pi}{\partial Q_1} = \frac{\partial TR_1}{\partial Q_1} - \frac{\partial TC_1}{\partial Q_1} = 0 \tag{11.37a}$$

$$\frac{\partial \pi}{\partial Q_2} = \frac{\partial TR_2}{\partial Q_2} - \frac{\partial TC_2}{\partial Q_2} = 0 \tag{11.37b}$$

which may be written as

$$MR_1 = MC_1 \tag{11.38a}$$

$$MR_2 = MC_2 \tag{11.38b}$$

We will assume that the second-order conditions for profit maximization are satisfied.

Equations (11.38) indicate that a profit-maximizing firm jointly producing two goods with independent demands that are jointly produced in variable proportions will equate the marginal revenue generated from the sale of each good to the marginal cost of producing each product.

Problem 11.14. Suppose Gizmo Brothers also produces Tommy Gunn action figures for boys ages 7 to 12, and Bonzey, a toy bone for pet dogs. Except for the molding phase, both products are made on the same assembly line. Denoting Tommy Gunn as Q_1 and Bonzey as Q_2, the company has estimated the following demand equations:

$$Q_1 = 10 - 0.5P_1$$

$$Q_2 = 20 - 0.2P_2$$

The total cost equation for producing the two products is

$$TC = Q_1^2 + 2Q_1Q_2 + 3Q_2^2 + 10$$

a. As before, Gizmo Brothers is a profit-maximizing firm. Give the profit-maximizing levels of output for Tommy Gunn and for Bonzey. How much should the firm charge for Tommy Gunn and Bonzey?
b. What is Gizmo Brothers's profit?

Solution

a. Solving the demand equations for P_1 and P_2, respectively, yields

$$P_1 = 20 - 2Q_1$$

$$P_2 = 100 - 5Q_2$$

Gizmo Brothers's profit equation is

$$\pi = TR_1(Q_1) + TR_2(Q_2) - TC_1(Q_1, Q_2) = P_1Q_1 + P_2Q_2 - TC_1(Q_1, Q_2)$$

Substituting the demand equations into the profit equation yield

$$\pi = (20 - 2Q_1)Q_1 + (100 - 5Q_2)Q_2 - (Q_1^2 + 2Q_1Q_2 + 3Q_2^2 + 10)$$
$$= -10 + 20Q_1 + 100Q_2 - 3Q_1^2 - 8Q_2^2 - 2Q_1Q_2$$

The first-order conditions for profit maximization are

$$\frac{\partial\pi}{\partial Q_1} = 20 - 6Q_1 - 2Q_2 = 0$$

$$\frac{\partial\pi}{\partial Q_2} = 100 - 16Q_2 - 2Q_1 = 0$$

The second-order conditions for profit maximization are

$$\frac{\partial^2 \pi}{\partial Q_1^2} < 0$$

$$\frac{\partial^2 \pi}{\partial Q_2^2} < 0$$

$$\left(\frac{\partial^2 \pi}{\partial Q_1^2}\right)\left(\frac{\partial^2 \pi}{\partial Q_1^2}\right) - \left(\frac{\partial^2 \pi}{\partial Q_1 \partial Q_2}\right)^2 > 0$$

The appropriate second-partial derivatives are

$$\frac{\partial^2 \pi}{\partial Q_1^2} = -6 < 0$$

$$\frac{\partial^2 \pi}{\partial Q_2^2} = -16 < 0$$

$$\frac{\partial^2 \pi}{\partial Q_1 \partial Q_2} = -2$$

$$(-6)(-16) - (-2)^2 = 96 - 4 = 92 > 0$$

Thus, the second-order conditions for profit maximization are satisfied. Solving the first-order conditions for Q_1 and Q_2 yields

$$6Q_1 + 2Q_2 = 20$$

$$2Q_1 + 16Q_2 = 100$$

which may be solved simultaneously to yield

$$Q_1^* = 1.304$$

$$Q_2^* = 6.087$$

Substituting these results into the price equations yields

$$P_1^* = 20 - 2(1.304) = \$17.39$$

$$P_2^* = 100 - 2(6.087) = \$69.66$$

b. Gizmo Brothers's profit is

$$\pi = 20(1.304) + 100(6.087) - 2(1.304)(6.087) - 3(1.304)^2 - 8(6.087)^2 - 10$$
$$= \$88.17$$

OPTIMAL PRICING OF TWO OR MORE PRODUCTS WITH INDEPENDENT DEMANDS JOINTLY PRODUCED IN FIXED PROPORTIONS

Now, let us assume that a firm jointly produces two goods in fixed proportions but with independent demands. In many cases, the second product is a by-product of the first, such as beef and hides. With joint production in fixed proportions, it is conceptually impossible to consider two separate products, since the production of one good automatically determines the quantity produced of the other.

Suppose that the demand functions for two goods produced jointly are given as Equations (11.33). The total cost equation is given as Equation (11.13).

$$TC(Q) = TC(Q_1 + Q_2) \tag{11.13}$$

The analysis differs, however, in that Q_1 and Q_2 are in direct proportion to each other, that is,

$$Q_2 = kQ_1 \tag{11.39}$$

where the constant $k > 0$. Solving Equation (11.33) for P_1 and P_2 yields

$$P_1 = h_1(Q_1) \tag{11.40a}$$

$$P_2 = h_2(Q_2) \tag{11.40b}$$

Substituting Equation (11.39) into Equations (11.13) and (11.40b) yields

$$P_1 = h_1(Q_1)$$

$$P_2 = h_2(Q_1) \tag{11.41}$$

$$TC(Q) = TC(Q_1) \tag{11.42}$$

Substituting Equations (11.39), (11.40a), (11.41), and (11.42) into Equation (11.36) yields the firm's profit equation:

$$\begin{aligned}\pi &= P_1Q_1 + P_2(kQ_1) - TC(Q_1)\\ &= h_1(Q_1)Q_1 + h_2(Q_1)(kQ_1) - TC(Q_1)\end{aligned} \tag{11.43}$$

Stated another way, the firm's total profit function is

$$\pi(Q_1) = TR_1(Q_1) + TR_2(Q_1) - TC(Q_1) \tag{11.44}$$

Equation (11.44) indicates that total profit is a function of the single decision variable, Q_1. Equation (11.44) may also be written

$$\pi(Q_2) = TR_1(Q_2) + TR_2(Q_2) - TC(Q_2) \tag{11.45}$$

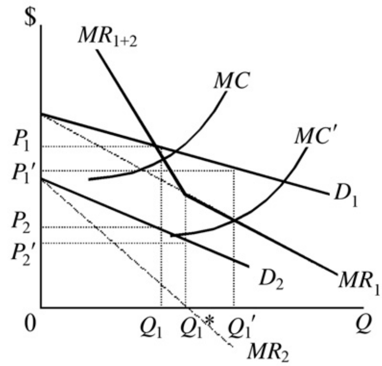

FIGURE 11.6 Optimal pricing of two goods jointly produced in fixed proportions with independent demands.

From Equation (11.44), the first-order condition for profit maximization is

$$\frac{d\pi}{dQ_1} = \frac{dTR_1}{dQ_1} + \frac{dTR_2}{dQ_1} - \frac{dTC_1}{dQ_1} = 0 \tag{11.46}$$

Equation (11.46) may be rewritten

$$\frac{dTR_1}{dQ_1} + \frac{dTR_2}{dQ_1} = \frac{dTC_1}{dQ_1}$$

$$MR_1(Q_1) + MR_2(Q_1) = MC(Q_1) \tag{11.47}$$

Equation (11.47) says that a profit-maximizing firm that jointly produces two goods in fixed proportions with independent demands will equate the sum of the marginal revenues of both products expressed in terms of one of the products with the marginal cost of jointly producing both products expressed in terms of the same product. This situation is depicted diagrammatically in Figure 11.6.

In Figure 11.6 the marginal cost curve is labeled MC. According to Equation (11.47) the firm should produce Q_1 units where marginal cost is equal to the sum of MR_1 and MR_2. The amount of Q_2 produced is proportional to Q_1. At that output level the firm charges P_1 for Q_1 and P_2 for Q_2. It should be noted that beyond output level Q_1* in Figure 11.6, MR_2 becomes negative and MR_{1+2} becomes simply MR_1.

Suppose that marginal cost increases to MC'. In this case, the firm should produce Q_1', but still only sell Q_1* units. Any output in excess of Q_1* should be disposed of, since the firm's marginal revenue beyond Q_1* is negative. The amount of Q_2 produced will be in fixed proportion to Q_1'. The price of Q_1* is P_2' and the price of Q_2 is P_1'.

Problem 11.15. Suppose that a firm produces two units of Q_2 for each unit of Q_1. Suppose further that the demand equations for these two goods are

$$Q_1 = 10 - 0.5P_1$$

$$Q_2 = 20 - 0.2P_2$$

The total cost of production is

$$TC = 10 + 5Q^2$$

a. What are the profit-maximizing output levels and prices for Q_1 and Q_2?
b. At the profit-maximizing output levels, what is the firm's total profit?

Solution

a. Solving the demand equations for P_1 and P_2 yields

$$P_1 = 20 - 2Q_1$$

$$P_2 = 100 - 5Q_2$$

The firm's total profit equation is

$$\begin{aligned}\pi &= P_1Q_1 + P_2Q_2 - TC(Q_1 + Q_2)\\ &= (20 - 2Q_1)Q_1 + (100 - 5Q_2)Q_2 - (10 + 5Q^2)\\ &= 20Q_1 - 2Q_1^2 + 100Q_2 - 5Q_2^2 - 10 - 5(Q_1 + Q_2)^2\end{aligned}$$

Since $Q_2 = 2Q_1$, this may be rewritten as

$$\begin{aligned}\pi &= 20Q_1 - 2Q_1^2 + 100(2Q_1) - 5(2Q_1)^2 - 10 - 5(Q_1 + 2Q_1)^2\\ &= -10 - 220Q_1 - 67Q_1^2\end{aligned}$$

The first-order condition for profit maximization is

$$\frac{d\pi}{dQ_1} = 220 - 134Q_1 = 0$$

The second-order condition for profit maximization is

$$\frac{d^2\pi}{dQ_1^2} < 0$$

Since $d^2\pi/dQ_1^2 = -137$ the second-order condition is satisfied. Solving the first-order condition for Q_1 yields

$$Q_1{}^* = 1.64$$

The profit-maximizing level of Q_2 is

$$Q_2{}^* = 2Q_1{}^* = 3.28$$

Substituting these results into the price equations yield

$$P_1{*} = 20 - 2(1.64) = \$16.72$$

$$P_2{*} = 100 - 5(3.28) = \$83.60$$

b. The firm's total profit is

$$\pi = 220(1.64) - 67(1.64)^2 - 10 = 360.80 - 180.20 - 10 = \$170.60$$

Problem 11.16. Suppose that a firm jointly produces two goods. Good *B* is a by-product of the production of good *A*. The demand equations for the two goods are

$$Q_A = 200 - 10P_A$$

$$Q_B = 120 - 5P_B$$

The firm's total cost equation is

$$TC = 500 + 15Q + 0.05Q^2$$

a. What is the profit-maximizing price for each product?
b. What is the firm's total profit?

Solution

a. Solving the demand equation for price yields

$$P_A = 20 - 0.1Q_A$$

$$P_B = 24 - 0.2Q_B$$

The respective total and marginal revenue equations are

$$TR_A = 20Q_A - 0.1Q_A^2$$

$$MR_A = 20 - 0.2Q_A$$

$$TR_B = 24Q_B - 0.2Q_B^2$$

$$MR_B = 24 - 0.4Q_B$$

The firm's marginal revenue equation is

$$MR = MR_A + MR_B = 20 - 0.2Q_A + 24 - 0.4Q_B = 44 - 0.6Q$$

The firm's marginal cost equation is

$$MC = \frac{dTC}{dQ} = 15 + 0.1Q$$

The profit-maximizing rate of output is

$$MR = MC$$

$$44 - 0.6Q = 15 + 0.1Q$$

$$Q^* = 41.43$$

The profit-maximizing prices for the two goods are

$$P_A{}^* = 20 - 0.1(41.43) = 20 - 4.14 = \$15.86$$

$$P_B{}^* = 20 - 0.2(41.43) = 24 - 8.29 = \$15.71$$

b. The firm's total profit is

$$\begin{aligned}\pi^* &= P_A{}^*Q^* + P_B{}^*Q^* - (500 + 15Q^* + 0.05Q^{*2})\\ &= 15.86(41.43) + 15.71(41.43) - \left[500 - 15(41.43) + 0.05(41.43)^2\right]\\ &= \$1,343.57\end{aligned}$$

PEAK-LOAD PRICING

In many markets the demand for a service is higher at certain times than at others. The demand for electric power, for example, is higher during the day than at night, and during summer and winter than during spring and fall. The demand for theater tickets is greater at night and on the weekends or for midweek matinees. Toll bridges have greater traffic during rush hours than at other times of the day. The demand for airline travel is greater during holiday seasons than at other times. During such "peak" periods it becomes difficult, if not impossible, to satisfy the demands of all customers. Thus the profit-maximizing firm will charge a higher price for the product during "peak" periods and a lower price during "off-peak" periods. This kind of pricing scheme is known as *peak-load pricing*.

Definition: Peak-load pricing is the practice of charging a higher price for a service when demand is high and capacity is fully utilized and a lower price when demand is low and capacity is underutilized.

Figure 11.7 illustrates an example of peak-load pricing for a profit-maximizing firm. Here the marginal cost of providing a service is assumed to be constant until capacity is reached at a peak output level of O_p. At the peak output level the marginal cost curve becomes vertical. This reflects the fact that to satisfy additional demand at O_p, the firm must increase its capacity, by building a new bridge, installing a new hydroelectric generator, or other high-cost measure.

The short-run production function is typically defined in terms of a time interval over which certain factors of production are "fixed." Strictly speaking, this assertion is incorrect. In principle, virtually any factor may be varied if the derived benefits are great enough. It is certainly the case, however, that some factors of production are more easily varied that others. It is clearly easier and less expensive to hire an additional worker at a moment's

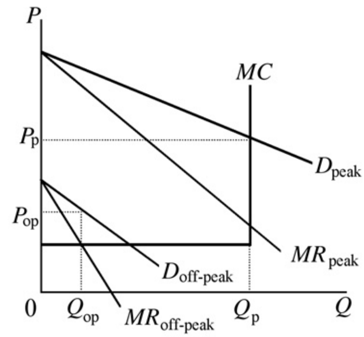

FIGURE 11.7 Peak-load pricing.

notice than to build a new bridge. Thus, it is reasonable to assume that the short-run marginal cost of expanding bridge traffic or increasing hydroelectric capacity is infinite. For that reason, the marginal cost curve at Q_p is assumed to be vertical.

To maximize profits subject to capacity limitations, the firm will charge different prices at different times. Off-peak prices are determined by equating marginal revenue to marginal operating costs. Peak prices, on the other hand, are determined by equating marginal revenue to the marginal cost of increasing capacity. In Figure 11.7, for example, $MR = MC$ for off-peak users at output level Q_{op}. At that output level the firm will charge off-peak users a price of P_{op}. On the other hand, the profit-maximizing level of output for peak users is at the firm's capacity, which in Figure 11.7 occurs at output level Q_p. At that output level the marginal cost curve of producing the service becomes vertical. The profit-maximizing price at that output level is P_p.

Peak-load pricing suggests that users of, say, congested bridges during rush hours, ought to be charged a higher toll than users during non–rush hour periods when there is excess capacity. Since peak-period demand strains capacity, the cost of additional capital investment ought to be borne by peak-period users. This tends to run contrary to the common practice on trains and toll bridges of offering multiple-use discounts to commuters traveling during rush hour, such as lower per-ride prices for, say, monthly tickets on commuter railways.

Problem 11.17. The Gotham Bridge and Tunnel Authority (GBTA) has estimated the following demand equations for peak and off-peak automobile users of the Frog's Neck Bridge:

$$\text{Peak: } T_p = 10 - 0.02Q_p$$

$$\text{Off-peack: } T_{op} = 5 - 0.05Q_{op}$$

where T is the toll charged for a one-minute trip (Q) across the bridge. The marginal cost of operating the bridge has been estimated at \$2 per automobile bridge crossing. The peak capacity of the Frog's Neck Bridge has been estimated a 50 automobiles per minute. What toll should the GBTA charge peak and off-peak users of the bridge?

Solution. This is a problem of peak-load pricing. If the GBTA is a profit maximizer, then off-peak drivers should be charged a price consistent with the first-order condition for profit maximization, $MC = MR$. The total revenue equation for off-peak users of the bridge is given as

$$TR_{op} = T_{op}Q_{op} = (5 - 0.05Q_{op})Q_{op} = 5Q - 0.05Q_{op}^2$$

The marginal revenue equation is

$$MR_{op} = \frac{dTR}{dQ_{op}} = 5 - 0.1Q_{op}$$

Equating marginal revenue to marginal cost yields

$$MR_{op} = MC_{op}$$

$$5 - 0.1Q_{op} = 2$$

$$Q_{op} = 30$$

Substituting this result into the off-peak demand equation yields the toll charged to off-peak automobile users of the bridge:

$$T_{op} = 5 - 0.05Q_{op} = 5 - 0.05(30) = \$3.50$$

At a bridge capacity of 50 automobiles per minute, the marginal cost curve is vertical. Substituting bridge capacity into the peak demand equation yields the toll that should be charged to peak automobile users of the bridge:

$$T_p = 10 - 0.02(50) = \$9$$

Peak users of the bridge should be charged \$9 per crossing.

TRANSFER PRICING

In recent years, the growth of large, conglomerate corporations producing a multitude of products has been accompanied by the parallel development of semiautonomous profit centers or subsidiaries. The creation of these "companies within a company" was an attempt to control rising production costs that accompanied the burgeoning managerial and administrative superstructure necessary to coordinate the activities of multiple corporate divisions.

Often the output of a division or subsidiary of a parent company is used as a productive input in the manufacture of the output of another division. A subsidiary of a large, multinational firm, for example, might assemble automobiles, while another subsidiary manufactures automobile bodies. Still another subsidiary might produce air and oil filters, while yet another produces electronic ignition systems, all of which are used in the production of automobiles.

Transfer pricing concerns itself with the correct pricing of intermediate products that are produced and sold between divisions of a parent company. For example, what price should one division of a company that produces, say, ignition systems, charge another division that assembles automobiles. The optimal pricing of intermediary goods is important because the organizational objective of each division is to maximize profit. What is more, the price charged for the output of one division that is used as an input in the production of another division affects not only each division's profits but also profits of the parent company as a whole.

Definition: Transfer pricing involves the optimal pricing of the output of one subsidiary of a parent company that is sold as an intermediate good to another subsidiary of the same parent company.

The literature dealing with transfer pricing typically focuses on three possible scenarios. In the first scenario, there is no external market for the output of the division or subsidiary producing the intermediate good. In other words, the division producing the final product is the sole customer for the output of the division producing the intermediate good. In the second scenario there exists a perfectly competitive external market for the intermediate good. In the third scenario the division or subsidiary operates in an industry that may be characterized as imperfectly competitive.

TRANSFER PRICING WITH NO EXTERNAL MARKET

Assume that a parent company comprises two subsidiary companies. One subsidiary sells its output, Q_1, exclusively to the other subsidiary that is used in the production of Q_2, for final sale in an external market. Assume further that there exists no other demand for Q_1; that is, there is no external market for the intermediate good. Finally assume that one unit of Q_1 is used to produce one unit of Q_2.

Since the parent company comprises only two subsidiaries, the marginal cost of producing Q_2 for final sale must include the marginal cost of producing Q_1. The rationale for this is straightforward. Although the company has been divided into separate profit centers, in the final analysis the company is in the business of producing and selling Q_2 for final sale. The

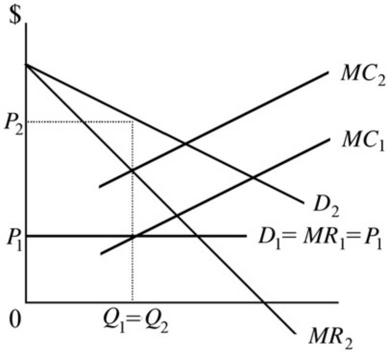

FIGURE 11.8 Transfer pricing with no external market for the intermediate good.

marginal cost of producing Q_2 must, of course, include the marginal cost of producing Q_1. If we assume that the parent company is a profit maximizer, it will produce at the output level where $MR_2 = MC_2$. This situation is illustrated in Figure 11.8.

Since Q_1 and Q_2 are used on a one-to-one basis, the output level that maximizes profit of Q_2 will be the same output level as Q_1. The selling price of Q_2 is P_2. Since the output level of Q_1 has been determined by the output level of Q_2, the profit-maximizing price for Q_1 must be P_1 (i.e., where $MC_1 = MR_1$). Thus, the correct transfer price for the intermediate good Q_1 must be P_1. It should be noted that any increase in the marginal cost of producing Q_1 will result in an increase in the marginal cost of producing Q_2, which will further result in an increase in the selling price of Q_2 and an increase in the transfer price (i.e., the price of the intermediate good that is sold between divisions).

On the other hand, suppose that the marginal cost curve of producing Q_1 remains unchanged, but the marginal cost curve of producing Q_2 shifts upward, perhaps because of an increase in factor or intermediate goods prices purchased in the external market. The result will be an increase in P_2, a decline in the output of Q_2 and Q_1, and, assuming an upward sloping marginal cost curve for Q_1, a fall in the transfer price. When the marginal cost of producing Q_1 is constant, the transfer price remains unaffected by the additional increase in the marginal cost of producing Q_2.

Problem 11.18. Parallax Corporation produces refractive telescopes for amateur astronomers. The demand equation for Parallax telescopes was estimated by the operations research department as

$$Q_T = 2,000 - 20P_T$$

Parallax's total cost equation was estimated as

$$TC_T = 100 + 2Q_T^2$$

Although the company procures most of its components from outside vendors, each Parallax telescope requires three highly polished lenses that are manufactured on site. Because these components are manufactured to exact specifications, there is no outside market for Parallax lenses. The total cost equation for producing Parallax lenses is

$$TC_L = 200 + 0.025Q_L^2$$

Because of the rapid growth of the company in the 1980s, Parallax management decided to divide the company into two separate profit centers to control costs—the telescope division and the lens division.

a. What is the profit-maximizing price and quantity for Parallax telescopes?
b. What is Parallax's total profit?
c. What transfer price should the lens division charge the telescope division?

Solution

a. Solving the demand equation for P_T yields

$$P_T = 100 - 0.05Q_T$$

The corresponding total revenue equation for Parallax telescopes is

$$TR_T = P_T Q_T = (100 - 0.05Q_T)Q_T = 100Q_T - 0.05Q_T^2$$

The total profit equation for Parallax telescopes is

$$\pi_T = TR_T - TC_T = (100Q_T - 0.05Q_T^2) - (100 + 2Q_T^2)$$
$$= -100 + 100Q_T - 2.05Q_T^2$$

The first-order condition for profit maximization is $d\pi/dQ_T = 0$. Taking the first derivative of the profit equation yields

$$\frac{d\pi}{dQ} = 100 - 4.1Q_T = 0$$

Solving, we have

$$Q_T{}^* = 24.39$$

The second-order condition for profit maximization is $d\pi/dQ_T < 0$. After taking the second derivative of the profit equation, we obtain

$$\frac{d^2\pi}{dQ^2} = -4.1 < 0$$

which guarantees that this output level represents a local maximum.

The profit-maximizing price, therefore, is

$$P_T^* = 100 - 0.05(24.39) = \$98.78$$

b. Parallax's profit at the profit-maximizing price and quantity is

$$\pi = -100 + 100(24.39) - 2.05(24.39)^2 = \$1,119.51$$

c. Since there is no external market for Parallax lenses, the transfer price is equal to the marginal cost of producing the lenses at the profit-maximizing output level. Parallax's marginal cost equation for producing lenses is

$$MC_L = \frac{dTC_1}{dQ_L} = 0.05Q_L$$

Since Parallax needs three lenses for every telescope produced, the total number of lenses required by the telescope division is 73.17 lenses (3 × 23.39 telescopes). The marginal cost of producing these lenses, therefore, is

$$MC_L = 0.05(70.17) = \$3.51 = P_L$$

The transfer price of the lenses, therefore, is $3.51 per lens.

TRANSFER PRICING WITH A PERFECTLY COMPETITIVE EXTERNAL MARKET

We will now consider the situation in which there exists an external market for the intermediate good. That is, the division or subsidiary producing the final product has the option of purchasing the intermediate good either from a subsidiary of its own parent company or from an outside vendor. If the intermediate good is purchased from within, what will its transfer price be? The answer to this question will depend on whether the external market for the intermediate good is or is not perfectly competitive. We will begin by assuming that there exists a perfectly competitive external market for the intermediate good produced by the subsidiary.

Since both divisions are assumed to be profit maximizers, it stands to reason that the division producing the final good will pay no more for the intermediate good than it would pay in the perfectly competitive external market. Similarly, the division producing the intermediate good will sell its output for nothing less than the perfectly competitive external market price. Thus, the transfer price for the intermediate good is the perfectly competitive price in the external market. This situation is depicted in Figure 11.9, where the price for the intermediate good is the same price depicted in Figure 11.8.

It should be noted that because the price of the intermediate good in Figure 11.9 is assumed to be the same price depicted in Figure 11.8, the mar-

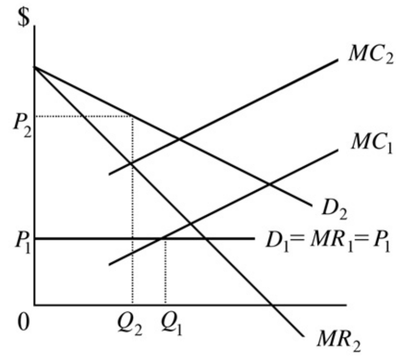

FIGURE 11.9 Transfer pricing with a perfectly competitive external market for intermediate good.

ginal cost of producing the final good remains unchanged. Thus, the profit-maximizing price and the quantity for the final good remain unchanged at P_2 and Q_2. Unlike the situation depicted in Figure 11.8, the amount of output produced by the intermediate good division no longer needs to equal the output of the final good division. Moreover, the transfer price for the intermediate good is the perfectly competitive price in the external market. In the preceding case, with no external market for the intermediate good, the transfer price was determined by the level of output that maximized profits for the final goods division.

In the situation depicted in Figure 11.9, the marginal cost of producing the intermediate good is lower than that depicted in Figure 11.8. The profit-maximizing, intermediate good division will produce at an output level at which $MC_1 = MR_1$. This occurs at an output level that is greater than Q_2. The division producing the intermediate good will sell Q_2 to the division producing the final good and will sell the surplus output of $Q_1 - Q_2$ in the external market at the perfectly competitive price, P_1.

Problem 11.19. Suppose that in the Parallax telescope example of Problem 11.18 the lenses produced by the subsidiary are of a standard variety produced by a perfectly competitive firm. Suppose further that the market-determined price of these lenses is $4.

a. Find the profit-maximizing price and quantity for Parallax telescopes. What is Parallax's total profit?
b. What transfer price should the lens division charge the telescope division?
c. How many lenses will the lens division produce? Will the number of lenses produced be sufficient to satisfy the requirements of the telescope division? If not, what should the telescope division do? If the lens division produces more lenses than the telescope division requires, what should the overproducing division do?

Solution

a. Since there is no change in the demand for Parallax telescopes, there is no change in the firm's total revenue function. However, since lenses are now \$0.49 more expensive than before, Parallax must spend \$1.47 more to produce each telescope. Parallax's total cost equation for telescopes is now

$$TC_{\mathrm{T}} = 100 - 2Q_{\mathrm{T}}^2 + 1.47Q_T$$

Parallax's total profit equation is

$$\begin{aligned}\pi = TR - TC_{\mathrm{T}} &= (100Q_{\mathrm{T}} - 0.05Q_{\mathrm{T}}^2) - (100 + 2Q_{\mathrm{T}}^2 + 1.47Q_{\mathrm{T}})\\ &= -100 + 100Q_{\mathrm{T}} - 2.05Q_{\mathrm{T}}^2 - 1.47Q_{\mathrm{T}} = -100 + 98.53Q_{\mathrm{T}} - 2.05Q_{\mathrm{T}}^2\end{aligned}$$

The first-order condition for profit maximization is $d\pi/dQ_{\mathrm{T}} = 0$. Taking the first derivative of the profit equation yields

$$\frac{d\pi}{dQ} = 98.53 - 4.1Q_{\mathrm{T}} = 0$$

Solving, we have

$$Q_{\mathrm{T}}{}^* = 24.03$$

The second-order condition for profit maximization is $d\pi/dQ_{\mathrm{T}} < 0$. Taking the second derivative of the profit equation yields

$$\frac{d^2\pi}{dQ^2} = -4.1 < 0$$

which guarantees that this output level represents a local maximum. The profit-maximizing price, therefore, is

$$P_{\mathrm{T}}{}^* = 100 - 0.05(24.03) = \$98.80$$

Parallax's total profit at the profit-maximizing price and quantity is

$$\pi = -100 + 98.53(24.03) - 2.05(24.03)^2 = \$1{,}083.93$$

b. The transfer price for lenses is the price set in the perfectly competitive market (i.e., $P_{\mathrm{L}} = \$4$).

c. The lens division will maximize profit by setting the marginal cost of producing lenses equal to the marginal revenue of selling lenses, that is,

$$MC_{\mathrm{L}} = MR_{\mathrm{L}}$$

Since lens production takes place in a perfectly competitive industry, the marginal revenue from selling lenses is \$4 per lens. The marginal cost equation of lens production is

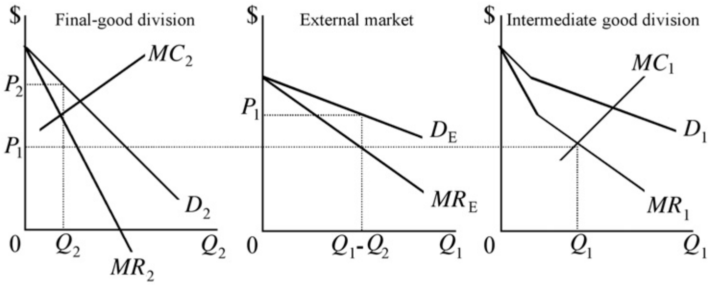

FIGURE 11.10 Transfer pricing with an imperfectly competitive external market for the intermediate good.

$$MC_L = \frac{dTC_L}{dQ_L} = 0.05Q_L$$

Substitution yields

$$4 = 0.05Q_L$$

$$Q_L^* = 80$$

The lens division should produce 80 lenses. Since the telescope division needs 72.09 lenses (3×24.03 telescopes), the lens division should sell the remaining 7.91 lenses in the external market.

TRANSFER PRICING WITH AN IMPERFECTLY COMPETITIVE EXTERNAL MARKET

Finally, let us consider an imperfectly competitive external market for the intermediate good. In this case, the price charged by the intermediate good division to the final good division will differ from the price of the intermediate good in the imperfectly competitive external market. The prices charged internally and externally by the intermediate good division become a matter of third-degree price discrimination. Consider Figure 11.10.

In Figure 11.10, the intermediate good division faces a downward-sloping demand curve for its output. The total demand for Q_1 includes the demand for the intermediate good by the final-good division and the demand by the external market. This demand curve is labeled D_1. Once again, the marginal cost of producing the final good Q_2 includes the marginal cost of producing Q_1. The profit-maximizing level of output for the intermediate good division is Q_1. The corresponding $MR_1 = MC_1$ will determine the selling price of the intermediate good to the final good division. This will be the transfer price of the intermediate good.

The amount of Q_1 that will be sold to the final good division will be determined by the profit-maximizing level of output Q_2, since $Q_1 = Q_2$. This leaves $Q_1 - Q_2$ units of Q_1 available for sale in the external market. The intermediate good division will maximize its profits by charging a price in the external market such that $MC_1 = MR_E$. In Figure 11.10, the intermediate good division engages in third-degree price discrimination by charging more in the external market than it charges the final good division.

OTHER PRICING PRACTICES

This chapter has so far focused on the pricing behavior of profit-maximizing firms operating under somewhat unique circumstances. In each case, the firm's pricing practices were predicated on subtle economic concepts. It was also assumed that management had complete information about the realities of the market in which the firm operated. In practice however, a firm's pricing practices are much looser in the sense that they are based less on detailed mathematical analysis than on perception, custom, and intuition. The remainder of this chapter is devoted to a review of five of these alternative pricing practices-*price leadership*, *price skimming*, *penetration pricing*, *prestige pricing*, and *psychological pricing*.

PRICE LEADERSHIP

Price leadership is a phenomenon that is likely to be observed in oligopolistic industries. It was noted in Chapter 10 that oligopolistic industries are characterized by the interdependence of managerial decisions between and among the firms in the industry. Firms in oligopolistic industries are keenly aware that the pricing and output decisions of any individual firm will provoke a reaction by competing firms. A consequence of this interdependence is relatively infrequent price changes.

Definition: Price leadership occurs when a dominant company in an industry establishes the selling price of a product for the rest of the firms in the industry. Two forms of price leadership are barometric price leadership and dominant price leadership.

Barometric Price Leadership

We saw in our discussion of the kinked demand curve that in oligopolistic industries, marginal cost may fluctuate within a fairly narrow range without evoking a price change. The reason for this is the discontinuity in the firm's marginal revenue curve. As a result, prices are relatively stable at the "kink" in the demand curve. What happens, however, when cost conditions for the typical firm in the industry increase significantly because of some exogenous shock? How will the increased cost of production manifest itself in the selling price of the product when, for example, the United Auto Workers negotiate higher wages and benefits for union workers in all firms in the U.S. automobile industry, or OPEC production cutbacks result in higher energy prices?

Definition: Barometric price leadership occurs when a price change by one firm in an oligopolistic industry, usually in response to perceived changes in macroeconomic or market conditions, is quickly followed by price changes by other firms in the industry.

In an oligopolistic industry characterized by firms of roughly the same size, price changes may sometimes be explained by *barometric price leadership*. In this case, a typical firm in the industry initiates, say, a price increase based on management's belief that changes in macroeconomic or market conditions will have a uniform impact on all other firms in the industry. If other firms believe that the firm's interpretations of economic events are correct, they will quickly follow suit. If they disagree, the firm initiating the price increase will be forced to reevaluate its decision and may modify or repeal the price increase. If the price increase is modified, the evaluation process begins again. Ultimately, member firms in the industry will form a consensus and a new, stable, price will be established.

An example of this type of price leadership can be seen in the commercial banking industry. Based on its reading of macroeconomic conditions, a leading money-center commercial bank, such as Citibank, may announce its decision to raise or lower the prime rate (the interest rate on loans to its best customers). If the rest of the industry agrees with Citibank's interpretation of macroeconomic conditions, other money-center commercial banks will quickly follow suit. If not, they will not raise their prime rates and Citibank will quietly lower its prime rate to a level consistent with the sentiments of the industry.

Dominant Price Leadership

Some industries are characterized by a single, dominant firm and many smaller competitors. The dominant firm may be the industry leader because of its leadership in product innovation, or because of economies of scale. If the firm is large enough or efficient enough, it may be able to force smaller competitors out of business by undercutting their prices, or it may simply buy them out. Such behavior, however, often incurs the wrath of the U.S. Department of Justice, which is charged with enforcing federal antitrust legislation.

Definition: Dominant price leadership occurs when one firm in the industry is able to establish the industry price as a result of its profit-maximizing behavior. Once a price has been established by the dominant firm, the remaining firms in the industry become price takers and face a perfectly elastic demand curve for their output.

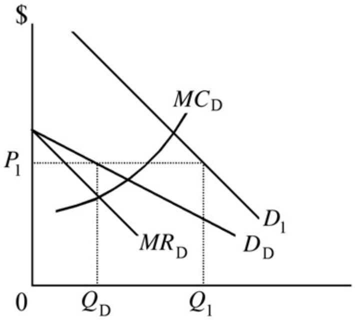

FIGURE 11.11 Dominant price leadership.

Industries dominated by a single large firm are characterized by price stability. The reason for this is that the dominant firm establishes the selling price of the product, and the smaller firms quickly adjust their price and output decisions accordingly. This situation is illustrated in Figure 11.11, which indicates that the dominant firm in the industry will behave like a monopolist by producing at output level Q_D, where its marginal cost is equal to its marginal revenue, $MC_D = MR_D$. The profit-maximizing price P_I will then serve as the industry standard. The amount of output provided by the rest of the industry will be $Q_I - Q_D$.

What is interesting about this analysis is that once the industry price has been established by the dominant firm, the remaining firms take on the appearance of a perfectly competitive industry in which the demand curve for their product is perfectly elastic. The other words, other firms in the industry are price takers. If entry and exit into and from the industry are relatively easy, the existence of above-normal profits will attract new firms into the industry, while below-normal profits will provide an incentive for firms to leave. It is speculative whether the influx or outflow of firms into the industry weakens or strengthens the market power of the dominant firm. In large part, this will depend on the circumstances explaining the firm's rise to industry dominance, and whether those factors are sufficient to maintain the firm's preeminent position.

PRICE SKIMMING

If a firm is first to market with a new product, it may engage in a form of first-degree price discrimination called price skimming. As with first-degree price discrimination, price skimming is an attempt to extract consumer surplus. During the interval between the firm's introduction of a new product and the competition's development of their own versions of the new product, the innovating firm is a virtual monopoly. If the innovating firm wants to extract consumer surplus, however, it must act fast.

Definition: Price skimming is an attempt by a firm that introduces a new product to extract consumer surplus through differential pricing before competitors develop their own versions of the new product.

The firm begins by initially charging a very high price for its product. Consumers willing and able to pay this price will buy first. Before competitors have a chance to sell their versions of the new product, the innovating firm will lower its price just a bit to attract the next, lower tier of consumers. This process is continued until the price charged equals the marginal cost of production. Pricing in this manner will enable the innovating firm to extract consumer surplus to enhance profits. Of course, for this pricing scheme to be successful the firm must have knowledge of the demand curve of the product. Management is unlikely to have such knowledge, however, because of the novel nature of the product. The firm could, of course, conduct consumer surveys, market experiments, and so on, to develop information regarding the demand for the product, but care would have to be taken not to "tip off" the competition.

PENETRATION PRICING

Penetration pricing occurs when a firm entering a new market charges a price that is below the prevailing market price to gain a foothold in the industry. A form of penetration pricing is *dumping*. Dumping is defined by the U.S. Department of Commerce as selling a product at less than fair market value. The most egregious form of this kind of pricing behavior is *predatory dumping*, the attempt by a foreign producer to gain control of the market in another country by selling a product there at less than fair market value with the goal of driving out domestic producers.

Definition: Penetration pricing is the practice of charging a price that is lower than the prevailing market price to gain a foothold in the industry.

PRESTIGE PRICING

Prestige pricing is essentially an attempt by firms to increase sales of certain products by capitalizing on snob appeal. Many consumers derive a degree of personal identity from the ostentatious display of certain brand-name items. For them, an enhanced personal image from the conspicuous consumption of upscale products is as valuable, and sometimes more so, than the usefulness or quality of the product itself. The mere fact that a product sells for a higher price often conveys the impression of higher quality, which may or may not be supported by reality. Prestige pricing is an attempt by some firms to exploit this perception by charging higher prices because of the increased prestige that they believe ownership of their products confers. An often cited example is the luxury automobile market, where higher priced automobiles are perceived to be superior to lower priced automobiles of similar quality.

Definition: Prestige pricing is the practice of charging a higher price for a product to exploit the belief by some consumers that a higher price means better quality, which in turn confers on the owner greater prestige.

PSYCHOLOGICAL PRICING

Finally, psychological pricing is a marketing ploy designed to create the illusion in the mind of the consumer that a product is being sold at a significantly lower price when, in fact, the price differential is inconsequential. Retailer sale of a product for $4.99 instead of $5.00 is psychological pricing. Retailers who engage in psychological pricing are attempting to exploit consumers' initial impressions or their lack of familiarity with the product. The effects of psychological pricing tend to be transitory, however, as initial impressions wear off or the consumer becomes more knowledgeable about the product.

Definition: Psychological pricing is a marketing ploy designed to create the illusion in the mind of the consumer that a product is being sold at a significantly lower price when, in fact, the price differential is inconsequential.

CHAPTER REVIEW

Earlier chapters discussed output and pricing decisions under some very simplistic assumptions. We assumed, for example, that profit-maximizing firms produce a single good or service, that production takes place in a single location, that these firms sell their products in a well-defined market, that managements have perfect information about production, revenue, and cost functions, and that the firms sell their output at a uniform price to all customers. In reality, these assumptions are rarely valid. For that reason, we considered alternative pricing practices, which in some cases are derivatives of the more general cases already encountered.

In *price discrimination*, a firm sells identical products in two or more markets at different prices. Economists have identified three degrees of price discrimination. *First-degree price discrimination* occurs when a firm charges each buyer a different price based on what he or she is willing to pay. In practice, first-degree price discrimination is virtually impossible.

In *second-degree price discrimination*, often referred to as volume discounting, different prices are charged for different blocks of units, or different products are bundled and sold at a package price. An example of second-degree price discrimination is block pricing, in which there are different prices for different blocks of goods and services. Second-degree price discrimination requires that a firm be able to closely monitor the level of services consumed by individual buyers.

In *third-degree price discrimination*, which is by far the most frequently practiced type of price discrimination, firms segment the market for a particular good or service into easily identifiable groups and then charge each group a different price. Such market segregation may be based on such factors as geography, age, product use, or income. For third-degree price discrimination to be successful, firms must be able to prevent resale of the good or service across segregated markets.

Cost-plus pricing, also known as *markup* or *full-cost pricing*, is an example of *nonmarginal pricing*. Firms that engage in nonmarginal pricing are unable or unwilling to devote the resources required to accurately estimate the total revenue and total cost equations, or do not know enough about demand and cost conditions to determine the profit-maximizing price and output levels. In cost-plus pricing, a firm sets the selling price of its product as a markup above its *fully allocated per-unit cost* of production. One criticism of cost-plus pricing is that it is insensitive to demand conditions. In practice, however, the size of a firm's markup tends to be inversely related to the price elasticity of demand for a good or service.

Multi product pricing involves optimal pricing strategies of firms producing and selling more than one good or service. Firms that independently produce two products with interrelated demands will maximize profits by producing at a level at which marginal cost is equal to the change in total revenue derived from the sale of the product itself, plus the change in total revenue derived from the sale of the related product. A profit-maximizing firm selling two goods with independent demands that are jointly produced in variable proportions will equate the marginal revenue generated from the sale of each good to the marginal cost of producing each product. Finally, a profit-maximizing firm that jointly produces two goods in fixed proportions with independent demands will equate the sum of the marginal revenues of both products expressed in terms of one of the products with the marginal cost of jointly producing both products expressed in terms of the same product.

Peak-load pricing occurs when a profit-maximizing firm charges a one price for a service when capacity is fully utilized and a lower price when capacity is underutilized. Off-peak prices are determined by equating marginal revenue to marginal operating costs. Peak prices, on the other hand, are determined by equating marginal revenue to the marginal cost of increasing capacity.

Price leadership appears when an oligopolist establishes a price that is quickly adopted by other firms in the industry. There are two types of price leadership: *barometric price leadership* and *dominant price leadership*.

Barometric price leadership exists when a price change by one firm in an oligopolistic industry, usually in response to perceived changes in macroeconomic or market conditions, is quickly followed by price changes by other firms in the industry.

Dominant price leadership exists when the largest firm in the industry establishes the industry price as a result of its profit-maximizing behavior. Once the industry price has been established, the remaining firms become price takers in the sense that they face a perfectly elastic demand curve for their output.

Other important pricing practices include transfer pricing, price skimming, penetration pricing, prestige pricing, and psychological pricing.

Transfer pricing is a method of correctly pricing a product as it is transferred from one stage of production to another.

Price skimming is the practice of taking advantage of weak or nonexistent competition to change a higher price for a new product than is justified by economic analysis. While competitors are trying to catch up, the firm may have monopoly pricing power.

Penetration pricing is found when a firm entering a new market charges less than its competitors to gain a foothold in the industry.

Prestige pricing is the setting of a high price for a product in the belief that demand will be higher because of the prestige that ownership bestows on the buyer.

Finally, *psychological pricing* is a marketing ploy designed to create the illusion in the mind of the consumer that a product is being sold at a significantly lower price when, in fact, the price differential is inconsequential. A retailer that sells a product for $4.99 instead of $5.00 is engaging in psychological pricing. The effect of psychological pricing tends to be transitory.

KEY TERMS AND CONCEPTS

Barometric price leadership A price change by one firm in an oligopolistic industry, usually in response to perceived changes in macroeconomic or market conditions, quickly followed by price changes by other firms in the industry.

Block pricing A form of second-degree price discrimination. It involves charging different prices for different "blocks" of goods and services to enhance profits by extracting at least some consumer surplus.

Commodity bundling Like block pricing, a form or second-degree price discrimination. Commodity bundling involves the combining of two or more different products into a single package, which is sold at a single price. Like block pricing, commodity bundling is an attempt to enhance profits by extracting at least some consumer surplus.

Consumer surplus The value of benefits received per unit of output consumed minus the product's selling price.

Cost-plus pricing The most popular form of nonmarginal pricing, cost-plus pricing is the practice of adding a predetermined "markup" to a firm's estimated per-unit cost of production at the time of setting the selling price of its product. Cost-plus pricing is given by the expression $P = ATC(1 + m)$, where m is the percentage markup and ATC is the fully allocated per-unit cost of production. The percentage markup may also be expressed as $m = (P - ATC)/ATC$.

Differential pricing Another term for price discrimination. It involves charging different prices to different groups, for different prices for different blocks of goods or services.

Dominant price leadership Establishment of the industry price by the dominant firm in the industry, as a result of its profit-maximizing behavior. Once the industry price has been established, the remaining firms in the industry become price takers and face a perfectly elastic demand curve for their output.

Dumping Third-degree price discrimination practiced in foreign trade. An exporting company that sells its product at a different, usually lower, price in the foreign market than it does in the home market is practicing dumping.

Dumping margin The difference between the price charged for a product sold by a firm in a foreign market and the price charged in the domestic market.

First-degree price discrimination The changing of a different price for each unit purchased. The price charged for any unit, which is based on the seller's knowledge of the individual buyer's demand curve, reflects the consumer's valuation of each unit purchased. The purpose of first-degree price discrimination is to maximize profits by extracting from each consumer the full amount of consumer surplus.

Full-cost pricing Another term for cost-plus pricing.

Fully allocated per-unit cost The sum of the estimated average variable cost of producing a good or service and a per-unit allocation for fixed cost. It is an approximation of average total cost.

Markup pricing Another term for cost-plus pricing.

Multiproduct pricing Optimal pricing strategies of a firm producing and selling more than one good under a number of alternative scenarios, including pricing of two or more goods with interdependent demands, pricing of two or more goods with independent demands produced in variable proportions, pricing of two or more goods with independent demands jointly produced in fixed proportions, and pricing of two or more goods given capacity limitations.

Nonmarginal pricing The profit maximizing price and output level are determined by equating marginal cost with marginal revenue. Management will often practice nonmarginal pricing, however, when the firm's total cost and total revenue equations are difficult or impossible to estimate. The most popular form of nonmarginal pricing is cost-plus pricing, also known as markup on full-cost pricing.

Peak-load pricing The practice of charging one price for a service when demand is high and capacity is fully utilized and a lower price for the service when demand is low and capacity is underutilized.

Penetration pricing The practice of charging less than the prevailing market price to gain a foothold in the industry; a strategy sometimes selected by firms entering a new market.

Prestige pricing The practice of charging a high price for a product to exploit the belief by some consumers that a high price tag means better quality, which confers upon the owner greater prestige.

Price discrimination The management, by a profit-maximizing firm, to charge different individuals or groups different prices for the same good or service.

Price leadership Seen when a dominant company in an industry establishes the selling price of a product for the rest of the firms in the industry. Two forms of price leadership are barometric price leadership and dominant price leadership.

Price skimming An attempt by a firm that introduces a new product to extract consumer surplus through differential pricing before the firm's competitiors develop their own versions of the new product.

Psychological pricing A marketing ploy designed to create the illusion in the mind of the consumer that a product is being sold at a significantly lower price when, in fact, the price reduction is inconsequential. Retailer sale of a product for $4.99 instead of $5.00 represents psychological pricing.

Relationship between the markup and the price elasticity of demand The size of a firm's markup tends to be inversely related to the price elasticity of demand for a good or service. When the demand for a product is low, the markup tends to be high, and vice versa. This relationship may be expressed as $m = -1/(\varepsilon_p + 1)$.

Second-degree price discrimination Similar in principle to first-degree price discrimination, it involves products in "blocks" or "bundles" rather than one unit at a time.

Third-degree price discrimination Segmenting the market for a particular good or service into easily identifiable groups, with a different price for each group.

Transfer pricing The optimal pricing of the output of one subsidiary of a parent company that is sold as an intermediate good to another subsidiary of the same parent company.

Two-part pricing A variation of second-degree price discrimination, two-part pricing is an attempt to enhance a firm's profits by charging a fixed fee for the right to purchase a good or service, plus a per-unit charge.

The per-unit charge is set equal to the marginal cost of providing the product, while the fixed fee is used to extract maximum consumer surplus, which is pure profit.

Volume discounting A form of second-degree price discrimination.

CHAPTER QUESTIONS

11.1 Explain each of the following pricing practices.

a. First-degree price discrimination

b. Second-degree price discrimination

c. Third-degree price discrimination

11.2 What is consumer surplus?

11.3 An important objective of firms engaged that practice price discrimination is the extraction of consumer surplus. Do you agree? Explain.

11.4 First-degree price discrimination is a relatively common practice, especially by firms dealing directly with the public, such as restaurants and retail outlets. Do you agree with this statement? If not, then why not?

11.5 What is the difference between block pricing and commodity bundling?

11.6 Sales of frankfurter rolls in packages of eight or beer in six-packs are examples of what pricing practice? What is the objective of the firm?

11.7 Explain the use of block pricing by amusement parks. Why do some amusement parks engage in block pricing while other, usually older, amusement parks do not?

11.8 The pricing of private goods is fundamentally different from the pricing of public goods because of the properties of excludability and depletability. Explain.

11.9 Explain how block pricing by amusement parts is similar to block pricing by cable television companies.

11.10 What is two-part pricing? Provide examples.

11.11 Explain how the practice of commodity bundling may give to a firm an unfair competitive advantage over its rivals.

11.12 Explain cost-plus (markup) pricing. Markup pricing suffers from what theoretical weakness? What are the advantages and disadvantages of markup pricing?

11.13 The more price elastic is the demand for a good or service, the higher will be the price markup over the marginal cost of production. Do you agree with this statement? Explain.

11.14 A firm producing two goods with interrelated demands, such as personal computers and modems, will maximize profits by equating the marginal revenue generated from the sale of each good separately to the marginal cost of producing each good. Do you agree with this statement? Explain.

11.15 A firm producing in variable proportions two goods with independent demands, such as automobile taillight and flashlight bulbs, will maximize profits by equating the marginal cost of producing each good separately to the combined marginal revenue generated from the sale of both goods. Do you agree with this statement? Explain.

11.16 A firm producing two goods with independent demands, which are produced in fixed proportions, will maximize profits by equating the sum of the marginal revenues generated from the sale of both goods, expressed in terms of one of the goods, to the marginal cost of jointly producing both goods. Do you agree? Explain.

11.17 Identify situations in which peak-load pricing may be appropriate. What is the distinguishing characteristic of short-run production functions in these situations?

11.18 Peak-load pricing suggests that users of commuter railroads be charged higher fares during off-peak hours to compensate the company for lost revenues arising from fewer riders. Do you agree with this statement? Explain.

11.19 Suppose a firm that produces a product for sale in the market also produces a vital component of that good for which there is no outside market. How should the firm "price" this component?

11.20 Explain each of the following pricing practices.

a. Barometric price leadership
b. Dominant price leadership
c. Price skimming
d. Penetration pricing
e. Prestige pricing
f. Psychological pricing

CHAPTER EXERCISES

11.1 Assume that an individual's demand for a product is

$$Q = 20 - 0.5P$$

Suppose that the market price of the product $10.

a. Approximate the value of this individual's consumer surplus for $\triangle Q = 1$.
b. What is value of consumer surplus as $\triangle Q \to 0$?

11.2 An amusement park has estimated the following demand equation for the average park guest

$$Q = 16 - 2P$$

where Q represents the number or rides per guest, and P the price per ride. The total cost of providing a ride is characterized by the equation

$$TC = 2 + 0.5Q$$

a. How much should the park charge on a per-ride basis to maximize its profit? What is the amusement park's total profit per customer?
b. Suppose that the amusement park decides to charge a one-time admission fee. What admission fee will maximize the park's profit? What is the estimated average profit per park guest?

11.3 A firm sells its product in two separable and identifiable markets. The firm's total cost of production is

$$TC = 5 + 5Q$$

The demand equations for its product in the two markets are

$$Q_1 = 10 - \frac{P_1}{2}$$

$$Q_2 = 20 - \frac{P_2}{5}$$

where $Q = Q_1 + Q_2$.

a. Calculate the firm's profit-maximizing price and output level in each market.
b. Verify that the demand for the product is less elastic in the market with the higher price.
c. Find the firm's total profit at the profit-maximizing prices and output levels.

11.4 Ned Bayward practices third-degree price discrimination when selling barrels of Eastfarthing Leaf in the isolated villages of Toadmorton and Forlorn. The reason for this is that the residents of Toadmorton have a particular preference for Eastfarthing Leaf, while the people in Forlorn can either take it or leave it. Ned's total cost of producing Eastfarthing Leaf is given by the equation

$$TC = 10 + 0.5Q^2$$

The respective demand equations in Forlorn and Toadmorton are

$$Q_1 = 50 - \frac{P_1}{4.5}$$

$$Q_2 = 75 - \frac{P_2}{7.5}$$

where $Q = Q_1 + Q_2$.

a. Calculate Ned's profit-maximizing price and output level in each market.
b. Verify that the demand for Eastfarthing Leaf is less elastic in the Toadmorton than in Forlorn. What does your answer imply about Ned's pricing policy?
c. Find the firm's total profit at the profit-maximizing prices and output levels.

11.5 Suppose a company has estimated the average variable cost of producing its product to be \$10. The firms total fixed cost is \$100,000.

a. If the company produces 1,000 units and its standard pricing practice is to add a 35% markup, what price should the company charge?
b. Verify that the selling price calculated in part a represents a 35% markup over the estimated average cost of production.

11.6 What is the estimated percentage markup over the fully allocated per-unit cost of production for the following price elasticities of demand?

a. $\varepsilon_p = -10$
b. $\varepsilon_p = -6$
c. $\varepsilon_p = -3$
d. $\varepsilon_p = -2.3$
e. $\varepsilon_p = -1.8$

11.7 A company produces two products, *I* and *F*. The demand equation for *F* is

$$Q_F = 1,500 - 15P_F$$

The total cost equation is

$$TC_F = 100 + 2Q_F^2$$

The company produces product *I* exclusively as an intermediate good in the production of product *F*. The total cost equation for producing good *I* is

$$TC_1 = 50 - 0.02Q_I^2$$

The company is divided into two semiautonomous profit centers: *I* division and the *F* division.

a. What is the profit-maximizing price and quantity for *F* division?
b. What is *F* division's total profit?
c. What transfer price should *I* division charge *F* division?

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