CHAPTER 1

What Is Operations Research?

Chapter Guide. The first formal activities of Operations Research (OR) were initiated in England during World War II, when a team of British scientists set out to make scientifically based decisions regarding the best utilization of war materiel. After the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector.

This chapter will familiarize you with the basic terminology of operations research, including mathematical modeling, feasible solutions, optimization, and iterative computations. You will learn that defining the problem correctly is the most important (and most difficult) phase of practicing OR. The chapter also emphasizes that, while mathematical modeling is a cornerstone of OR, intangible (unquantifiable) factors (such as human behavior) must be accounted for in the final decision. As you proceed through the book, you will be presented with a variety of applications through solved examples and chapter problems. In particular, Chapter 24 (on the CD) is entirely devoted to the presentation of fully developed case analyses. Chapter materials are cross-referenced with the cases to provide an appreciation of the use of OR in practice.

1.1 OPERATIONS RESEARCH MODELS

Imagine that you have a 5-week business commitment between Fayetteville (FYV) and Denver (DEN). You fly out of Fayetteville on Mondays and return on Wednesdays. A regular round-trip ticket costs $400, but a 20% discount is granted if the dates of the ticket span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should you buy the tickets for the 5-week period?

We can look at the situation as a decision-making problem whose solution requires answering three questions:

1. What are the decision **alternatives?**
2. Under what **restrictions** is the decision made?
3. What is an appropriate **objective criterion** for evaluating the alternatives?

Three alternatives are considered:

1. Buy five regular FYV-DEN-FYV for departure on Monday and return on Wednesday of the same week.
2. Buy one FYV-DEN, four DEN-FYV-DEN that span weekends, and one DEN-FYV.
3. Buy one FYV-DEN-FYV to cover Monday of the first week and Wednesday of the last week and four DEN-FYV-DEN to cover the remaining legs. All tickets in this alternative span at least one weekend.

The restriction on these options is that you should be able to leave FYV on Monday and return on Wednesday of the same week.

An obvious objective criterion for evaluating the proposed alternative is the price of the tickets. The alternative that yields the smallest cost is the best. Specifically, we have

$$\text{Alternative 1 cost} = 5 \times 400 = \$2000$$

$$\text{Alternative 2 cost} = .75 \times 400 + 4 \times (.8 \times 400) + .75 \times 400 = \$1880$$

$$\text{Alternative 3 cost} = 5 \times (.8 \times 400) = \mathbf{\$1600}$$

Thus, you should choose alternative 3.

Though the preceding example illustrates the three main components of an OR model—alternatives, objective criterion, and constraints—situations differ in the details of how each component is developed and constructed. To illustrate this point, consider forming a maximum-area rectangle out of a piece of wire of length L inches. What should be the width and height of the rectangle?

In contrast with the tickets example, the number of alternatives in the present example is not finite; namely, the width and height of the rectangle can assume an infinite number of values. To formalize this observation, the alternatives of the problem are identified by defining the width and height as continuous (algebraic) variables.

Let

$$w = \text{width of the rectangle in inches}$$

$$h = \text{height of the rectangle in inches}$$

Based on these definitions, the restrictions of the situation can be expressed verbally as

1. Width of rectangle + Height of rectangle = Half the length of the wire
2. Width and height cannot be negative

These restrictions are translated algebraically as

1. $2(w + h) = L$
2. $w \geq 0, h \geq 0$

The only remaining component now is the objective of the problem; namely, maximization of the area of the rectangle. Let z be the area of the rectangle, then the complete model becomes

$$\text{Maximize } z = wh$$

subject to

$$2(w + h) = L$$
$$w, h \geq 0$$

The optimal solution of this model is $w = h = \frac{L}{4}$, which calls for constructing a square shape.

Based on the preceding two examples, the general OR model can be organized in the following general format:

> Maximize or minimize **Objective Function**
>
> subject to
>
> **Constraints**

A solution of the mode is **feasible** if it satisfies all the constraints. It is **optimal** if, in addition to being feasible, it yields the best (maximum or minimum) value of the objective function. In the tickets example, the problem presents three feasible alternatives, with the third alternative yielding the optimal solution. In the rectangle problem, a feasible alternative must satisfy the condition $w + h = \frac{L}{2}$ with w and h assuming nonnegative values. This leads to an infinite number of feasible solutions and, unlike the tickets problem, the optimum solution is determined by an appropriate mathematical tool (in this case, differential calculus).

Though OR models are designed to "optimize" a specific objective criterion subject to a set of constraints, the quality of the resulting solution depends on the completeness of the model in representing the real system. Take, for example, the tickets model. If one is not able to identify all the dominant alternatives for purchasing the tickets, then the resulting solution is optimum only relative to the choices represented in the model. To be specific, if alternative 3 is left out of the model, then the resulting "optimum" solution would call for purchasing the tickets for $1880, which is a **suboptimal** solution. The conclusion is that "the" optimum solution of a model is best only for *that* model. If the model happens to represent the real system reasonably well, then its solution is optimum also for the real situation.

PROBLEM SET 1.1A

1. In the tickets example, identify a fourth feasible alternative.
2. In the rectangle problem, identify two feasible solutions and determine which one is better.
3. Determine the optimal solution of the rectangle problem. (*Hint*: Use the constraint to express the objective function in terms of one variable, then use differential calculus.)

4. Amy, Jim, John, and Kelly are standing on the east bank of a river and wish to cross to the west side using a canoe. The canoe can hold at most two people at a time. Amy, being the most athletic, can row across the river in 1 minute. Jim, John, and Kelly would take 2, 5, and 10 minutes, respectively. If two people are in the canoe, the slower person dictates the crossing time. The objective is for all four people to be on the other side of the river in the shortest time possible.
   - **(a)** Identify at least two feasible plans for crossing the river (remember, the canoe is the only mode of transportation and it cannot be shuttled empty).
   - **(b)** Define the criterion for evaluating the alternatives.
   - ***(c)**[1] What is the smallest time for moving all four people to the other side of the river?

***5.** In a baseball game, Jim is the pitcher and Joe is the batter. Suppose that Jim can throw either a fast or a curve ball at random. If Joe correctly predicts a curve ball, he can maintain a .500 batting average, else if Jim throws a curve ball and Joe prepares for a fast ball, his batting average is kept down to .200. On the other hand, if Joe correctly predicts a fast ball, he gets a .300 batting average, else his batting average is only .100.
   - **(a)** Define the alternatives for this situation.
   - **(b)** Define the objective function for the problem and discuss how it differs from the familiar optimization (maximization or minimization) of a criterion.

6. During the construction of a house, six joists of 24 feet each must be trimmed to the correct length of 23 feet. The operations for cutting a joist involve the following sequence:

Operation	Time (seconds)
1. Place joist on saw horses	15
2. Measure correct length (23 feet)	5
3. Mark cutting line for circular saw	5
4. Trim joist to correct length	20
5. Stack trimmed joist in a designated area	20

Three persons are involved: Two loaders must work simultaneously on operations 1, 2, and 5, and one cutter handles operations 3 and 4. There are two pairs of saw horses on which untrimmed joists are placed in preparation for cutting, and each pair can hold up to three side-by-side joists. Suggest a good schedule for trimming the six joists.

1.2 SOLVING THE OR MODEL

In OR, we do not have a single general technique to solve all mathematical models that can arise in practice. Instead, the type and complexity of the mathematical model dictate the nature of the solution method. For example, in Section 1.1 the solution of the tickets problem requires simple ranking of alternatives based on the total purchasing price, whereas the solution of the rectangle problem utilizes differential calculus to determine the maximum area.

The most prominent OR technique is **linear programming**. It is designed for models with linear objective and constraint functions. Other techniques include **integer programming** (in which the variables assume integer values), **dynamic programming**

[1]An asterisk (*) designates problems whose solution is provided in Appendix C.

(in which the original model can be decomposed into more manageable subproblems), **network programming** (in which the problem can be modeled as a network), and **nonlinear programming** (in which functions of the model are nonlinear). These are only a few among many available OR tools.

A peculiarity of most OR techniques is that solutions are not generally obtained in (formulalike) closed forms. Instead, they are determined by **algorithms.** An algorithm provides fixed computational rules that are applied repetitively to the problem, with each repetition (called **iteration**) moving the solution closer to the optimum. Because the computations associated with each iteration are typically tedious and voluminous, it is imperative that these algorithms be executed on the computer.

Some mathematical models may be so complex that it is impossible to solve them by any of the available optimization algorithms. In such cases, it may be necessary to abandon the search for the *optimal* solution and simply seek a *good* solution using **heuristics** or *rules of thumb.*

1.3 QUEUING AND SIMULATION MODELS

Queuing and simulation deal with the study of waiting lines. They are not optimization techniques; rather, they determine measures of performance of the waiting lines, such as average waiting time in queue, average waiting time for service, and utilization of service facilities.

Queuing models utilize probability and stochastic models to analyze waiting lines, and simulation estimates the measures of performance by imitating the behavior of the real system. In a way, simulation may be regarded as the next best thing to observing a real system. The main difference between queuing and simulation is that queuing models are purely mathematical, and hence are subject to specific assumptions that limit their scope of application. Simulation, on the other hand, is flexible and can be used to analyze practically any queuing situation.

The use of simulation is not without drawbacks. The process of developing simulation models is costly in both time and resources. Moreover, the execution of simulation models, even on the fastest computer, is usually slow.

1.4 ART OF MODELING

The illustrative models developed in Section 1.1 are true representations of real situations. This is a rare occurrence in OR, as the majority of applications usually involve (varying degrees of) approximations. Figure 1.1 depicts the levels of abstraction that characterize the development of an OR model. We abstract the assumed real world from the real situation by concentrating on the dominant variables that control the behavior of the real system. The model expresses in an amenable manner the mathematical functions that represent the behavior of the assumed real world.

To illustrate levels of abstraction in modeling, consider the Tyko Manufacturing Company, where a variety of plastic containers are produced. When a production order is issued to the production department, necessary raw materials are acquired from the company's stocks or purchased from outside sources. Once the production batch is completed, the sales department takes charge of distributing the product to customers.

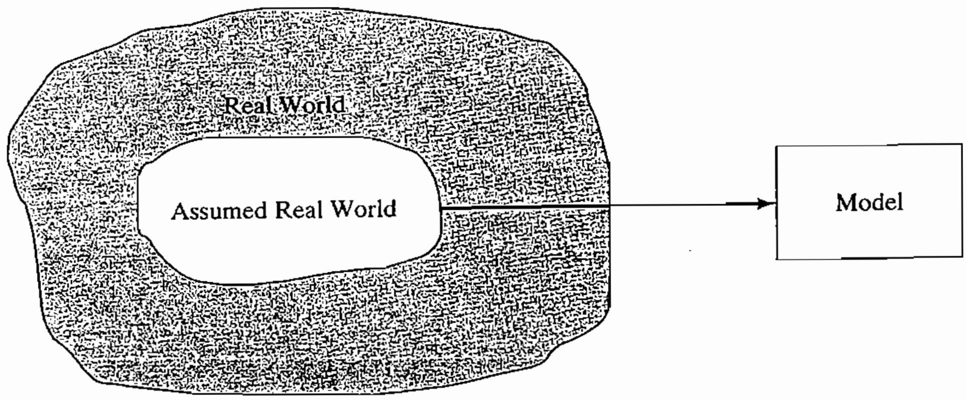

FIGURE 1.1
Levels of abstraction in model development

A logical question in the analysis of Tyko's situation is the determination of the size of a production batch. How can this situation be represented by a model?

Looking at the overall system, a number of variables can bear directly on the level of production, including the following (partial) list categorized by departments.

1. *Production Department:* Production capacity expressed in terms of available machine and labor hours, in-process inventory, and quality control standards.
2. *Materials Department:* Available stock of raw materials, delivery schedules from outside sources, and storage limitations.
3. *Sales Department:* Sales forecast, capacity of distribution facilities, effectiveness of the advertising campaign, and effect of competition.

Each of these variables affects the level of production at Tyko. Trying to establish explicit functional relationships between them and the level of production is a difficult task indeed.

A first level of abstraction requires defining the boundaries of the assumed real world. With some reflection, we can approximate the real system by two dominant variables:

1. Production rate.
2. Consumption rate.

Determination of the production rate involves such variables as production capacity, quality control standards, and availability of raw materials. The consumption rate is determined from the variables associated with the sales department. In essence, simplification from the real world to the assumed real world is achieved by "lumping" several real-world variables into a single assumed-real-world variable.

It is easier now to abstract a model from the assumed real world. From the production and consumption rates, measures of excess or shortage inventory can be established. The abstracted model may then be constructed to balance the conflicting costs of excess and shortage inventory—i.e., to minimize the total cost of inventory.

1.5 MORE THAN JUST MATHEMATICS

Because of the mathematical nature of OR models, one tends to think that an OR study is *always* rooted in mathematical analysis. Though mathematical modeling is a cornerstone of OR, simpler approaches should be explored first. In some cases, a "common sense" solution may be reached through simple observations. Indeed, since the human element invariably affects most decision problems, a study of the psychology of people may be key to solving the problem. Three illustrations are presented here to support this argument.

1. Responding to complaints of slow elevator service in a large office building, the OR team initially perceived the situation as a waiting-line problem that might require the use of mathematical queuing analysis or simulation. After studying the behavior of the people voicing the complaint, the psychologist on the team suggested installing full-length mirrors at the entrance to the elevators. Miraculously the complaints disappeared, as people were kept occupied watching themselves and others while waiting for the elevator.

2. In a study of the check-in facilities at a large British airport, a United States-Canadian consulting team used queuing theory to investigate and analyze the situation. Part of the solution recommended the use of well-placed signs to urge passengers who were within 20 minutes from departure time to advance to the head of the queue and request immediate service. The solution was not successful, because the passengers, being mostly British, were "conditioned to very strict queuing behavior" and hence were reluctant to move ahead of others waiting in the queue.

3. In a steel mill, ingots were first produced from iron ore and then used in the manufacture of steel bars and beams. The manager noticed a long delay between the ingots production and their transfer to the next manufacturing phase (where end products were manufactured). Ideally, to reduce the reheating cost, manufacturing should start soon after the ingots left the furnaces. Initially the problem was perceived as a line-balancing situation, which could be resolved either by reducing the output of ingots or by increasing the capacity of the manufacturing process. The OR team used simple charts to summarize the output of the furnaces during the three shifts of the day. They discovered that, even though the third shift started at 11:00 P.M., most of the ingots were produced between 2:00 and 7:00 A.M. Further investigation revealed that third-shift operators preferred to get long periods of rest at the start of the shift and then make up for lost production during morning hours. The problem was solved by "leveling out" the production of ingots throughout the shift.

Three conclusions can be drawn from these illustrations:

1. Before embarking on sophisticated mathematical modeling, the OR team should explore the possibility of using "aggressive" ideas to resolve the situation. The solution of the elevator problem by installing mirrors is rooted in human psychology rather than in mathematical modeling. It is also simpler and less costly than any recommendation a mathematical model might have produced. Perhaps this is the reason OR teams usually include the expertise of "outsiders" from nonmathematical fields (psychology in the case of the elevator problem). This point was recognized and implemented by the first OR team in Britain during World War II.

2. Solutions are rooted in people and not in technology. Any solution that does not take human behavior into account is apt to fail. Even though the mathematical solution of the British airport problem may have been sound, the fact that the consulting team was not aware of the cultural differences between the United States and Britain (Americans and Canadians tend to be less formal) resulted in an unimplementable recommendation.

3. An OR study should never start with a bias toward using a specific mathematical tool before its use can be justified. For example, because linear programming is a successful technique, there is a tendency to use it as the tool of choice for modeling "any" situation. Such an approach usually leads to a mathematical model that is far removed from the real situation. It is thus imperative that we first analyze available data, using the simplest techniques where possible (e.g., averages, charts, and histograms), with the objective of pinpointing the source of the problem. Once the problem is defined, a decision can be made regarding the most appropriate tool for the solution.[2] In the steel mill problem, simple charting of the ingots production was all that was needed to clarify the situation.

1.6 PHASES OF AN OR STUDY

An OR study is rooted in *teamwork*, where the OR analysts and the client work side by side. The OR analysts' expertise in modeling must be complemented by the experience and cooperation of the client for whom the study is being carried out.

As a decision-making tool, OR is both a science and an art. It is a science by virtue of the mathematical techniques it embodies, and it is an art because the success of the phases leading to the solution of the mathematical model depends largely on the creativity and experience of the operations research team. Willemain (1994) advises that "effective [OR] practice requires more than analytical competence: It also requires, among other attributes, technical judgement (e.g., when and how to use a given technique) and skills in communication and organizational survival."

It is difficult to prescribe specific courses of action (similar to those dictated by the precise theory of mathematical models) for these intangible factors. We can, however, offer general guidelines for the implementation of OR in practice.

The principal phases for implementing OR in practice include

1. Definition of the problem.
2. Construction of the model.

[2]Deciding on a specific mathematical model before justifying its use is like "putting the cart before the horse," and it reminds me of the story of a frequent air traveler who was paranoid about the possibility of a terrorist bomb on board the plane. He calculated the probability that such an event could occur, and though quite small, it wasn't small enough to calm his anxieties. From then on, he always carried a bomb in his briefcase on the plane because, according to his calculations, the probability of having *two* bombs aboard the plane was practically zero!

3. Solution of the model.
4. Validation of the model.
5. Implementation of the solution.

Phase 3, dealing with *model solution*, is the best defined and generally the easiest to implement in an OR study, because it deals mostly with precise mathematical models. Implementation of the remaining phases is more an art than a theory.

Problem definition involves defining the scope of the problem under investigation. This function should be carried out by the entire OR team. The aim is to identify three principal elements of the decision problem: (1) description of the decision alternatives, (2) determination of the objective of the study, and (3) specification of the limitations under which the modeled system operates.

Model construction entails an attempt to translate the problem definition into mathematical relationships. If the resulting model fits one of the standard mathematical models, such as linear programming, we can usually reach a solution by using available algorithms. Alternatively, if the mathematical relationships are too complex to allow the determination of an analytic solution, the OR team may opt to simplify the model and use a heuristic approach, or they may consider the use of simulation, if appropriate. In some cases, mathematical, simulation, and heuristic models may be combined to solve the decision problem, as the case analyses in Chapter 24 demonstrate.

Model solution is by far the simplest of all OR phases because it entails the use of well-defined optimization algorithms. An important aspect of the model solution phase is *sensitivity analysis.* It deals with obtaining additional information about the behavior of the optimum solution when the model undergoes some parameter changes. Sensitivity analysis is particularly needed when the parameters of the model cannot be estimated accurately. In these cases, it is important to study the behavior of the optimum solution in the neighborhood of the estimated parameters.

Model validity checks whether or not the proposed model does what it purports to do—that is, does it predict adequately the behavior of the system under study? Initially, the OR team should be convinced that the model's output does not include "surprises." In other words, does the solution make sense? Are the results intuitively acceptable? On the formal side, a common method for checking the validity of a model is to compare its output with historical output data. The model is valid if, under similar input conditions, it reasonably duplicates past performance. Generally, however, there is no assurance that future performance will continue to duplicate past behavior. Also, because the model is usually based on careful examination of past data, the proposed comparison is usually favorable. If the proposed model represents a new (nonexisting) system, no historical data would be available. In such cases, we may use simulation as an independent tool for verifying the output of the mathematical model.

Implementation of the solution of a validated model involves the translation of the results into understandable operating instructions to be issued to the people who will administer the recommended system. The burden of this task lies primarily with the OR team.

1.7 ABOUT THIS BOOK

Morris (1967) states that "the teaching of models is not equivalent to the teaching of modeling." I have taken note of this important statement during the preparation of the eighth edition, making an effort to introduce the art of modeling in OR by including realistic models throughout the book. Because of the importance of computations in OR, the book presents extensive tools for carrying out this task, ranging from the tutorial aid TORA to the commercial packages Excel, Excel Solver, and AMPL.

A first course in OR should give the student a good foundation in the mathematics of OR as well as an appreciation of its potential applications. This will provide OR users with the kind of confidence that normally would be missing if training were concentrated only on the philosophical and artistic aspects of OR. Once the mathematical foundation has been established, you can increase your capabilities in the artistic side of OR modeling by studying published practical cases. To assist you in this regard, Chapter 24 includes 15 fully developed and analyzed cases that cover most of the OR models presented in this book. There are also some 50 cases that are based on real-life applications in Appendix E on the CD. Additional case studies are available in journals and publications. In particular, *Interfaces* (published by INFORMS) is a rich source of diverse OR applications.

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