3. Select data input and type **A1:A36** as the Input Range.
4. Type **80** for the Hypothesized mean and select the "less than" Alternative.
5. Select *z* test and click [OK].

The result of the procedure is shown next.

Hypothesis Test: Mean vs. Hypothesized Value

```
80.000  Hypothesized value
75.000  Mean data
19.161  Standard deviation
 3.193  Standard error
    36  n

 −1.57  z
0.0587  P-value (one-tailed, lower)
```

8–3 *t* Test for a Mean

Objective 6

Test means when σ is unknown, using the *t* test.

When the population standard deviation is unknown, the *z* test is not normally used for testing hypotheses involving means. A different test, called the *t test,* is used. The distribution of the variable should be approximately normal.

As stated in Chapter 7, the *t* distribution is similar to the standard normal distribution in the following ways.

1. It is bell-shaped.
2. It is symmetric about the mean.
3. The mean, median, and mode are equal to 0 and are located at the center of the distribution.
4. The curve never touches the *x* axis.

The *t* distribution differs from the standard normal distribution in the following ways.

1. The variance is greater than 1.
2. The *t* distribution is a family of curves based on the *degrees of freedom,* which is a number related to sample size. (Recall that the symbol for degrees of freedom is d.f. See Section 7–2 for an explanation of degrees of freedom.)
3. As the sample size increases, the *t* distribution approaches the normal distribution.

The *t* test is defined next.

The ***t* test** is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed, and σ is unknown.

The formula for the *t* test is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. $= n - 1$.

The formula for the *t* test is similar to the formula for the *z* test. But since the population standard deviation σ is unknown, the sample standard deviation *s* is used instead.

The critical values for the *t* test are given in Table F in Appendix C. For a one-tailed test, find the α level by looking at the top row of the table and finding the appropriate column. Find the degrees of freedom by looking down the left-hand column.

Notice that the degrees of freedom are given for values from 1 through 30, then at intervals above 30. When the degrees of freedom are above 30, some textbooks will tell you to use the nearest table value; however, in this textbook, you should always round down to the nearest table value. For example, if d.f. = 59, use d.f. = 55 to find the critical value or values. This is a conservative approach.

As the degrees of freedom get larger, the critical values approach the z values. Hence the bottom values (large sample size) are the same as the z values that were used in the last section.

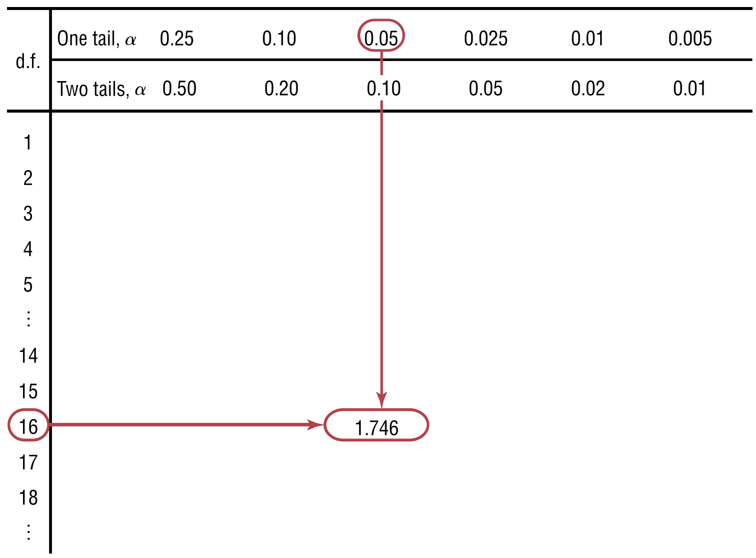
Example 8–8

Find the critical t value for $\alpha = 0.05$ with d.f. = 16 for a right-tailed t test.

Solution

Find the 0.05 column in the top row and 16 in the left-hand column. Where the row and column meet, the appropriate critical value is found; it is +1.746. See Figure 8–21.

Figure 8–21
Finding the Critical Value for the *t* Test in Table F (Example 8–8)



Example 8–9

Find the critical t value for $\alpha = 0.01$ with d.f. = 22 for a left-tailed test.

Solution

Find the 0.01 column in the row labeled One tail, and find 22 in the left column. The critical value is −2.508 since the test is a one-tailed left test.

Example 8–10

Find the critical values for $\alpha = 0.10$ with d.f. = 18 for a two-tailed t test.

Solution

Find the 0.10 column in the row labeled Two tails, and find 18 in the column labeled d.f. The critical values are +1.734 and −1.734.

Example 8–11

Find the critical value for $\alpha = 0.05$ with d.f. = 28 for a right-tailed t test.

Solution

Find the 0.05 column in the One-tail row and 28 in the left column. The critical value is +1.701.

Assumptions for the *t* Test for a Mean When σ Is Unknown

1. The sample is a random sample.
2. Either $n \geq 30$ or the population is normally distributed if $n < 30$.

When you test hypotheses by using the *t* test (traditional method), follow the same procedure as for the *z* test, except use Table F.

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from Table F.

Step 3 Compute the test value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

Remember that the t test should be used when the population is approximately normally distributed and the population standard deviation is unknown.

Examples 8–12 through 8–14 illustrate the application of the *t* test.

Example 8–12

Hospital Infections

A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at $\alpha = 0.05$?

Source: Based on information obtained from Pennsylvania Health Care Cost Containment Council.

Solution

Step 1 H_0: $\mu = 16.3$ (claim) and H_1: $\mu \neq 16.3$.

Step 2 The critical values are $+2.262$ and -2.262 for $\alpha = 0.05$ and d.f. $= 9$.

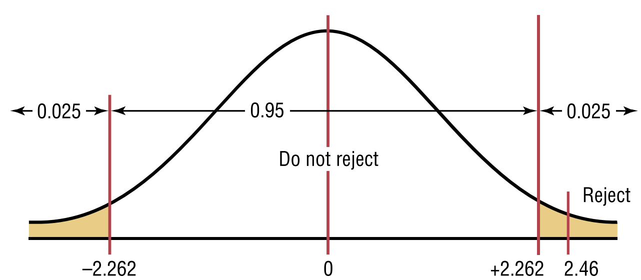
Step 3 The test value is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{17.7 - 16.3}{1.8/\sqrt{10}} = 2.46$$

Step 4 Reject the null hypothesis since $2.46 > 2.262$. See Figure 8–22.

Figure 8–22

Summary of the *t* Test of Example 8–12



Step 5 There is enough evidence to reject the claim that the average number of infections is 16.3.

Example 8–13 Substitute Teachers' Salaries

An educator claims that the average salary of substitute teachers in school districts in Allegheny County, Pennsylvania, is less than \$60 per day. A random sample of eight school districts is selected, and the daily salaries (in dollars) are shown. Is there enough evidence to support the educator's claim at $\alpha = 0.10$?

60 56 60 55 70 55 60 55

Source: *Pittsburgh Tribune-Review.*

Solution

Step 1 H_0: $\mu = \$60$ and H_1: $\mu < \$60$ (claim).

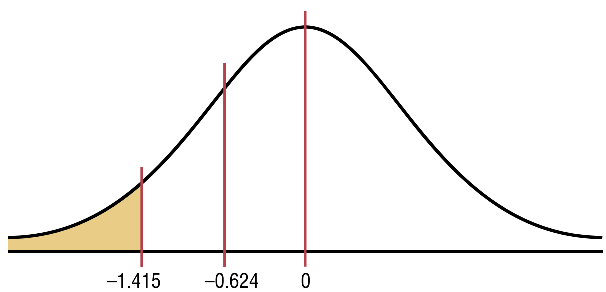
Step 2 At $\alpha = 0.10$ and d.f. $= 7$, the critical value is -1.415.

Step 3 To compute the test value, the mean and standard deviation must be found. Using either the formulas in Chapter 3 or your calculator, $\overline{X} = \$58.88$, and $s = 5.08$, you find

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{58.88 - 60}{5.08/\sqrt{8}} = -0.624$$

Step 4 Do not reject the null hypothesis since -0.624 falls in the noncritical region. See Figure 8–23.

Figure 8–23
Critical Value and Test Value for Example 8–13



Step 5 There is not enough evidence to support the educator's claim that the average salary of substitute teachers in Allegheny County is less than \$60 per day.

The *P*-values for the *t* test can be found by using Table F; however, specific *P*-values for *t* tests cannot be obtained from the table since only selected values of α (for example, 0.01, 0.05) are given. To find specific *P*-values for *t* tests, you would need a table similar to Table E for each degree of freedom. Since this is not practical, only *intervals* can be found for *P*-values. Examples 8–14 to 8–16 show how to use Table F to determine intervals for *P*-values for the *t* test.

Example 8–14

Find the *P*-value when the *t* test value is 2.056, the sample size is 11, and the test is right-tailed.

Solution

To get the *P*-value, look across the row with 10 degrees of freedom (d.f. $= n - 1$) in Table F and find the two values that 2.056 falls between. They are 1.812 and 2.228. Since this is a right-tailed test, look up to the row labeled One tail, α and find the two α values corresponding to 1.812 and 2.228. They are 0.05 and 0.025, respectively. See Figure 8–24.

Figure 8–24

Finding the *P*-Value for Example 8–14

d.f.	Confidence intervals	50%	80%	90%	95%	98%	99%
	One tail, α	0.25	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.50	0.20	0.10	0.05	0.02	0.01
1		1.000	3.078	6.314	12.706	31.821	63.657
2		0.816	1.886	2.920	4.303	6.965	9.925
3		0.765	1.638	2.353	3.182	4.541	5.841
4		0.741	1.533	2.132	2.776	3.747	4.604
5		0.727	1.476	2.015	2.571	3.365	4.032
6		0.718	1.440	1.943	2.447	3.143	3.707
7		0.711	1.415	1.895	2.365	2.998	3.499
8		0.706	1.397	1.860	2.306	2.896	3.355
9		0.703	1.383	1.833	2.262	2.821	3.250
10		0.700	1.372	1.812 *	2.228	2.764	3.169
11		0.697	1.363	1.796	2.201	2.718	3.106
12		0.695	1.356	1.782	2.179	2.681	3.055
13		0.694	1.350	1.771	2.160	2.650	3.012
14		0.692	1.345	1.761	2.145	2.624	2.977
15		0.691	1.341	1.753	2.131	2.602	2.947
⋮		⋮	⋮	⋮	⋮	⋮	⋮
(z) ∞		0.674	1.282	1.645	1.960	2.326	2.576

*2.056 falls between 1.812 and 2.228.

Hence, the P-value would be contained in the interval $0.025 < P\text{-value} < 0.05$. This means that the P-value is between 0.025 and 0.05. If α were 0.05, you would reject the null hypothesis since the P-value is less than 0.05. But if α were 0.01, you would not reject the null hypothesis since the P-value is greater than 0.01. (Actually, it is greater than 0.025.)

Example 8–15

Find the P-value when the t test value is 2.983, the sample size is 6, and the test is two-tailed.

Solution

To get the P-value, look across the row with d.f. = 5 and find the two values that 2.983 falls between. They are 2.571 and 3.365. Then look up to the row labeled Two tails, α to find the corresponding α values.

In this case, they are 0.05 and 0.02. Hence the P-value is contained in the interval $0.02 < P\text{-value} < 0.05$. This means that the P-value is between 0.02 and 0.05. In this case, if $\alpha = 0.05$, the null hypothesis can be rejected since $P\text{-value} < 0.05$; but if $\alpha = 0.01$, the null hypothesis cannot be rejected since $P\text{-value} > 0.01$ (actually $P\text{-value} > 0.02$).

Note: **Since many of you will be using calculators or computer programs that give the specific *P*-value for the *t* test and other tests presented later in this textbook, these specific values, in addition to the intervals, will be given for the answers to the examples and exercises.**

The P-value obtained from a calculator for Example 8–14 is 0.033. The P-value obtained from a calculator for Example 8–15 is 0.031.

To test hypotheses using the *P*-value method, follow the same steps as explained in Section 8–2. These steps are repeated here.

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the *P*-value.

Step 4 Make the decision.

Step 5 Summarize the results.

This method is shown in Example 8–16.

Example 8–16

Jogger's Oxygen Uptake

A physician claims that joggers' maximal volume oxygen uptake is greater than the average of all adults. A sample of 15 joggers has a mean of 40.6 milliliters per kilogram (ml/kg) and a standard deviation of 6 ml/kg. If the average of all adults is 36.7 ml/kg, is there enough evidence to support the physician's claim at $\alpha = 0.05$?

Solution

Step 1 State the hypotheses and identify the claim.

H_0: $\mu = 36.7$ and H_1: $\mu > 36.7$ (claim)

Step 2 Compute the test value. The test value is

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{40.6 - 36.7}{6/\sqrt{15}} = 2.517$$

Step 3 Find the *P*-value. Looking across the row with d.f. $= 14$ in Table F, you see that 2.517 falls between 2.145 and 2.624, corresponding to $\alpha = 0.025$ and $\alpha = 0.01$ since this is a right-tailed test. Hence, *P*-value > 0.01 and *P*-value < 0.025, or $0.01 <$ *P*-value < 0.025. That is, the *P*-value is somewhere between 0.01 and 0.025. (The *P*-value obtained from a calculator is 0.012.)

Step 4 Reject the null hypothesis since *P*-value < 0.05 (that is, *P*-value $< \alpha$).

Step 5 There is enough evidence to support the claim that the joggers' maximal volume oxygen uptake is greater than 36.7 ml/kg.

Interesting Fact

The area of Alaska contains $\frac{1}{6}$ of the total area of the United States.

Students sometimes have difficulty deciding whether to use the *z* test or *t* test. The rules are the same as those pertaining to confidence intervals.

1. If σ is known, use the *z* test. The variable must be normally distributed if $n < 30$.
2. If σ is unknown but $n \geq 30$, use the *t* test.
3. If σ is unknown and $n < 30$, use the *t* test. (The population must be approximately normally distributed.)

These rules are summarized in Figure 8–25.