1. From the toolbar, select Add-Ins, **MegaStat>Hypothesis Tests>Proportion vs. Hypothesized Value.** *Note:* You may need to open MegaStat from the MegaStat.xls file on your computer's hard drive.
2. Type **0.37** for the Observed proportion, *p*.
3. Type **0.40** for the Hypothesized proportion, *p*.
4. Type **100** for the sample size, *n*.
5. Select the "not equal" Alternative.
6. Click [OK].

The result of the procedure is shown next.

Hypothesis Test for Proportion vs. Hypothesized Value

Observed	Hypothesized	
0.37	0.4	*p* (as decimal)
37/100	40/100	*p* (as fraction)
37.	40.	*X*
100	100	*n*
	0.049	standard error
	−0.61	*z*
	0.5403	*p*-value (two-tailed)

8–5 χ^2 Test for a Variance or Standard Deviation

Objective 8

Test variances or standard deviations, using the chi-square test.

In Chapter 7, the chi-square distribution was used to construct a confidence interval for a single variance or standard deviation. This distribution is also used to test a claim about a single variance or standard deviation.

To find the area under the chi-square distribution, use Table G in Appendix C. There are three cases to consider:

1. Finding the chi-square critical value for a specific α when the hypothesis test is right-tailed.
2. Finding the chi-square critical value for a specific α when the hypothesis test is left-tailed.
3. Finding the chi-square critical values for a specific α when the hypothesis test is two-tailed.

Example 8–21

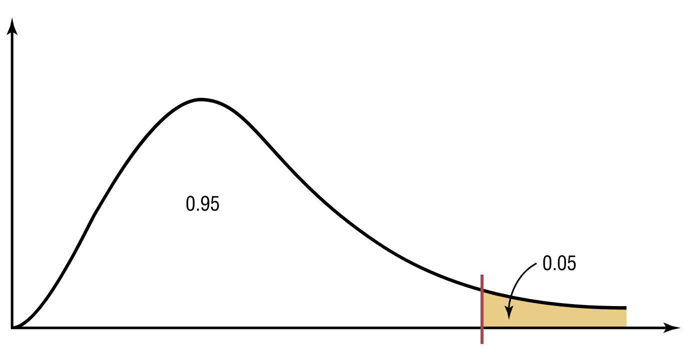
Find the critical chi-square value for 15 degrees of freedom when $\alpha = 0.05$ and the test is right-tailed.

Solution

The distribution is shown in Figure 8–30.

Figure 8–30

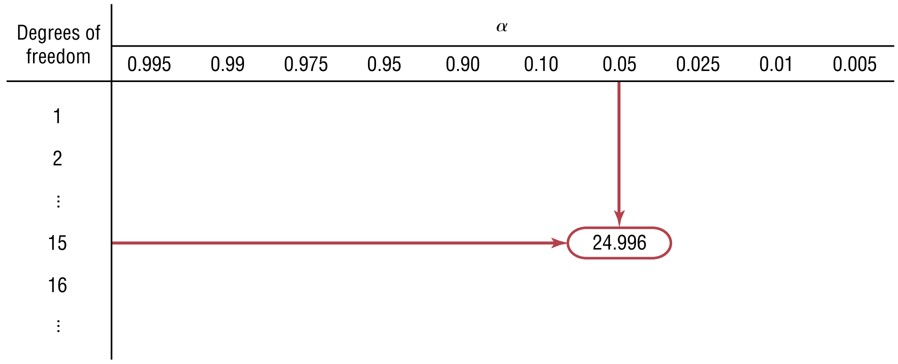
Chi-Square Distribution for Example 8–21



Find the α value at the top of Table G, and find the corresponding degrees of freedom in the left column. The critical value is located where the two columns meet—in this case, 24.996. See Figure 8–31.

Figure 8–31

Locating the Critical Value in Table G for Example 8–21



Example 8–22

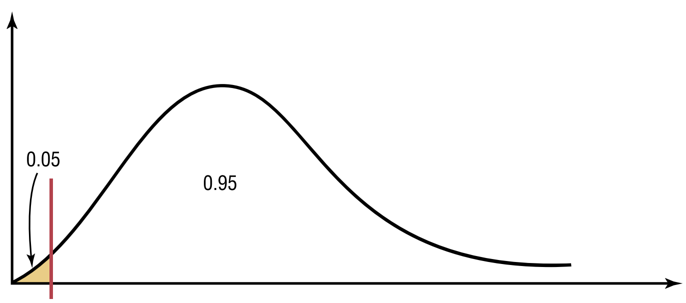
Find the critical chi-square value for 10 degrees of freedom when $\alpha = 0.05$ and the test is left-tailed.

Solution

This distribution is shown in Figure 8–32.

Figure 8–32

Chi-Square Distribution for Example 8–22

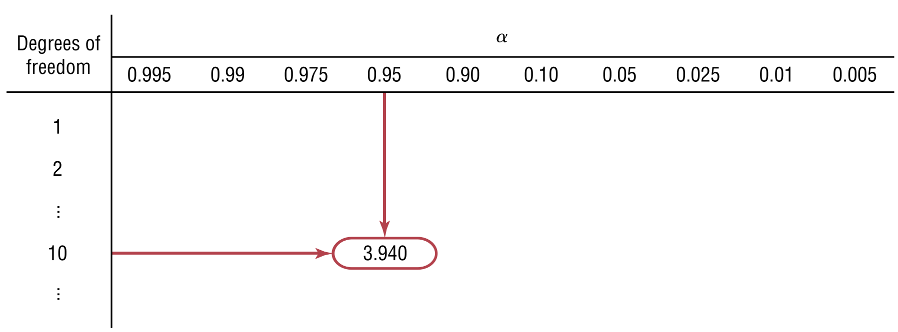


When the test is left-tailed, the α value must be subtracted from 1, that is, $1 - 0.05 = 0.95$. The left side of the table is used, because the chi-square table gives the area to the right of the critical value, and the chi-square statistic cannot be negative. The table is set up so that it gives the values for the area to the right of the critical value. In this case, 95% of the area will be to the right of the value.

For 0.95 and 10 degrees of freedom, the critical value is 3.940. See Figure 8–33.

Figure 8–33

Locating the Critical Value in Table G for Example 8–22



Example 8–23

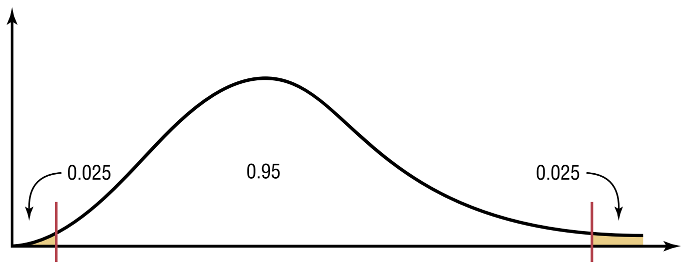
Find the critical chi-square values for 22 degrees of freedom when $\alpha = 0.05$ and a two-tailed test is conducted.

Solution

When a two-tailed test is conducted, the area must be split, as shown in Figure 8–34. Note that the area to the right of the larger value is 0.025 (0.05/2 or $\alpha/2$), and the area to the right of the smaller value is 0.975 ($1.00 - 0.05/2$ or $1 - \alpha/2$).

Figure 8–34

Chi-Square Distribution for Example 8–23



Remember that chi-square values cannot be negative. Hence, you must use α values in the table of 0.025 and 0.975. With 22 degrees of freedom, the critical values are 36.781 and 10.982, respectively.

After the degrees of freedom reach 30, Table G gives values only for multiples of 10 (40, 50, 60, etc.). When the exact degrees of freedom sought are not specified in the table, the closest smaller value should be used. For example, if the given degrees of freedom are 36, use the table value for 30 degrees of freedom. This guideline keeps the type I error equal to or below the α value.

When you are testing a claim about a single variance using the **chi-square test,** there are three possible test situations: right-tailed test, left-tailed test, and two-tailed test.

If a researcher believes the variance of a population to be greater than some specific value, say, 225, then the researcher states the hypotheses as

$$H_0\colon \sigma^2 = 225 \qquad \text{and} \qquad H_1\colon \sigma^2 > 225$$

and conducts a right-tailed test.

If the researcher believes the variance of a population to be less than 225, then the researcher states the hypotheses as

$$H_0\colon \sigma^2 = 225 \qquad \text{and} \qquad H_1\colon \sigma^2 < 225$$

and conducts a left-tailed test.

Finally, if a researcher does not wish to specify a direction, she or he states the hypotheses as

$$H_0\colon \sigma^2 = 225 \qquad \text{and} \qquad H_1\colon \sigma^2 \neq 225$$

and conducts a two-tailed test.

Formula for the Chi-Square Test for a Single Variance

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

with degrees of freedom equal to $n - 1$ and where

n = sample size
s^2 = sample variance
σ^2 = population variance

You might ask, Why is it important to test variances? There are several reasons. First, in any situation where consistency is required, such as in manufacturing, you would like to have the smallest variation possible in the products. For example, when bolts are manufactured, the variation in diameters due to the process must be kept to a minimum, or the nuts will not fit them properly. In education, consistency is required on a test. That is, if the same students take the same test several times, they should get approximately the same grades, and the variance of each of the student's grades should be small. On the other hand, if the test is to be used to judge learning, the overall standard deviation of all the grades should be large so that you can differentiate those who have learned the subject from those who have not learned it.

Three assumptions are made for the chi-square test, as outlined here.

Unusual Stat

About 20% of cats owned in the United States are overweight.

Assumptions for the Chi-Square Test for a Single Variance

1. The sample must be randomly selected from the population.
2. The population must be normally distributed for the variable under study.
3. The observations must be independent of one another.

The traditional method for hypothesis testing follows the same five steps listed earlier. They are repeated here.

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s).

Step 3 Compute the test value.

Step 4 Make the decision.

Step 5 Summarize the results.

Examples 8–24 through 8–26 illustrate the traditional hypothesis-testing procedure for variances.

Example 8–24 Variation of Test Scores

An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance ($\sigma^2 = 225$) at $\alpha = 0.05$? Assume that the scores are normally distributed.

Solution

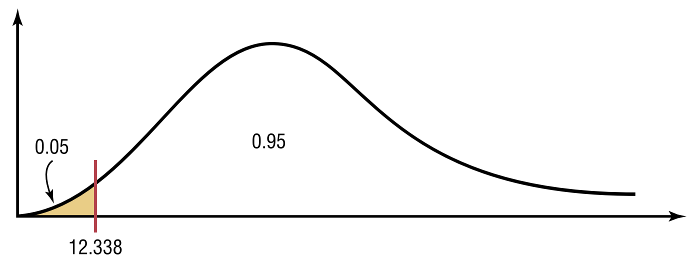
Step 1 State the hypotheses and identify the claim.

$$H_0\colon \sigma^2 = 225 \qquad \text{and} \qquad H_1\colon \sigma^2 < 225 \text{ (claim)}$$

Step 2 Find the critical value. Since this test is left-tailed and $\alpha = 0.05$, use the value $1 - 0.05 = 0.95$. The degrees of freedom are $n - 1 = 23 - 1 = 22$. Hence, the critical value is 12.338. Note that the critical region is on the left, as shown in Figure 8–35.

Figure 8–35

Critical Value for Example 8–24



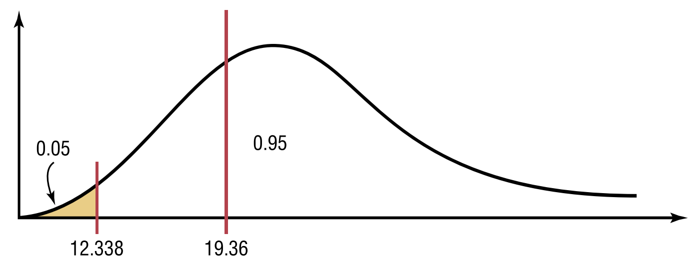
Step 3 Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(23-1)(198)}{225} = 19.36$$

Step 4 Make the decision. Since the test value 19.36 falls in the noncritical region, as shown in Figure 8–36, the decision is to not reject the null hypothesis.

Figure 8–36

Critical and Test Values for Example 8–24



Step 5 Summarize the results. There is not enough evidence to support the claim that the variation in test scores of the instructor's students is less than the variation in scores of the population.

Example 8–25

Outpatient Surgery

A hospital administrator believes that the standard deviation of the number of people using outpatient surgery per day is greater than 8. A random sample of 15 days is selected. The data are shown. At $\alpha = 0.10$, is there enough evidence to support the administrator's claim? Assume the variable is normally distributed.

25	30	5	15	18
42	16	9	10	12
12	38	8	14	27

Solution

Step 1 State the hypotheses and identify the claim.

H_0: $\sigma = 8$ and H_1: $\sigma > 8$ (claim)

Since the standard deviation is given, it should be squared to get the variance.

Step 2 Find the critical value. Since this test is right-tailed with d.f. of $15 - 1 = 14$ and $\alpha = 0.10$, the critical value is 21.064.

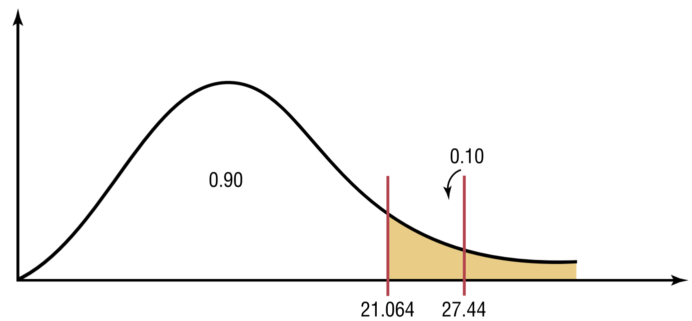
Step 3 Compute the test value. Since raw data are given, the standard deviation of the sample must be found by using the formula in Chapter 3 or your calculator. It is $s = 11.2$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(15-1)(11.2)^2}{64} = 27.44$$

Step 4 Make the decision. The decision is to reject the null hypothesis since the test value, 27.44, is greater than the critical value, 21.064, and falls in the critical region. See Figure 8–37.

Figure 8–37

Critical and Test Value for Example 8–25



Step 5 Summarize the results. There is enough evidence to support the claim that the standard deviation is greater than 8.

Example 8–26

Nicotine Content of Cigarettes

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?

Solution

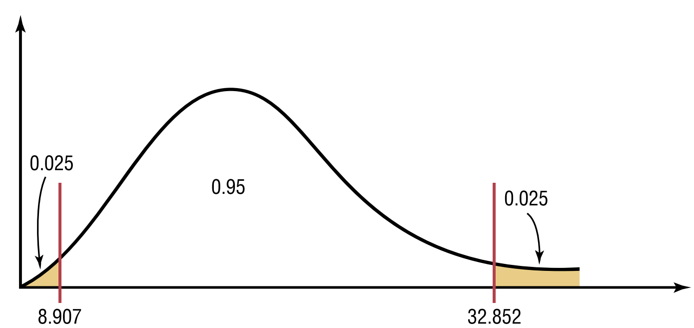
Step 1 State the hypotheses and identify the claim.

H_0: $\sigma^2 = 0.644$ (claim) and H_1: $\sigma^2 \neq 0.644$

Step 2 Find the critical values. Since this test is a two-tailed test at $\alpha = 0.05$, the critical values for 0.025 and 0.975 must be found. The degrees of freedom are 19; hence, the critical values are 32.852 and 8.907, respectively. The critical or rejection regions are shown in Figure 8–38.

Figure 8–38

Critical Values for Example 8–26



Step 3 Compute the test value.

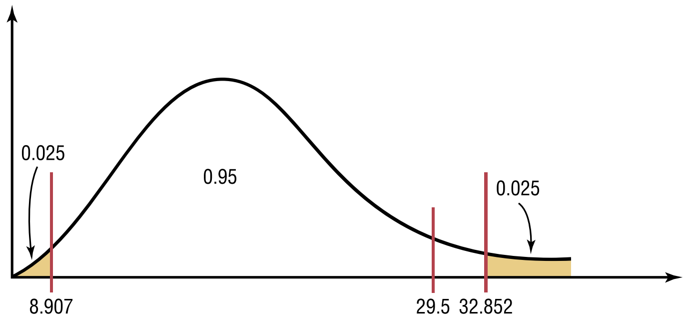
$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)(1.0)^2}{0.644} = 29.5$$

Since the standard deviation s is given in the problem, it must be squared for the formula.

Step 4 Make the decision. Do not reject the null hypothesis, since the test value falls between the critical values ($8.907 < 29.5 < 32.852$) and in the noncritical region, as shown in Figure 8–39.

Figure 8–39

Critical and Test Values for Example 8–26



Step 5 Summarize the results. There is not enough evidence to reject the manufacturer's claim that the variance of the nicotine content of the cigarettes is equal to 0.644.

Approximate *P*-values for the chi-square test can be found by using Table G in Appendix C. The procedure is somewhat more complicated than the previous procedures for finding *P*-values for the z and t tests since the chi-square distribution is not exactly symmetric and χ^2 values cannot be negative. As we did for the t test, we will determine an *interval* for the *P*-value based on the table. Examples 8–27 through 8–29 show the procedure.

Example 8–27

Find the *P*-value when $\chi^2 = 19.274$, $n = 8$, and the test is right-tailed.

Solution

To get the *P*-value, look across the row with d.f. $= 7$ in Table G and find the two values that 19.274 falls between. They are 18.475 and 20.278. Look up to the top row and find the α values corresponding to 18.475 and 20.278. They are 0.01 and 0.005, respectively. See Figure 8–40. Hence the *P*-value is contained in the interval $0.005 < P\text{-value} < 0.01$. (The *P*-value obtained from a calculator is 0.007.)

Figure 8–40

***P*-Value Interval for Example 8–27**

Degrees of freedom	α									
	0.995	**0.99**	**0.975**	**0.95**	**0.90**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475*	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

*19.274 falls between 18.475 and 20.278

Example 8–28

Find the P-value when $\chi^2 = 3.823$, $n = 13$, and the test is left-tailed.

Solution

To get the P-value, look across the row with d.f. $= 12$ and find the two values that 3.823 falls between. They are 3.571 and 4.404. Look up to the top row and find the values corresponding to 3.571 and 4.404. They are 0.99 and 0.975, respectively. When the χ^2 test value falls on the left side, each of the values must be subtracted from 1 to get the interval that P-value falls between.

$$1 - 0.99 = 0.01 \qquad \text{and} \qquad 1 - 0.975 = 0.025$$

Hence the P-value falls in the interval

$$0.01 < P\text{-value} < 0.025$$

(The P-value obtained from a calculator is 0.014.)

When the χ^2 test is two-tailed, both interval values must be doubled. If a two-tailed test were being used in Example 8–28, then the interval would be $2(0.01) < P\text{-value} < 2(0.025)$, or $0.02 < P\text{-value} < 0.05$.

The P-value method for hypothesis testing for a variance or standard deviation follows the same steps shown in the preceding sections.

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the P-value.

Step 4 Make the decision.

Step 5 Summarize the results.

Example 8–29 shows the P-value method for variances or standard deviations.

Example 8–29

Car Inspection Times

A researcher knows from past studies that the standard deviation of the time it takes to inspect a car is 16.8 minutes. A sample of 24 cars is selected and inspected. The standard deviation is 12.5 minutes. At $\alpha = 0.05$, can it be concluded that the standard deviation has changed? Use the P-value method.

Solution

Step 1 State the hypotheses and identify the claim.

$$H_0\colon \sigma = 16.8 \qquad \text{and} \qquad H_1\colon \sigma \neq 16.8 \text{ (claim)}$$

Step 2 Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(24-1)(12.5)^2}{(16.8)^2} = 12.733$$

Step 3 Find the P-value. Using Table G with d.f. $= 23$, the value 12.733 falls between 11.689 and 13.091, corresponding to 0.975 and 0.95, respectively. Since these values are found on the left side of the distribution, each value must be subtracted from 1. Hence $1 - 0.975 = 0.025$ and $1 - 0.95 = 0.05$. Since this is a two-tailed test, the area must be doubled to obtain the P-value interval. Hence $0.05 < P\text{-value} < 0.10$, or somewhere between 0.05 and 0.10. (The P-value obtained from a calculator is 0.085.)

Step 4 Make the decision. Since $\alpha = 0.05$ and the P-value is between 0.05 and 0.10, the decision is to not reject the null hypothesis since P-value $> \alpha$.

Step 5 Summarize the results. There is not enough evidence to support the claim that the standard deviation has changed.

Applying the Concepts 8–5

Testing Gas Mileage Claims

Assume that you are working for the Consumer Protection Agency and have recently been getting complaints about the highway gas mileage of the new Dodge Caravans. Chrysler Corporation agrees to allow you to randomly select 40 of its new Dodge Caravans to test the highway mileage. Chrysler claims that the Caravans get 28 mpg on the highway. Your results show a mean of 26.7 and a standard deviation of 4.2. You support Chrysler's claim.

1. Show why you support Chrysler's claim by listing the P-value from your output. After more complaints, you decide to test the variability of the miles per gallon on the highway. From further questioning of Chrysler's quality control engineers, you find they are claiming a standard deviation of 2.1.
2. Test the claim about the standard deviation.
3. Write a short summary of your results and any necessary action that Chrysler must take to remedy customer complaints.
4. State your position about the necessity to perform tests of variability along with tests of the means.

See pages 469 and 470 for the answers.

Exercises 8–5

1. Using Table G, find the critical value(s) for each, show the critical and noncritical regions, and state the appropriate null and alternative hypotheses. Use $\sigma^2 = 225$.

a. $\alpha = 0.05$, $n = 18$, right-tailed
b. $\alpha = 0.10$, $n = 23$, left-tailed
c. $\alpha = 0.05$, $n = 15$, two-tailed
d. $\alpha = 0.10$, $n = 8$, two-tailed
e. $\alpha = 0.01$, $n = 17$, right-tailed
f. $\alpha = 0.025$, $n = 20$, left-tailed
g. $\alpha = 0.01$, $n = 13$, two-tailed
h. $\alpha = 0.025$, $n = 29$, left-tailed

2. (ans) Using Table G, find the P-value interval for each χ^2 test value.

a. $\chi^2 = 29.321$, $n = 16$, right-tailed
b. $\chi^2 = 10.215$, $n = 25$, left-tailed
c. $\chi^2 = 24.672$, $n = 11$, two-tailed
d. $\chi^2 = 23.722$, $n = 9$, right-tailed
e. $\chi^2 = 13.974$, $n = 28$, two-tailed
f. $\chi^2 = 10.571$, $n = 19$, left-tailed
g. $\chi^2 = 12.144$, $n = 6$, two-tailed
h. $\chi^2 = 8.201$, $n = 23$, two-tailed

For Exercises 3 through 9, assume that the variables are normally or approximately normally distributed. Use the traditional method of hypothesis testing unless otherwise specified.

3. Calories in Pancake Syrup A nutritionist claims that the standard deviation of the number of calories in 1 tablespoon of the major brands of pancake syrup is 60. A sample of major brands of syrup is selected, and the number of calories is shown. At $\alpha = 0.10$, can the claim be rejected?

53	210	100	200	100	220
210	100	240	200	100	210
100	210	100	210	100	60

Source: Based on information from *The Complete Book of Food Counts* by Corrine T. Netzer, Dell Publishers, New York.

4. High Temperatures in January Daily weather observations for southwestern Pennsylvania for the first three weeks of January show daily high temperatures as follows: 55, 44, 51, 59, 62, 60, 46, 51, 37, 30, 46, 51, 53, 57, 57, 39, 28, 37, 35, and 28 degrees Fahrenheit. The normal standard deviation in high temperatures for this time period is usually no more than 8 degrees. A meteorologist believes that with the unusual trend in