## University of Sargodha

## Department of Mechanical Engineering

## Assignment

Program: BSc Mechanical Engineering
Course: Complex Variable and Transform
Maximum Marks: 30
Semester: $4^{\text {rth }}$ (Fall-19)

Teacher Name: Ms. Farhat Imtiaz<br>Course Code: BScMEF18<br>Late of Submission: 19/04/2020<br>Student ID:<br>$\qquad$

## Question No. 1

(a) Evaluate $i^{-2 i}$
(b) Show that $\log (1+i)^{2}=2 \log (1+i)$
(c) Prove that $e^{z_{1} z_{2}}=e^{z_{1}+z_{2}}$
(d) Find $f^{v i}(z), f(z)=\frac{\left(1+z^{2}\right)^{4}}{z^{2}}$.
(e) Determine the principal value of argument $7 \pm 7 i$ and find the all roots of $\sqrt[5]{-1}$

## Question No. 2

Let u and v denote the real and imaginary components of the function f defined by means of the equations $f(x)= \begin{cases}\bar{z}^{2} / z & \text { when } z=0, \\ 0 & \text { when } z \neq 0 .\end{cases}$
Verify that the Cauchy-Riemann equations are satisfied at the origin $\mathrm{z}=(0,0)$. Compare with differential formula, where it is shown that $f^{\prime}(0)$ nevertheless fails to exist.

## Question No. 3

Show that $\mathrm{u}(\mathrm{x}, \mathrm{y})$ is harmonic in some domain and find a harmonic conjugate $\mathrm{v}(\mathrm{x}, \mathrm{y})$ when

$$
\mathrm{u}(\mathrm{x}, \mathrm{y})=\frac{y}{x^{2}+y^{2}} .
$$

## Question No. 4

(a) Define analytic function, limit, continuity and differentiation with example.
(b) Write the Cauchy-Riemann equations in Cartesian and Polar form,
(c) Find $f^{\prime \prime}(\mathrm{z}), f(z)=e^{-\theta} \cos (\ln \mathrm{r})+i \sin (\operatorname{lnr})$,
(d) Find the value of z such that $\exp (2 \mathrm{z}-1)=1$
(e) Solve that $e^{2+\pi i}=-e^{z}$

